

# Moral Hazard and Dynamics of Insider Ownership Stakes<sup>†</sup>

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**ABSTRACT.** In this paper, I analyze the ownership dynamics of  $N$  strategic risk-averse corporate insiders facing a moral hazard problem. A solution for the equilibrium share price and the dynamics of the aggregate insider stake is obtained in two cases: when agents can credibly commit to an optimal ownership policy and when they cannot commit (time-consistent case). In the latter case, the aggregate stake gradually adjusts towards the competitive allocation. The speed of adjustment increases with  $N$  when outside investors are risk-averse, and does not depend on it when investors are risk-neutral. Predictions of the model are consistent with recent empirical findings.

**Classification Codes:** G14, G32, D43.

**Key Words:** Corporate Insiders, Moral Hazard, Ownership Dynamics

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## Introduction

The optimal structure and concentration of corporate ownership is a hotly debated issue of corporate finance and political economics even today, seventy years after the publication of the influential Berle and Means (1932) book. The last two decades witnessed a significant progress in the modeling of the behavior of “large shareholders”, i.e., shareholders with a stake large enough to have the ability and motivation to monitor and improve company performance.<sup>1</sup> Until recently, however, theoretical literature typically downplayed two important aspects of strategic agents’ behavior: the agents’ motivation to trade, as well as the strategic interactions between multiple large shareholders. Empirically, Mikkelsen et al. (1997) (for the U.S.) and Franks et al. (2002) (for the U.K) document a significant and steady decrease in the aggregate insider ownership stake both before and after the Initial Public Offering (IPO). Urošević (2002) confirms (for the U.S.) that the average aggregate insider ownership stake gradually declines after the IPO. In addition, he finds that the majority of companies have several insiders with large stakes, and that the speed of the aggregate insider stake adjustment depends upon the composition of the insider stake: the decline tends to be higher for companies that have multiple insiders with large stakes than for companies that have only one such insider, even though this effect is not always significant.

The aim of this paper is to model the dynamics of corporate ownership for companies with multiple large shareholders. In doing so, I build upon the dynamic moral hazard model of DeMarzo and Urošević (2001) which explicitly demonstrates gradual adjustment towards the competitive equilibrium in the case of one strategic agent.<sup>2</sup> Extending their model to incorporate the strategic behavior of multiple large shareholders allows me to explore how the make up of the aggregate insider stake influences the nature and the speed of its adjustment. Importantly, my model leads to predictions broadly consistent with the available empirical evidence.

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<sup>1</sup> See Shleifer and Vishny (1997) for a review of the corporate governance literature, and La Porta et al (1999) for a survey of corporate ownership concentration around the world.

<sup>2</sup> Most of the related literature incorporates only one strategic agent. Basak (1996), for example, considers a commitment strategy for an agent who recognizes that his portfolio choices influence state prices including, e.g., the risk-free rate in the economy, but who cannot influence the dividends process. The paper notices but does not address the lack of time consistency of such solution. Kihlstrom (1998) considers a three-period model similar to Basak’s but solves, in addition, for the time-consistent strategy of the large shareholder and shows that the problem exhibits Coasian dynamics (see Coase (1972); DeMarzo and Bizer (1993) establish the connection between the durable goods monopolist problem and securities markets). Stoughton and Zechner (1998) solve a one-agent two-period model in the special case of linear moral hazard (see Section 2). Those models build, in turn, on the influential one-period models of Admati et al (1994) and Lindenberg (1979). Other important one-period models include Bolton and von Thadden (1998), Burkart et al (1997) and (1998), Demsetz and Lehn (1985), Demsetz (1986), Grinblatt and Ross (1985), Kahn and Winton (1998), Leland and Pyle (1977), and Maug (1998), among the others. There are also papers in which price process is exogenous (i.e. markets do not necessarily clear) such as Cuoco and Cvitanic (1997), El Karoui et al (1997), and Jarrow (1992), among others.

In an economy consisting of one risky firm and a risk-less bond, there are  $N$  equally risk-averse agents that I refer to as insiders,<sup>3</sup> and a continuum of small outside investors, all with CARA preferences. There are two competing forces driving the results of the model. On one hand, insiders are facing a moral hazard problem so that the expected firm cash flows and, therefore, the company value increases with insider holdings, *ceteris paribus*. On the other hand, high company stakes imply a significant risk exposure to insiders who, as a result, have the incentive to decrease their stakes over time. Importantly, the ownership policy of each insider influences the share price, and, therefore, every other agent's future ownership decisions. The equilibrium in the economy simultaneously specifies the optimal ownership policies of each insider, as well as the corresponding share price process.

I consider the case in which each insider can commit to an optimal ownership policy, and the case in which such commitment is impossible. Note that the commitment policy is time-inconsistent since after the initial sale, as long as their marginal valuation is below that of the outside investors, each insider will be tempted to trade again since she will no longer internalize the capital loss on the shares just sold. Put in another way, insiders' risk-aversion creates a wedge between their valuation of company shares and the value placed on the company shares by the outside investors (the market). The optimal time-consistent policy, therefore, is for insiders to gradually adjust their stakes in the company until the perfect risk sharing allocation is achieved. The intuition for that result is quite similar to the one agent model (see DeMarzo and Urošević (2001)).

When investors are risk-averse there is an additional strategic reason for a dynamic stake adjustment in this model vis-à-vis the one-agent case. Namely, a decrease in the aggregate insider stake raises the market risk premium and thus lowers the company valuation. Therefore, by selling more today, each insider hopes to decrease the incentive for others to sell in the future (since they will receive a lower share price). This creates among the insiders a "race to diversify". As a result, in the unique subgame-perfect equilibrium, the speed of adjustment toward the perfect risk sharing allocation increases with the number of company insiders  $N$ . Intuitively, as  $N$  increases, the asset price more quickly becomes competitive, though the adjustment towards the long-run equilibrium is gradual.

When investors are risk-neutral, the speed of adjustment of the aggregate insider stake loses its dependence on  $N$ . In that case, the multi-agent setting formally coincides with the one-agent results of DeMarzo and Urošević (2001). As in the one-agent case, the speed of adjustment of the aggregate stake increases with cash flow volatility and decreases in marginal monitoring incentives, *ceteris paribus*.

Independently of this work, Pritsker (2002) develops a related model. He also constructs a model of multiple "large traders" in a CARA/normal setting and borrows, like this paper, some modeling techniques from DeMarzo and Urošević (2001). In many important ways, however, our models are quite different. The key difference is in the economic environment that the two papers portray. This paper aims to explain the evolution of the aggregate corporate insider stake in a given company. In contrast,

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<sup>3</sup> Here, and in the rest of the paper, I use the word "insider" to denote anyone who files SEC insider forms and not necessarily someone who "trades on information" as in the Kyle (1985) model, for example.

Pritsker (2002) models the trading behavior of large institutional traders. Consequently, while both of our papers incorporate diversification as a motivation to trade, Pritsker (2002) studies market liquidity, shock transmission and market manipulation in an economy with no moral hazard, whereas I focus on the moral hazard problem, instead.<sup>4</sup> Moral hazard is likely to play a significant role in the ownership decisions of corporate insiders.<sup>5</sup> DeMarzo and Urošević (2001) show that the moral hazard provides a natural explanation for the gradual adjustment of a large shareholder stake. This paper confirms that intuition in a more realistic setting that incorporates multiple strategic agents. In addition, this paper offers a simple explanation of the fact that the speed of adjustment tends to (weakly) increase with the number of large shareholders in a company. Another difference between Pritsker (2002) and this paper is that the former considers an economy with multiple risky assets whereas I consider only one risky asset. Such choices are sensible, given the environments that the papers aim to describe: large institutional traders can impact the prices of multiple assets; on the other hand, the majority of executives, at any given time, are insiders in only one company.

This paper is organized as follows. In Section 1, I describe the model. In Section 2, I perform some preliminary analysis and find the benchmark commitment and price-taking allocations. In Section 3, I formulate the time-consistent equilibrium problem and obtain the solution in terms of a coupled set of recursive relations. In Section 4, I obtain the numerical solution to these equations and find important comparative statics in the case when investors are risk-averse. In addition, I solve the problem exactly when the outside investors are risk-neutral. In Section 5, I discuss the results. Section 6 contains my conclusions and suggestions for future research. Section 7 contains the list of references. Proofs are relegated to the Appendix.

## 1. The Model

I consider a going-concern publicly traded firm with a supply of shares that is normalized to one, and with a cumulative free cash flow process described by the following diffusion process

$$dD = \hat{\mu}dt + \hat{\sigma}dZ,$$

where  $Z$  is standard Brownian motion. Consequently, the cash flows in each period are normally distributed and independent across periods. Normality is important for the tractability of the model and while unlimited liability is not desirable, it does not play an important role in the forces driving my results. Inter-temporal independence of cash flows means that there is no learning from the past. Shares of the firm trade in the market at the price  $V$  that needs to be determined in equilibrium. In addition to this firm there

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<sup>4</sup> Another interesting recent addition to the literature on the trading behavior of multiple strategic traders is Brunnermeier and Pedersen (2002). They focus on the issue of predatory trading and market liquidity in a continuous time setting. Thus, their paper is closer in spirit to Pritsker (2002) than to my paper.

<sup>5</sup> Brav and Gompers (2001) provide an empirical evidence on the link between the moral hazard problem and insider ownership dynamics by studying the role of lock-up mechanisms in IPOs. They show that the primary reason for introducing a lockup, i.e. a mechanism that prevents insider trading in a company stock for a period of time after the IPO, is to alleviate the moral hazard problem.

exists a risk-less investment that pays a continuously compounded return of  $r$ , with a perfectly elastic supply.

The firm pays out all of its cash flows as dividends. This assumption is made for convenience only since if the firm can re-invest any retained cash flows only in marketed securities, i.e. in the risk-free asset and the company stock, the optimal ownership policy does not depend on the payout ratio. The expression for the share price, on the other hand, does depend on the dividend policy. In particular, full payout implies that the share price is a deterministic function of time. On the other hand, for less than a full payout, the expression for the share price would become stochastic. (For more details, see the discussion in DeMarzo and Urošević (2001)).

There are  $N > 1$  agents in the model with the ability to monitor the firm and affect decisions within the firm. I refer to them as insiders. This extends the single “large shareholder” model of DeMarzo and Urošević (2001) by incorporating strategic interactions between corporate insiders. For simplicity I assume that the insiders may have different initial company stakes but are otherwise identical. In addition to these insiders, there exists a continuum of competitive outside investors, with measure  $F$ .<sup>6</sup> All individuals in the economy have standard CARA utility so that on date  $t$ , they optimize

$$E_t \int_t^\infty e^{-r(\tau-t)} u^i(c_\tau) d\tau,$$

where  $u^i(c) = -e^{-\gamma^i c}$  and  $\gamma^i$  is the individual’s coefficient of absolute risk aversion. DeMarzo and Urošević (2001) show that restricting the rate of time preference to the risk-free rate  $r$  is without loss of generality. Without risking confusion, hopefully, I denote by  $\gamma$  the risk aversion of the insiders, and by  $\gamma^i$  the risk aversion of the outside investor  $i$ .

All trades occur in a competitive market.<sup>7</sup> That means that the insiders in the economy trade with competitive outside investors. Let  $\alpha^l(t) \in [0,1]$  ( $l=1:N$ ), be the fraction of the firm held by the insider  $l$  at time  $t$  and let the vector of insiders’ holdings be  $\equiv (\alpha^1, \dots, \alpha^N)$ . I restrict each component  $\alpha^l(t)$  to be right-continuous, and interpret  $\alpha^l(t-) \equiv \lim_{\tau \uparrow t} \alpha^l(\tau)$  as the shares held at the “start” of period  $t$  by insider  $l$ ; thus,  $\alpha^l(t) - \alpha^l(t-)$  is the discrete number of shares purchased by the insider  $l$  in period  $t$  or, since the two are interchangeable in this model, the change in the insider’s  $l$  ownership stake at time  $t$ . The insider  $l$  has an initial endowment  $\alpha^l(0-) = \alpha^{l-}$ . By market clearing, in equilibrium the investors’ holdings at time  $t$  are given by the expression  $1 - A(t)$ , where  $A(t) \equiv \sum_l \alpha^l(t)$  is the aggregate insiders’ stake at time  $t$ . The initial aggregate stake is

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<sup>6</sup> The insiders’ trading opportunities depend, in principal, on the liquidity of the market that is, in turn, endogenous to their trading behavior due to the microstructure effects (adverse selection, inventory cost etc). In order to simplify the analysis I ignore these effects. In addition, I assume that infinitesimal shareholders cannot form coalitions or in any other way behave strategically (see Zwiebel (1995) on a sufficient condition for preventing small investors from forming coalitions).

<sup>7</sup> In contrast, Vayanos (1999) considers a model with  $N$  strategic traders in the absence of competitive investors.

given, therefore, by the following expression:  $A^- \equiv \sum_l \alpha^{l-}$ . (An important note:

Whenever confusion would not occur, I drop the superscript  $l$  writing, instead, an insider stake simply as  $\alpha$  and the corresponding initial stake as  $\alpha^-$ ). I assume that the initial stakes are exogenous to the model and above the perfect risk sharing allocation level. Endogenous treatment of the initial stakes can be found, for example, in Stoughton and Zechner (1998) and DeMarzo and Urošević (2001) in a one-agent moral hazard setting.

Insiders are facing a moral hazard problem: insider  $l$ 's costly monitoring effort  $e_l, l=1, \dots, N$  affects the expected free cash flow in a linear fashion, namely,

$\hat{\mu}(e_1, \dots, e_N) = \mu_0 + \sum_{l=1}^N e_l$ . One way to interpret this expression is to think of each insider as

working on a separate task within the firm without interacting with other insiders. Such interactions would be represented, for example, by the terms of the type  $e_i e_j, i \neq j$  in the expression for  $\hat{\mu}(e_1, \dots, e_N)$ . Quantity  $\mu_0$  represents the expected cash flow that the firm receives each period based purely on the existing capital in place (and not on the insiders' effort). Since it impacts only, in a straightforward fashion, the expression for the share price but does not have an impact on the determination of the optimal ownership policies, I normalize it to zero. I assume further that the variance of the firm's cash flows cannot be altered by the actions or holdings of the insiders. All of the parameters in the model are constant. Note that while insiders' do not affect the efforts of other insiders directly, they do so indirectly through by recognizing their own, and other insiders' impact on the process of share price formation (see below).

Each insider's problem is symmetric so, without loss of generality, I can pick an insider and denote her stake as  $\alpha$  and her effort choice as  $e$ . The insider's cost of effort is quadratic in  $e$ , i.e.  $f(e) = e^2 / (2\mu_1)$ , where parameter  $\mu_1$  is identical for each insider. Thus, the cost of an effort is independent of other agents' effort choices. Since the effort in this economy cannot be contracted on, each insider's effort choice must be incentive compatible. I refer to such a model as a linear hazard model.<sup>8</sup> In addition, because of the CARA/normal setting, each insider's problem can be expressed in terms of her certainty equivalent. Thus, in each instant, each insider chooses her effort in order to maximize the instantaneous certainty equivalent of her payoff,<sup>9</sup>

$$z \equiv \max_e \alpha \hat{\mu}(e_1, \dots, e_N) - f(e) - \frac{1}{2} \alpha^2 \gamma r \sigma^2 \quad (1)$$

The expression (1) is quite intuitive. The instantaneous certainty equivalent is equal to the total expected dividends received by the insider, net of her cost of effort and adjusted for the insider's risk aversion. Here,  $\alpha^2 \sigma^2$  captures the variance of the dividends received,  $\gamma$  is the insider's risk aversion, and the scaling by the interest rate  $r$  appears since the insider can smooth shocks over time. Given  $\hat{\mu}$  and  $f$ , this is solved by  $e = \alpha \mu_1$  for each

<sup>8</sup> With only one agent in the economy, such a model gained prominence in Holmstrom and Milgrom (1987) and has been used extensively in the literature (see Admati et al (1994), Stoughton and Zechner (1998), and DeMarzo and Urošević (2000), among many others).

<sup>9</sup> Rigorous derivation in DeMarzo and Urošević (2000) goes through without change in the multiple agent case.

insider. Thus, the certainty equivalent payoff “flow” to each insider can be re-written entirely in terms of her and other insiders’ holdings:

$$z(\alpha, \beta) = (\mu_1 - \gamma r \sigma^2) \alpha^2 / 2 + \mu_1 \alpha \beta \quad (2)$$

In the above, I introduce  $\beta \equiv A - \alpha$  as the aggregate holdings of all but that particular insider, and explicitly note that the instantaneous certainty equivalent of an insider is a function of that insider’s holdings, given the aggregate holdings of all other insiders. Here,  $\mu_1 \geq 0$  measures the expected free cash flow sensitivity with respect to the change in corporate ownership, and, thus, parameterizes the importance of the moral hazard problem in this model.

Equation (2) determines the total risk-adjusted payoff to an insider from holding a fraction  $\alpha$  of the firm. It is useful to restate this in terms of the marginal value of ownership to the insider:

$$z_\alpha(\alpha, \beta) = \mu_1 A - \alpha \gamma r \sigma^2 \quad (3)$$

That is, the marginal value of a share to the insider is simply the expected dividend per share,  $\mu_1 A$ , adjusted by the insider’s “risk premium” given holdings  $\alpha$ . In fact it is sufficient to know  $z_\alpha$ , since  $z(\alpha, \beta) = \int_0^\alpha d\hat{\alpha} z_\alpha(\hat{\alpha}, \beta)$ .

An analogous expression can be derived for outside investors. Investors consume and trade shares of the firm and the risk-less security continuously and competitively. Given CARA utility, investors can be aggregated into a single representative investor with an aggregate risk aversion coefficient  $\gamma^I \equiv [\int_i 1/\gamma^i dF(i)]^{-1}$ . The marginal value of a share to this aggregate investor is then given by

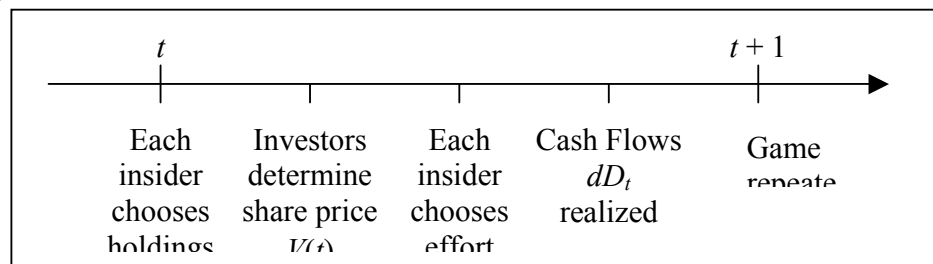
$$v(A) = \mu_1 A - (1 - A) \gamma^I r \sigma^2 \quad (4)$$

Equations (3) and (4) summarize the primitives of the model. Extending the results of DeMarzo and Urošević (2001) (their Section 3.3) to the multi-agent setting, one can prove that the transformation of the problem from the explicit moral hazard formulation expressed in terms of insiders’ efforts (1) into the reduced formulation expressed in terms of insiders’ holdings (4) is without loss of generality whenever both the expected cash flow and volatility can be represented as a sum of  $N$  single-agent functions. The linear moral hazard model is an obvious special case. The moral hazard of the insiders is, therefore, reflected in the dependence of the dividend on their aggregate holdings  $A$ , whereas the motivation for trading is provided by the difference in risk premiums across insiders.

In the model, insiders live infinitely but trade and actively monitor the company only for a finite period of time. In particular, while insiders may consume, make effort choices, and trade in the risk-less security continuously, I assume that they are restricted to trade shares of the firm on a finite set of dates  $\mathcal{T}$  common to all insiders. Technically, a finite number of trading periods is necessary for the uniqueness of the sub-game perfect equilibrium. In practice, companies frequently impose “windows” within which company

insiders can trade say, every quarter (e.g., Urošević (2002) finds that the average interval between two successive trades by a corporate insider is 109 days, roughly corresponding to quarterly trading windows; see, also, Seyhun (1998)). Since trading windows are often narrow this justifies both the assumption of discreteness as well as the assumption of commonality of trading dates. Finally, in order to justify the finiteness of the trading horizon, note that some large shareholders such as venture capitalists have an obligation to divest within a certain period of time (say 10 years). After the partnership dissolves, partners may still be invested in the company, but do not usually monitor their investment closely (see Cumming and MacIntosh (2001) for modes of VC exists).

The timing in the model is as follows:



**Figure 1: Timing in the model**

I assume that diversification is the primary motive for the reduction of insider stakes. A growing body of empirical evidence suggests that insiders are, indeed, increasingly influenced in their trading decisions by diversification and/or liquidity considerations and less by the desire to capitalize on inside information.<sup>10</sup> Muelbroek (2000), in particular, finds that for highly volatile stocks like those in the high technology sector in the U.S., the desire to diversify is one of the key motivations for insider trading.

Another important assumption is that insiders' decisions are sequentially rational and that each agent is playing a multi-period simultaneous-move game with the other insiders in the company. In particular, each of the insiders knows her own, as well as the other insiders', past trades. They also know that their trading decisions today affect the share price and, consequently, the future trading decisions of all insiders in the economy (including her own). Though the outside investors trade competitively as price-takers, they are aware of the strategic interaction among the insiders and the fact that the insiders' current trading decisions have an impact on the insiders' current and future trading decisions. Investors are rational, and make their demands for shares after they observe the insiders' trading decisions for that time period. Note, in particular, that in this model there is no asymmetric information about the dividend process or about the insiders' trading decisions. In other words, all information about the company and insider

<sup>10</sup> It appears that informational motive may be less important in the 1990's (see Carpenter and Remmers (2001), Lakonishok and Lee (2001), Jeng et al. (2000), among others) than in the past (Lorie and Niederhoffer (1968), Jaffe (1974), and Seyhun (1986), among others). Having said that, trading on inside information may still be important (see Seyhun (1998) and references therein).



trades is revealed instantaneously to the investment community.<sup>11</sup> Finally, I do not explicitly restrict investors to trade only on dates  $\mathcal{T}$  since, in equilibrium, investors would only trade when the insiders trade.

## 2. Commitment and Price-Taking Strategies

The setup in this model is similar to the one in DeMarzo and Urošević (2001) with some important differences: a) they consider only one strategic agent while I consider multiple agents and their strategic interactions; b) I restrict the analysis to the linear moral hazard problem where insiders cannot influence volatility and have no benefits of control; and c) the parameters in my model are assumed not to depend on time. As a result, I can adopt most of the results of their preliminary Sections 3.1 to 3.3 with some minor changes. For example, the equilibrium share price in this model is given by

$$V(\bar{\alpha}(t)) = \int_t^\infty e^{-r(\tau-t)} v(A(\tau)) d\tau. \quad (5)$$

The expression in (5) requires some discussion. In one sense, it is straightforward – the equilibrium share price is simply equal to the discounted risk-adjusted dividend flow to investors. Less trivially, future dividend flows depend upon the insiders' anticipated trading strategies (through both the expected dividends and the future risk premium), which must be determined in equilibrium. Indeed, (5) states that the share price at time  $t$  is determined by the aggregate insider holdings at time  $t$  since, in a unique sub-game perfect equilibrium (see Section 3), an insider's holdings today influence each insider's trading strategy and, thus, their holdings in the future. Therefore, the share price is given by the discounted risk-adjusted dividends calculated *at the equilibrium trading strategy*  $\bar{\alpha}(\tau)$ . (A formal proof of (5) can be obtained following the steps leading to **Proposition 5** in DeMarzo and Urošević (2001).) Due to the symmetry of the model, in the sub-game perfect equilibrium (Section 3), the share price  $V$  at time  $t$  is, in fact, a linear function of the aggregate insider holdings  $A(t)$ .

In order to determine insiders' trading strategies, I must formulate their optimization problems. Each insider's (total) certainty equivalent<sup>12</sup> is given by the following intuitive expression:

$$k(\bar{\alpha}(t)) = \int_{[t,\infty]} e^{-r(\tau-t)} [z(\alpha(\tau), \beta(\tau)) d\tau - d\alpha(\tau)V(\bar{\alpha}(\tau))] \quad (6)$$

<sup>11</sup> In contrast, Vayanos (2001) considers a model with one strategic trader, market makers, and noise traders in which the agent's trading decisions are his private information; Vayanos (1999), on the other hand, considers a model with  $N$  strategic players with private endowments and no noise traders or market makers. While these models are very different from each other (and from my model), agents' allocations converge in both models towards the competitive risk sharing allocation.

<sup>12</sup> I adopt set notation for the limits of integration to avoid ambiguity given discontinuities in  $\alpha$ . Thus,  $\int_{(t,T]} d\alpha^l(\tau) = \alpha^l(T) - \alpha^l(t)$  and  $\int_{[t,T)} d\alpha^l(\tau) = \alpha^l(T-) - \alpha^l(t-)$ , where  $\alpha^l(t-) = \lim_{\tau \uparrow t} \alpha^l(\tau)$ . I also define  $\alpha^{l-}$  to be the initial holdings of the agent  $l$ .

Note that due to the symmetry in the model the form of the expression in (6) is the same for each insider. The meaning of the right hand side in (6) is that each insider's certainty equivalent consists of her capitalized risk adjusted aggregate benefits from holding shares (the first term) and her capitalized trading gains from selling shares over time (the second term). These expressions depend on the insider's own future trading strategy as well as on the future strategies of all of the other insiders. These strategies need to be determined in equilibrium simultaneously with the share price process.

In general, each insider would trade at least once. Indeed, since relatively under-diversified insiders are typically more risk averse than the pool of outside investors, each insider would have an incentive to trade away from the initial allocation. Let me denote by  $\mathcal{T} = \{t_1 = 0, t_2, \dots, t_N = T\}$  the finite set of trading dates when each insider can trade. In this case,  $\alpha(t) = \alpha(t_i)$  for all  $t \in [t_i, t_{i+1})$ , where I define  $t_{N+1} = \infty$ . The following proposition describes a commitment Nash equilibrium when each insider's strategy can depend on time only; that means that, in particular, no insider is allowed to condition her trading decisions on the past insider trades.

**PROPOSITION 1.** Suppose that at time  $t$ , each agent announces a trading policy that depends only on time,  $\alpha^l(\tau)$ ,  $\tau \geq t$ , and **cannot be revised** in the future. Then, each agent's strategy is given by the following Nash equilibrium strategy:

$$\alpha^{l,c}(\tau) \equiv \arg \max_{\alpha^l(\tau)} z(\alpha^l(\tau), \beta(\tau)) + (\alpha^l(t^-) - \alpha^l(\tau))v(\alpha(\tau) + \beta(\tau)) \quad (7)$$

where and the aggregate equilibrium holdings are given by the following expression:

$$A^c = \frac{(\mu_1 + \gamma^l r \sigma^2)A^- + N\gamma^l r \sigma^2}{\mu_1 + (N+1)\gamma^l r \sigma^2 + \gamma r \sigma^2} \quad (8)$$

Since the parameters of the model do not depend on time, the commitment equilibrium allocation does not depend on time either. In addition, from (7) one can easily check that the first order conditions for the problem read:

$$\alpha^c = \frac{r\gamma^l \sigma^2}{\mu_1 + r(\gamma + 2\gamma^l)\sigma^2}(1 - \beta) + \frac{\mu_1 + r\gamma^l \sigma^2}{\mu_1 + r(\gamma + 2\gamma^l)\sigma^2} \alpha^- \quad (9)$$

(Here I have omitted, for brevity, superscript  $l$ ). Note that each insider's commitment equilibrium stake increases with her initial holdings. That means that whenever insiders can commit to an optimal allocation, those who initially hold larger stakes would tend to optimally choose a higher level of ownership to which to commit to. In addition, an insider's commitment holdings *decrease* with other insiders' holdings when investors are risk averse. That means that by holding a smaller company stake each insider aims to raise the market risk premium, thus lowering the stock price, and consequently, the motivation of the other insiders to trade down in the future. This effect will play an important role in the time-consistent model (see below). When the market puts no risk premium on the stock, i.e. when  $\gamma^l = 0$  in (9), this second effect disappears.

The expression (8) shows that the aggregate commitment allocation increases with the initial aggregate allocation  $A^-$ . When investors are risk-averse the aggregate

commitment allocation, in addition, increases with the number of insiders  $N$ . In particular, when  $N \rightarrow \infty$ , the aggregate insiders stake would approach unity. On the other hand, when investors are risk-neutral, (8) does not depend on  $N$  and is always below unity.

In the commitment equilibrium, each insider anticipates the impact that her trades will have, as well as those of other insiders, on the stock price. On the other hand, if insiders ignore such impact, i.e. if they take  $v$  and, consequently,  $V$  as given, the following Proposition holds:

**PROPOSITION 2.** Define for each insider  $\alpha^p = \gamma^l / (\gamma + N\gamma^l)$ . Then, if  $\mu_1 < r\gamma\sigma^2$ , a unique Walrasian equilibrium exists in which each insider is a price-taker, the equilibrium trading strategy for each insider is given by  $\alpha^p$ , and the aggregate price-taking equilibrium allocation is

$$A^p = \gamma^l / (\gamma + N\gamma^l) \quad (10)$$

From **Proposition 2** it follows that the price-taking equilibrium exists when the (beneficial) incentive effect is smaller than the insiders' aversion towards risk. The aggregate perfect risk sharing allocation  $A^p$  can be seen as the competitive allocation of one insider with  $N$  times higher risk tolerance. Note, also, that when  $N=1$ , (8) and (10) coincide with the commitment and the competitive allocations in DeMarzo and Urošević (2001) respectively (see their Section 3.4).

### 3. Optimal Trading Strategies Without Commitment

In the previous section, I solved for the optimal trading strategies assuming that all insiders could commit ex-ante to future trades. In that case, the current share price depends on all future trades that the insiders will make. Here, I no longer allow the insiders to commit to future trades. Thus, each insider's trading strategy must be time consistent. The previous results suggest that the commitment policy  $\bar{\alpha}^c$  is not time consistent. To see the intuition for this, note that from (9) it follows that  $\bar{\alpha}^c$  is increasing with the initial insider shareholdings. Therefore, once shares are initially sold, the insiders' decision-making process starts again, this time with smaller shareholdings but still above the competitive allocation. Thus, the insiders have an incentive to sell again. This second sale and the resulting change in effort impose a negative externality on the initial buyers of shares that the insiders do not consider when making a second sale. The same is true for each individual insider.

To solve for the equilibrium without commitment, note that the value of the shares at any time  $t$  must depend on the investors' expectations of the insiders' future trading decisions. Thus, investors must anticipate the insiders' ex-post incentives to trade. In addition, at each point in time, each insider recognizes that her trading decision today impacts not only her future trading decisions, but also those of all of the other insiders.

The problem shall be solved by backward induction. For that reason, consider, first, the insiders' decision-making process at time  $T$  (the last trading date). Recall that the insiders have the opportunity to trade only on the discrete dates  $\mathcal{T} = \{t_1=0, t_2, \dots, t_N=T\}$ .

Implicitly, the insiders commit not to trade during the intervals  $(t_i, t_{i+1})$ . For simplicity, I assume that time intervals  $\Delta$  between trades are constant and introduce the capitalization factor  $\delta_i$  such that  $\delta_i \equiv \delta \equiv (1 - e^{-r\Delta})/r$ , except in the case when  $t_{N+1} = \infty$ , where I set  $\delta_T = 1/r$ . At that time insiders, by assumption, commit not to trade again. At time  $t = T$ , one can, therefore, use the technique developed in Section 2. Denote an insider's holdings at time  $t = T$  as  $\alpha_T$  and by  $\beta_T$  the aggregate holdings of all of the other insiders. From (5) it follows that the share price at time  $T$  can be written as:

$$\begin{aligned} V_T(\alpha_T, \beta_T) &= v_{0T} + v_T A_T = v_{0T} + v_T(\alpha_T + \beta_T), \\ v_{0T} &= -\gamma^l \sigma^2, \quad v_T = \mu_1 / r + \gamma^l \sigma^2 \end{aligned} \quad (11)$$

Note that the share price is an affine function of the aggregate insider holdings. Note, also, that moral hazard is explicit in this expression: an increase in  $\mu_1$ , *ceteris paribus*, causes the terminal share price to rise.

Once one knows the terminal share price, one can determine the optimal share holdings for each insider at time  $t = T$  as a function of her, as well as other insiders', holdings at time  $t = T - 1$ . Namely, each insider chooses her holdings at time  $T$  in such a way as to maximize the certainty equivalent  $J_T$ :

$$J_T \equiv \max_{\alpha_T} z(\alpha_T, \beta_T) \delta_T + V_T(\alpha_T, \beta_T)(\alpha_{T-1} - \alpha_T) \quad (12)$$

From (12), the following first order conditions (FOCs) are obtained:

$$\frac{z_{\alpha_T}}{r} + \frac{\partial V_T}{\partial \alpha_T}(\alpha_{T-1} - \alpha_T) - V_T = 0$$

Using the fact that  $V_T(\alpha_T, \beta_T)$  is an affine function of  $\alpha_T$  and  $\beta_T$  and that  $z$  is a quadratic function in  $\alpha_T$  and linear in  $\beta_T$ , the first order conditions can be re-written as:

$$\begin{aligned} n_{1T} \alpha_T + n_{2T} \beta_T + n_{3T} \alpha_{T-1} + n_{4T} &= 0, \text{ where} \\ n_{1T} &= \mu_1 / r - \gamma \sigma^2 - 2v_T, \quad n_{2T} = \mu_1 / r - v_T, \quad n_{3T} = v_T, \quad n_{4T} = -v_{0T} \end{aligned} \quad (13)$$

Summing up the equations (13) and solving for the aggregate allocation, one can show that the aggregate insider stake at time  $T$  is an affine function of the aggregate insider holdings in the previous period:

$$\begin{aligned} A_T &= -\frac{n_{3T} A_{T-1}}{n_{5T}} - \frac{N n_{4T}}{n_{5T}} \\ n_{5T} &\equiv n_{1T} + (N - 1) n_{2T} \end{aligned} \quad (14)$$

Substituting (14) back into (13) leads to the following specification of  $\alpha_T$  and  $\beta_T$  in terms of the insiders' holdings at time  $T - 1$ :

$$\alpha_T = l_T^{\alpha, \alpha} \alpha_{T-1} + l_T^{\alpha, \beta} \beta_{T-1} + l_T^{\alpha, 0}, \quad \beta_T = l_T^{\beta, \alpha} \alpha_{T-1} + l_T^{\beta, \beta} \beta_{T-1} + l_T^{\beta, 0} \quad (15)$$

where coefficients  $l$  are defined as:

$$l_T^{\alpha,\alpha} = \frac{n_{3T}[n_{2T}(2-N) - n_{1T}]}{(n_{1T} - n_{2T})n_{5T}}, \quad l_T^{\alpha,\beta} = \frac{n_{2T}n_{3T}}{(n_{1T} - n_{2T})n_{5T}}, \quad l_T^{\alpha,0} = -\frac{n_{4T}}{n_{5T}}$$

$$l_T^{\beta,\alpha} = -\frac{n_{3T}}{n_{5T}} - l_T^{\alpha,\alpha}, \quad l_T^{\beta,\beta} = -\frac{n_{3T}}{n_{5T}} - l_T^{\alpha,\beta}, \quad l_T^{\beta,0} = -\frac{n_{4T}}{n_{5T}}N - l_T^{\alpha,0}$$

Therefore, the optimal holdings for each insider at time  $T$  depend on her own past holdings as well as on those of the other insiders. Importantly, this dependence is affine. In addition, each insider's optimal holdings are positively correlated with her own holdings at the preceding time period, i.e.  $l_T^{\alpha,\alpha} > 0$ , and negatively correlated with all of the other insiders' holdings at the preceding time period, i.e.  $l_T^{\alpha,\beta} < 0$ .

In order to proceed to  $t=T-1$ , note the two very important properties of the model. First, the share price at time  $t=T-1$  is, again, an affine function of the aggregate holdings. Indeed, from (5) it follows that  $V_{T-1}(\vec{\alpha}_{T-1}) = \delta v(A_{T-1}) + e^{-r\Delta}V_T(\vec{\alpha}_T(\vec{\alpha}_{T-1}))$ . Using the definition of  $v$ , and utilizing (11) and (14), one can again express  $V$  as a function of the aggregate holdings:

$$V_{T-1}(\alpha_{T-1}, \beta_{T-1}) = v_{0T-1} + v_{T-1}A_{T-1} = v_{0T-1} + v_{T-1}(\alpha_{T-1} + \beta_{T-1})$$

$$v_{0T-1} = -\delta\gamma^l\sigma^2r + e^{-r\Delta}[v_{0T} - Nv_Tn_{4T}/n_{5T}], \quad (16)$$

$$v_{T-1} = \delta(\mu_l + \gamma^l\sigma^2r) - e^{-r\Delta}(n_{3T}v_T/n_{5T})$$

This property generalizes for an arbitrary  $t \leq T$  and allows one to obtain a relatively simple recursive solution for the dynamic programming problem (see **Proposition 3**). Such specification of the equilibrium share price can be traced back to the choice of the moral hazard model (i.e. the linear symmetric moral hazard problem). The second important property of the model, which also generalizes for an arbitrary  $t \leq T$  (see **Proposition 3**), is that the value function  $J_T$ , obtained upon the substitution of the expressions for optimal holdings (15) into the objective function (12), is a quadratic form in  $(\alpha_{T-1}, \beta_{T-1})$ :

$$J_T = J_T^{\alpha,\alpha}\alpha_{T-1}\alpha_{T-1} + J_T^{\alpha,\beta}\alpha_{T-1}\beta_{T-1} + J_T^{\beta,\beta}\beta_{T-1}\beta_{T-1} + J_T^{\alpha,0}\alpha_{T-1} + J_T^{\beta,0}\beta_{T-1} + J_T^0 \quad (17)$$

Here, with some abuse of notation, I introduced on the right hand side *coefficients*  $J_s$  while on the right hand side  $J$  is the value function. In order to establish (17), it is sufficient to note that affine transformations map a quadratic form into another quadratic form. Then, (17) follows immediately from (11), (12) and (15).

So far, I have determined the optimal insiders' holdings at time  $T$  given their holdings as well as the holdings of all of the other insiders at time  $T-1$ . Clearly, at time  $t=T-1$ , insiders are facing a very similar problem, namely:

$$J_{T-1} \equiv \max_{\alpha_{T-2}} z(\alpha_{T-1}, \beta_{T-1})\delta + V_{T-1}(\alpha_{T-1}, \beta_{T-1})(\alpha_{T-2} - \alpha_{T-1}) + e^{-r\Delta}J_T \quad (18)$$

Note that the last term in (18) is a quadratic form in  $(\alpha_{T-1}, \beta_{T-1})$  (see (17)). Proceeding by backward induction one can obtain, eventually, the insiders' optimal holdings at time  $t=1$

as a function of their initial holdings. Given a vector of the initial insiders' holdings, then, the recursive solution is completely specified. Furthermore, under the assumptions of the model this recursive equilibrium is a unique sub-game perfect equilibrium. It is characterized by the following proposition:

**PROPOSITION 3.** For each  $t \leq T$ , the value function of the dynamic programming problem above is a quadratic form in variables  $(\alpha_t, \beta_t)$ :

$$J_{t+1} = J_{t+1}^{\alpha,\alpha} \alpha_t \alpha_t + J_{t+1}^{\alpha,\beta} \alpha_t \beta_t + J_{t+1}^{\beta,\beta} \beta_t \beta_t + J_{t+1}^{\alpha,0} \alpha_t + J_{t+1}^{\beta,0} \beta_t + J_{t+1}^0 \quad (19)$$

while the share price is an affine function in  $A_t = \alpha_t + \beta_t$

$$V_t = v_{0t} + v_t A_t = v_{0t} + v_t A_t \quad (20)$$

The optimal holdings of each insider at time  $t$  are determined as an affine transformation of her own and other insiders' holdings at time  $t-1$ :

$$\alpha_t = l_t^{\alpha,\alpha} \alpha_{t-1} + l_t^{\alpha,\beta} \beta_{t-1} + l_t^{\alpha,0}, \quad \beta_t = l_t^{\beta,\alpha} \alpha_{t-1} + l_t^{\beta,\beta} \beta_{t-1} + l_t^{\beta,0} \quad (21)$$

A complete set of recursive relations and the appropriate boundary conditions that determine the coefficients in (19)-(21) are given in the Appendix. Under the assumptions of the model (namely  $\mu_t \geq 0$  and a constant volatility), this equilibrium is the unique sub-game perfect equilibrium in the economy.

Note that **Assumptions A-C** in DeMarzo and Urošević (2000) under which they establish the existence and uniqueness of a sub-game perfect equilibrium for  $N=1$  coincide with  $\mu_t \geq 0$  and the constant volatility assumptions under which unique sub-game perfection exists for  $N>1$  in this model (see **Proposition 3**). A straightforward but tedious calculation shows that, when  $N \rightarrow 1$ , the solution given by the **Proposition 3** coincides with the solution in DeMarzo and Urošević (2001) in such a limit. Therefore, the equilibrium of the **Proposition 3** generalizes to the multi-agent setting the time-consistent equilibrium in DeMarzo and Urošević (2001).

## 4. The Solution and Comparative Statics

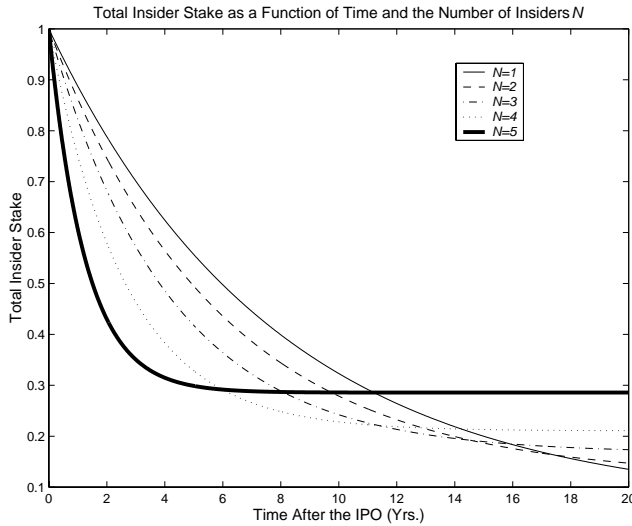
**Proposition 3** specifies the procedure for determining the optimal aggregate insider ownership policy when commitment is not possible. The solution is obtained in terms of a system of coupled recursive relations. In general, that system of equations does not simplify any further, so there is no explicit analytical solution for this problem except in special cases. The limiting case when  $N=1$  was briefly discussed above. Another special case is obtained when the outside investors are risk-neutral. In that case the problem effectively decouples into  $N$  single-agent problems, each of them equivalent to a problem discussed in DeMarzo and Urošević (2001) (this case shall be discussed later in this section). When outside investors are risk-averse, the solution can be analyzed numerically. This is what I do next. In particular, I demonstrate two important properties of the model:

Property 1: The aggregate stake gradually declines towards the perfect risk sharing allocation (the long-term equilibrium).

Property 2: When outside investors are risk-averse, both the long-term equilibrium allocation level and the speed of adjustment of the aggregate insider stake towards such a level increase with the number of insiders  $N$ , *ceteris paribus*

Property 1 follows from the fact that insiders cannot credibly commit to a level of ownership above the competitive allocation. The first part of the second property follows (see (8)) intuitively from the fact that with an increase in a number of insiders  $N$ , each insider's risk exposure diminishes. This allows them to absorb more risk in aggregate. In order to explain the second part of Property 2, note that a decrease in the aggregate insider stake raises the market risk premium and thus lowers the company valuation. Therefore, by selling more today, each insider hopes to decrease the incentive for others to sell in the future (since they will receive a lower share price). This creates among insiders a "race to diversify". As a result, in the unique sub-game perfect equilibrium, the speed of adjustment toward the perfect risk sharing allocation increases with the number of insiders in the company. Intuitively, as there are more strategic agents in the economy, prices more quickly become competitive, though the adjustment towards the long-run equilibrium is gradual.

The model confirms that intuition. In Figure 2, I present the dynamics of the aggregate insider stakes when the number of insiders varies from  $N=1$  to  $N=5$ .



**Figure 2: Aggregate Insider Ownership Policy Varies With the Number Of Insiders (Risk Averse Investors).** Here,  $\gamma^l \sigma^2 r = 5$ ,  $\gamma = 15\gamma^l$ ,  $\mu_1 = 100$ ,  $r = 4\%$ , and the number of insiders varies from  $N=1$  to  $N=5$ . Notice that the speed of adjustment increases as the number of insiders in the company increases. At the same time, the long-term aggregate equilibrium allocation also increases. As a result, as the number of insiders increases, the aggregate insider stake adjusts relatively quickly to a relatively high long-term equilibrium level. Trading is quarterly.

While an increase in the number of insiders, *ceteris paribus*, raises the speed of adjustment towards the competitive allocation (8), it raises that level as well. This leads to an interesting empirical prediction. Namely, if outside investors are risk averse one would expect that companies with a relatively large number of (identical) insiders, *ceteris paribus*, should have a relatively short period of steep insider ownership adjustment towards a relatively high aggregate insider ownership level thereafter. In contrast, when the number of insiders in a company is relatively small, *ceteris paribus*, one would expect to observe slower adjustment towards a relatively low level of the aggregate insider ownership stake. Thus, if outside investors are risk-averse, the dynamics of the aggregate insider stake depends on the number of corporate insiders in the company.

When investors are risk-neutral, the competitive allocation level vanishes (see (8)), and, thus, does not depend on the number of insiders. One would expect that, similarly, the speed of adjustment toward the long-term equilibrium does not depend on the number of insiders either. The reason for that is simple: since in this model the source of strategic interactions is the insiders' impact on the investors' risk premium, if the risk premium vanishes the problem effectively decouples into  $N$  single-insider problems which can be solved as in DeMarzo and Urošević (2001).<sup>13</sup> Instead of Property 2 stated above, when investors are risk neutral the following is true:

Property 3: When investors are risk-neutral, neither the long-term equilibrium allocation of the aggregate insider stake, nor the speed of adjustment towards such an allocation depends on the number of insiders  $N$ . In addition, the long-term equilibrium allocation vanishes.

More formally, the following Proposition holds:

**PROPOSITION 4.** If investors are risk-neutral, the time-consistent equilibrium given by **Proposition 3** is described, for all  $t \leq T$ , by the following system of recursive relations:

$$\begin{aligned} n_{2t} &= n_{4t} = v_{0t} = l_t^{\alpha,\beta} = l_t^{\beta,\alpha} = l_t^{\beta,0} = l_t^{\alpha,0} = 0 \\ J_t^\alpha &= J_t^\beta = J_t^0 \\ v_t &= \delta\mu_1 + \exp(-r\Delta) \frac{v_{t+1}^2}{v_{t+1} + \delta\gamma r\sigma^2} \\ l_t^{\alpha,\alpha} &= l_t^{\beta,\beta} = v_t / (v_t + \delta\gamma r\sigma^2), \end{aligned}$$

while the long-term equilibrium allocation is given by  $A^p = 0$ .

From **Proposition 4** it follows that the aggregate insider stake evolves over time according to:

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<sup>13</sup> Implicit in such reasoning is the assumption of additive separability, namely that both the mean and volatility can be represented in terms of a sum of single-agent terms, without any cross terms. Allowing interactions in the mean or volatility would lead to strategic interactions between the agents even in the absence of investors' risk aversion.



$$A_t = \frac{v_t}{v_t + \delta\gamma r\sigma^2} A_{t-1}, \quad A_0 = A^- \quad (22)$$

$$v_{t-1} = \delta\mu_1 + \exp(-r\Delta) \frac{v_t^2}{v_t + \delta\gamma r\sigma^2}, \quad v_T = \mu_1 / r + \gamma\sigma^2$$

Note that (22) does not depend on  $N$ . In addition, it depends on the initial aggregate insider allocation but not on the precise make up of that allocation (i.e. it does not depend on how the initial allocation is distributed among the insiders). When outside investors are risk neutral, the dynamics of the problem decouples into  $N$  independent problems. As a result, one can use the one-agent solution method of DeMarzo and Urošević (2001). (Note that their method is not applicable when investors are risk-averse or, more generally, whenever there are strategic interactions between the agents.) In particular, **Proposition 9** of DeMarzo and Urošević (2001) formally coincides with **Proposition 4** if one sets  $\gamma^I = 0$  and substitutes  $\alpha \leftrightarrow A$ . This implies that all of the empirical predictions of the one-insider model are also in the multi-insider case when investors are risk-neutral.

When the trading horizon grows without bound, and in the limit of continuous trading, the optimal aggregate insider allocation is given by the following explicit expressions.<sup>14</sup>

$$A(t) = A^- e^{-\lambda t}, \quad \lambda = (r\gamma\sigma^2) / (\mu_1 / r - \gamma\sigma^2) \quad (23)$$

From (23) it is clear that the speed of adjustment of the aggregate insider stake is positively correlated with cash flow volatility as well as the insiders' risk aversion coefficient, and negatively correlated with the insiders' monitoring incentives  $\mu_1$ . That result, in particular, implies that one would expect steeper speeds of adjustment towards the equilibrium for companies and industries that are more volatile (such as high tech stocks, for example), *ceteris paribus*. I check numerically that the same comparative statics are obtained when investors are risk averse.

## 5. Discussion of the Results

In this paper I model the evolution of the aggregate insider ownership stake in a company with multiple risk-averse insiders facing a moral hazard problem. Here, I summarize the most relevant insights and briefly discuss their empirical significance.

The first important finding is that the aggregate insider stake gradually adjusts toward the competitive risk-sharing allocation. This generalizes the one-agent result in DeMarzo and Urošević (2001) to the more realistic setting with multiple strategic agents: if, initially, the ownership of a company is concentrated in the hands of insiders, as a result of the interplay between the moral hazard problem and the insiders' risk aversion, the aggregate insider stake would gradually decline over time until the competitive allocation is reached. Several empirical studies document a significant, but gradual, decline of the aggregate insider stake in corporations as they evolve through time (see the Introduction).

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<sup>14</sup> Recall that we assume that the incentive effect overpowers the risk aversion effect, i.e. that  $\mu_1 / r > \gamma\sigma^2$ . In that case, the adjustment towards the competitive allocation is gradual even in the continuous time (see Section 5.1. of DeMarzo and Urošević (2001) for more details).

Another important question that I address is whether (and, if so, how) changing the number of company insiders impacts the speed of adjustment and the corresponding equilibrium share price. Given the symmetries of the model, the pivotal role in answering that question is played by the aggregate risk aversion of the *outside investors*: If investors are risk-averse, the speed of adjustment increases with the number of corporate insiders  $N$ ; otherwise, it does not depend on it. If the aggregate risk aversion of the outside investors is small but positive, the speed of adjustment of the aggregate stake would be positively (but weakly) correlated with the number of insiders in the company. This is exactly what U.S. insider trading data seems to indicate (see Urošević (2002), Chapter 2).

Finally, it is important to know the level to which the aggregate insider ownership stake would converge. That depends, again, on  $\gamma^I$ : the higher the outside investors risk aversion, the higher the eventual aggregate insider stake. In addition, when  $\gamma^I > 0$ , the steady-state level is positively correlated with the number of insiders. On the other hand, in the limit  $\gamma^I \rightarrow 0$ , the long-run aggregate equilibrium allocation is zero. That means that insiders would in that case sell off their entire company stake. In a recent study of the long-term evolution of corporate ownership in British companies over the past 100 years, Franks et al (2002) show that the aggregate insider ownership stakes steadily decline, decade after decade, but remains positive. For example, the median aggregate stake of corporate directors in their sample falls from 51% at the time of the IPO to only 5.3% thirty years after the IPO. This is consistent with the my predictions if outside investors are not very risk-averse (i.e.  $\gamma^I$  is a small, but positive, number).

In summary, this model is broadly consistent with the available empirical evidence if the aggregate outside investor coefficient of risk aversion is small but positive. That is a reasonable assumption if one recalls that the aggregate risk aversion is typically smaller than the risk aversion of individual investors.

## 6. Conclusions and Future Work

This paper develops a model of optimal ownership dynamics of risk-averse corporate insiders facing a moral hazard problem. In doing so, it expands both the literature on the optimal ownership structure and the literature on asset pricing under moral hazard problem. Extending the related one-agent model by DeMarzo and Urošević (2001) to the situation with multiple strategic insiders, a solution for the equilibrium share price and the dynamics of the aggregate insider ownership stake is derived in two cases: when insiders can credibly pre-commit not to deviate from their optimal ownership policies, and in the more realistic case when such a commitment is not credible (i.e., the time-consistent case). In the latter case, when outside investors are risk-averse there is an additional strategic reason for a dynamic aggregate stake adjustment. Namely, a decrease in the aggregate insider stake raises the market risk premium and thus lowers the company valuation. Therefore, by selling more today, each insider hopes to decrease the incentive for others to sell in the future (since they will receive a lower share price). This creates a “race to diversify” and in equilibrium, the speed of adjustment toward the perfect risk sharing allocation increases with the number of insiders in the company. This

strategic component of the game disappears when investors are risk-neutral. In that case, my results for the aggregate insider stake formally coincide with the results of the on-agent model of DeMarzo and Urošević (2001). In particular, the speed of adjustment is higher for more volatile companies, *ceteris paribus*.

The existing stylized empirical facts are broadly consistent with the main predictions of the model. That said, there are, of course, other plausible explanations for the same empirical results. For example, Franks (2002) finds that the very long-term changes that have taken place in the ownership structure in Great Britain are, primarily, the result of corporate acquisitions. Thus, the ownership of companies gets diluted at the same time as the number of insiders increases. This is at variance with one of the key assumptions of my model, namely that the number of insiders  $N$  is fixed. In the future, it would be interesting to relax this assumption by making  $N$  endogenous, for example.

The model rests on a number of other simplifying assumptions. For tractability, I assumed that the only strategic interaction between insiders occurs through their market risk premium impact. Including other types of interactions would make the model less tractable but, nevertheless, interesting to pursue.<sup>15</sup> DeMarzo and Urošević (2001) demonstrated that the private benefits of control may have a significant impact on the insider's dynamic trading policy. Incorporating private benefits of control would allow one to gain insight into the dynamics of the strategic corporate control issues.<sup>16</sup> Also, the model is mute on the issue of the initial insider stake creation. These stakes appear naturally in an IPO mechanism (see Stoughton and Zechner (1998) and DeMarzo and Urošević (2001)). It would be interesting to develop a similar approach in the case of, say, two insiders: a venture capitalist and a company manager. In order to model such situations more realistically, one would need to extend the present model to include heterogeneous insiders. Indeed, a CEO and a venture capitalist may differ both in their incentives and their risk aversion. A model with heterogeneous insiders who face a moral hazard problem should be tractable, even if technically involved. Following Pritsker (2002) it would be, then, interesting to explore the dynamics of the individual insider stakes and not just the aggregate stake. In particular, it would be instructive to study the possible effects of front-running and/or predatory trading among such insiders. These and many other interesting issues shall await further research.

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<sup>15</sup> For example, one could include in the expression for the expected dividends an interaction term proportional to  $e_1 e_2$ . In that case, even when investors are risk-neutral, one would expect for the model to exhibit a non-trivial strategic interaction between the agents.

<sup>16</sup> Classic papers on takeovers are Grossman and Hart (1980) and Shleifer and Vishny (1986). Stultz (1988) and, more recently, Gomes and Novaes (2001) study control mechanisms, among others.

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## Appendix

**PROOF OF THE PROPOSITION 1:** In this equilibrium, each agent maximizes her certainty equivalent taking into account other agents’ equilibrium allocations. Denoting for simplicity,  $\alpha^l = \alpha$  expression (7) follows from the expression for certainty equivalent (6) upon substituting the expression for the share price (5). Indeed:

$$\begin{aligned}
k(\bar{\alpha}(t)) &= \int_{[t,\infty]} e^{-r(\tau-t)} [z(\alpha(\tau), \beta(\tau)) d\tau - d\alpha(\tau)V(\bar{\alpha}(\tau))] \\
&= \int_{[t,\infty]} e^{-r(\tau-t)} z(\alpha(\tau), \beta(\tau)) d\tau - \int_{[t,\infty]} e^{-r(s-t)} \int_s^\infty e^{-r(\tau-s)} v(\alpha(\tau) + \beta(\tau)) d\tau d\alpha(s) \\
&= \int_{[t,\infty]} e^{-r(\tau-t)} z(\alpha(\tau), \beta(\tau)) d\tau - \int_t^\infty e^{-r(\tau-t)} \left( \int_{[t,\tau]} d\alpha(s) \right) v(\alpha(\tau) + \beta(\tau)) d\tau \\
&= \int_t^\infty e^{-r(\tau-t)} z(\alpha(\tau), \beta(\tau)) d\tau + \int_t^\infty e^{-r(\tau-t)} (\alpha(t-) - \alpha(\tau)) v(\alpha(\tau) + \beta(\tau)) d\tau ,
\end{aligned}$$

Given her own initial allocation  $\alpha^-$  and the other agents’ aggregate equilibrium allocation choice  $\beta$ , an agent’s best response is given by the first order condition:

$$\alpha^c = \frac{r\gamma^l \sigma^2}{\mu_1 + r(\gamma + 2\gamma^l)\sigma^2} (1 - \beta) + \frac{\mu_1 + r\gamma^l \sigma^2}{\mu_1 + r(\gamma + 2\gamma^l)\sigma^2} \alpha^- \quad (A1)$$

Summing up  $N$  identical equations (A1), taking into account that  $\sum_I \alpha^I = A$  and  $\sum_I \beta^I = (N-1)A$ , and solving for the aggregate allocation  $A^c$  one obtains (8). ♦

**PROOF OF THE PROPOSITION 2:** From (6), and a calculation similar to that in the proof of **Proposition 1**, it follows that if the price-taking equilibrium exists, the equilibrium allocation is given, for each insider, by the following expression (here, each insider is taking  $\beta$  and  $A^p$  as given):

$$\arg \max_{\alpha} z(\alpha, \beta) - \alpha v(A^p(\tau)).$$

This implies that, in such equilibrium, the optimality conditions  $\gamma \alpha^p - \gamma^I (1 - A^p) = 0$  need to be satisfied for each insider. In addition, an equilibrium condition is needed which states that the sum of each insider's holdings  $\alpha^p$  is equal to the aggregate holdings  $A^p$ . Summing up  $N$  identical equations and solving for  $A^p$  renders (10) as well as  $\alpha^p = A^p / N$ . The second order condition reads:  $z_{\alpha\alpha}(\alpha, \beta) = (\mu_1 - r\gamma\sigma^2) < 0$  which completes the proof. ♦

**PROOF OF THE PROPOSITION 3:** By backward induction. Let me first establish the relations (19)-(21) and (A2)-(A5). I have established them for  $t=T$ . Suppose that they are valid for  $t+1$ . One can easily see that, then, they are valid for  $t$  as well. Indeed, each insider's maximization problem reads:

$$J_t \equiv \max_{\alpha_t} \delta z(\alpha_t, \beta_t) + V_t(\alpha_t, \beta_t)(\alpha_{t-1} - \alpha_t) + e^{-r\Delta} J_{t+1}$$

where, by assumption, (19) and (20) hold. Consequently, the optimal insiders' holdings are easily seen to yield (21) as well as (A2)-(A3). Utilizing these expressions it is immediate to see that the equilibrium share price at time  $t-1$  is a linear function of the aggregate insider holdings at time  $t$  and that (A4). Using (21) and the linearity of the share price, value function  $J_t$  can be re-written as a quadratic form in variables  $(\alpha_{t-1}, \beta_{t-1})$ . Reading off the appropriate coefficients in  $J_t$  establishes (A5). In order to establish the second order conditions, note that they are equivalent to  $n_{it} < 0$ , where  $n_{it}$  is given by the first expression in (A3). Using (A2)-(A5), as well as the fact that  $\mu_1 \geq 0$  and that volatility and the insiders risk aversion are positive constants one shows, working backwards period by period, that  $n_{it} < 0$  and, thus, that the equilibrium is a unique sub-game perfect equilibrium under the assumptions of the model ♦

### **RECURSIVE RELATIONSHIPS AND BOUNDARY CONDITIONS USED IN PROOF OF THE PROPOSITION 3:**

The following is a list of the recursive relationship and the boundary conditions used in the proof of the **Proposition 3**.

The coefficients in (21) are determined as follows:

$$\begin{aligned}
l_t^{\alpha,\alpha} &= \frac{n_{3t}[n_{2t}(2-N)-n_{1t}]}{(n_{1t}-n_{2t})n_{5t}}, \quad l_t^{\alpha,\beta} = \frac{n_{2t}n_{3t}}{(n_{1t}-n_{2t})n_{5t}}, \quad l_t^{\alpha,0} = -\frac{n_{4t}}{n_{5t}} \\
l_t^{\beta,\alpha} &= -\frac{n_{3t}}{n_{5t}} - l_t^{\alpha,\alpha}, \quad l_t^{\beta,\beta} = -\frac{n_{3t}}{n_{5t}} - l_t^{\alpha,\beta}, \quad l_t^{\beta,0} = -\frac{n_{4t}}{n_{5t}}N - l_t^{\alpha,0}
\end{aligned} \tag{A2}$$

The coefficients  $n_{it}$  are defined by the following relations:

$$\begin{aligned}
n_{1t} &= \delta(\mu_1 - \gamma\sigma^2r) - 2v_t + 2e^{-r\Delta}J_{t+1}^{\alpha,\alpha}, \quad n_{2t} = \delta\mu_1 - v_t + e^{-r\Delta}J_{t+1}^{\alpha,\beta}, \\
n_{3t} &= v_t, \quad n_{4t} = -v_{0t} + e^{-r\Delta}J_{t+1}^{\alpha,0}, \quad n_{5t} = n_{1t} + (N-1)n_{2t}
\end{aligned} \tag{A3}$$

so that  $A_t = -\frac{n_{3t}}{n_{5t}}A_{t-1} - \frac{n_{4t}}{n_{5t}}$ .

The following are the recursive relations that define the coefficients in (20):

$$\begin{aligned}
v_{0t-1} &= -\delta\gamma^l\sigma^2r + e^{-r\Delta}[v_{0t} - Nv_t n_{4t} / n_{5t}], \\
v_{t-1} &= \delta(\mu_1 + \gamma^l\sigma^2r) - e^{-r\Delta}(n_{3t}v_t / n_{5t})
\end{aligned} \tag{A4}$$

The set of 6 recursive relations that defines the coefficients in (19) is listed below and denoted by (A5):

$$\begin{aligned}
J_t^{\alpha,\alpha} &= \delta(\mu_1 - \gamma\sigma^2r)(l_t^{\alpha,\alpha})^2 / 2 + \delta\mu_1 l_t^{\alpha,\alpha} l_t^{\beta,\alpha} + v_t(1 - l_t^{\alpha,\alpha})(l_t^{\alpha,\alpha} + l_t^{\beta,\alpha}) + \\
&+ e^{-r\Delta}[J_{t+1}^{\alpha,\alpha}(l_t^{\alpha,\alpha})^2 + J_{t+1}^{\alpha,\beta}l_t^{\alpha,\alpha}l_t^{\beta,\alpha} + J_{t+1}^{\beta,\beta}(l_t^{\beta,\alpha})^2]
\end{aligned}$$

$$\begin{aligned}
J_t^{\alpha,\beta} &= \delta(\mu_1 - \gamma\sigma^2r)l_t^{\alpha,\alpha}l_t^{\alpha,\beta} + \delta\mu_1(l_t^{\alpha,\alpha}l_t^{\beta,\beta} + l_t^{\alpha,\beta}l_t^{\beta,\alpha}) + v_t(l_t^{\alpha,\beta} + l_t^{\beta,\beta} - 2l_t^{\alpha,\alpha}l_t^{\alpha,\beta} - l_t^{\alpha,\alpha}l_t^{\beta,\beta} - l_t^{\alpha,\beta}l_t^{\beta,\alpha}) \\
&+ e^{-r\Delta}[2J_{t+1}^{\alpha,\alpha}l_t^{\alpha,\alpha}l_t^{\alpha,\beta} + J_{t+1}^{\alpha,\beta}(l_t^{\alpha,\alpha}l_t^{\beta,\beta} + l_t^{\alpha,\beta}l_t^{\beta,\alpha}) + 2J_{t+1}^{\beta,\beta}l_t^{\beta,\alpha}l_t^{\beta,\alpha}]
\end{aligned}$$



$$J_t^{\beta,\beta} = \delta(\mu_1 - \gamma\sigma^2 r)(l_t^{\alpha,\beta})^2 / 2 + \delta\mu_1 l_t^{\alpha,\beta} l_t^{\beta,\beta} - v_t(l_t^{\beta,\beta} l_t^{\alpha,\beta} + (l_t^{\alpha,\beta})^2) \\ + e^{-r\Delta} [J_{t+1}^{\alpha,\alpha} (l_t^{\alpha,\beta})^2 + J_{t+1}^{\alpha,\beta} l_t^{\alpha,\beta} l_t^{\beta,\beta} + J_{t+1}^{\beta,\beta} (l_t^{\beta,\beta})^2]$$

$$J_t^{\alpha,0} = \delta(\mu_1 - \gamma\sigma^2 r) l_t^{\alpha,\alpha} l_t^{\alpha,0} + \delta\mu_1 (l_t^{\alpha,\alpha} l_t^{\beta,0} + l_t^{\alpha,0} l_t^{\beta,\alpha}) + v_{0t} (1 - l_t^{\alpha,\alpha}) + v_t [l_t^{\alpha,0} (1 - l_t^{\beta,\alpha}) + l_t^{\beta,0} - 2l_t^{\alpha,\alpha} l_t^{\alpha,0} - l_t^{\alpha,\alpha} l_t^{\beta,0}] + \\ + e^{-r\Delta} [2J_{t+1}^{\alpha,\alpha} l_t^{\alpha,\alpha} l_t^{\alpha,0} + J_{t+1}^{\alpha,\beta} (l_t^{\alpha,\alpha} l_t^{\beta,0} + l_t^{\beta,\alpha} l_t^{\alpha,0}) + 2J_{t+1}^{\beta,\beta} l_t^{\beta,\alpha} l_t^{\beta,0} + J_{t+1}^{\alpha,\alpha} l_t^{\alpha,\alpha} + J_{t+1}^{\beta,\beta} l_t^{\beta,\alpha}]$$

$$J_t^{\beta,0} = \delta(\mu_1 - \gamma\sigma^2 r) l_t^{\alpha,\beta} l_t^{\alpha,0} + \delta\mu_1 (l_t^{\alpha,\beta} l_t^{\beta,0} + l_t^{\alpha,0} l_t^{\beta,\beta}) - v_{0t} l_t^{\alpha,\beta} - v_t [l_t^{\alpha,0} l_t^{\beta,\beta} + 2l_t^{\alpha,\beta} l_t^{\alpha,0} + l_t^{\alpha,\beta} l_t^{\beta,0}] + \\ + e^{-r\Delta} [2J_{t+1}^{\alpha,\beta} l_t^{\alpha,\beta} l_t^{\alpha,0} + J_{t+1}^{\alpha,\beta} (l_t^{\beta,\beta} l_t^{\alpha,0} + l_t^{\alpha,\beta} l_t^{\beta,0}) + 2J_{t+1}^{\beta,\beta} l_t^{\beta,\beta} l_t^{\beta,0} + J_{t+1}^{\alpha,\beta} l_t^{\alpha,\beta} + J_{t+1}^{\beta,\beta} l_t^{\beta,\beta}]$$

$$J_t^0 = \delta(\mu_1 - \gamma\sigma^2 r)(l_t^{\alpha,0})^2 / 2 + \delta\mu_1 l_t^{\alpha,0} l_t^{\beta,0} - v_{0t-1} l_t^{\alpha,0} - v_t ((l_t^{\alpha,0})^2 + l_t^{\alpha,0} l_t^{\beta,0}) \\ + e^{-r\Delta} [J_{t+1}^{\alpha,\alpha} (l_t^{\alpha,0})^2 + J_{t+1}^{\alpha,\beta} l_t^{\alpha,0} l_t^{\beta,0} + J_{t+1}^{\beta,\beta} (l_t^{\beta,0})^2 + J_{t+1}^{\alpha,\alpha} l_t^{\alpha,0} + J_{t+1}^{\beta,\beta} l_t^{\beta,0} + J_{t+1}^0]$$

(A5)

The boundary conditions read as follows: coefficients  $J_{t+1}$  vanish at time  $t=T$ ; the boundary values for the stock price coefficients are given by (11) and the initial insiders' allocations are given by an (exogenous) vector  $\alpha^-$ . ♦

**PROOF OF THE PROPOSITION 4:** By backward induction. When  $t=T$  and  $\gamma^l = 0$ , from formulas (A3) and (A4) in the Appendix it follows that

$$n_{2T} = n_{4T} = v_{0T} = 0 \\ l_T^{\alpha,\beta} = l_T^{\beta,\alpha} = l_T^{\beta,0} = l_T^{\alpha,0} = 0$$

Also, from (A2) and (A4), it is immediate that:

$$l_T^{\alpha,\alpha} = l_T^{\beta,\beta} = v_T / (v_T + \gamma\sigma^2), \\ v_{T-1} = \delta\mu_1 + \exp(-r\Delta) \frac{v_T^2}{v_T + \gamma\sigma^2}$$

Thus, the Proposition holds when  $t=T$ . Let me now assume that it holds at time  $t+1$ . Then from (A5) it is immediate that  $n_{2t} = J_t^{\alpha,\alpha} = J_t^{\beta,\beta} = J_t^0 = 0$ . In addition,  $J_{t+1}^{\alpha,\beta} = \delta\mu_1 l_{t+1}^{\alpha,\alpha} l_{t+1}^{\beta,\beta} + v_{t+1} l_{t+1}^{\beta,\beta} (1 - l_{t+1}^{\alpha,\alpha})$ . Plugging these expressions into the definition of  $n_{2t}$ , and utilizing the induction hypothesis one obtains that  $n_{2t} = 0$ . From (A4) it follows that  $v_{0t} = 0$  while (A2) implies that  $l_t^{\alpha,\beta} = l_t^{\beta,\alpha} = l_t^{\beta,0} = l_t^{\alpha,0} = 0$ . Finally, from the induction hypothesis and the first equation in (A5) it follows that  $l_t^{\alpha,\alpha} = l_t^{\beta,\beta} = v_t / (v_t + \delta\gamma r \sigma^2)$ . ♦