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Q-Monetary Transmission

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Abstract

We study the effects of monetary-policy-induced changes in Tobin's q on corporate investment and capital structure. We develop a theory of the mechanism, provide empirical evidence, evaluate the ability of the quantitative theory to match the evidence, and quantify the relevance for monetary transmission to aggregate investment.

Keywords: monetary transmission, stock prices, Tobin's q, investment, capital structure.
JEL classification: D83, E22, E44, E52, G12, G31, G32.

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1 Introduction

The chain of causal links that lie between monetary policy actions and their ultimate effects on macroeconomic variables is broadly referred to as *the monetary transmission mechanism.* Since the immediate effect of these actions is to influence a wide array of interest rates and prices of financial and non-financial assets, it is easy to imagine many ways in which monetary policy may affect economic decisions. Consequently, textbook treatments contain extensive taxonomies of a myriad of monetary transmission mechanisms.[1] The broadest classification typically consists of three main transmission channels: the (*direct* or *traditional*) *interest-rate channel*, the *asset-price channel*, and the *credit channel.*

The *interest-rate channel* is best described as a *user-cost channel*: Suppose there is an unexpected increase in the nominal policy rate, and that (as is usually the case) some of the increase passes through to real rates. Then, since the real rate is a key component of the user cost of capital, and the user cost of capital is a key determinant of the demand for capital (e.g., as in Jorgenson (1963)), investment should fall as a result of the monetary policy action.[2] The *asset-price channel* is best described as a *Tobin's q channel*: Suppose an unexpected decrease in the nominal policy rate causes stock prices to rise (as is well documented empirically, e.g., Bernanke and Kuttner (2005)) relative to the replacement cost of capital. Then, since the market yield of the stock is a key determinant of the cost of external financing in capital markets, equity-financed investment should increase as a result of the monetary policy action (e.g., as conjectured by Keynes (1936) and Tobin (1969)).[3] The *credit channel* is best described as an amplification mechanism associated with the other two channels: Suppose an unexpected increase in the nominal policy rate causes asset prices to fall (e.g., through either of the previous two channels), which in turn deteriorates borrowers' net worth. Then the resulting increase in external finance premia on debt (Bernanke and Gertler (1989)) or tightening of borrowing constraints (Kiyotaki and Moore (1997)) imply debt-financed investment should fall as a result of the monetary policy action.

The user-cost channel is well-understood and present in most quantitative models used for

[1]See, e.g., Mishkin (1995, 1996, 2001) and Boivin et al. (2010).

[2]Our focus here is on corporate investment, but all these channels have counterparts for household spending on consumption of durables and real estate.

[3]Keynes (1936, chap. 12, sec. 3) argued that stock-market (re)valuations "inevitably exert a decisive influence on the rate of current investment." Tobin (1969) elaborated on this idea by emphasizing stock-market revaluations driven by monetary policy—and introduced the now famous "q" to formalize this specific transmission mechanism.

policy analysis. The credit channel has received much attention in the past decade, and is now standard in theoretical and quantitative policy-oriented modelling. The asset-price channel is discussed in undergraduate textbooks and policy circles, but academic research on it is scant. In this paper we study the effects of changes in Tobin's q induced by monetary policy actions—a mechanism we dub *q-monetary transmission* or the *q-channel*—and take several steps toward (re-)establishing Tobin's q as a prominent causal link between monetary policy and the real economy. Specifically, we: (i) develop a model of the q-monetary transmission mechanism; (ii) provide identification and empirical evidence for the q-channel; (iii) evaluate the ability of the quantitative theory to match the evidence; and (iv) quantify the effect of q-monetary transmission on firms' investment and capital structure.

On the theory front, we develop a model that clarifies the roles that financial constraints, the stock market, and money, play in the transmission of monetary policy to firms' investment and financing decisions, through stock prices. Stock-market turnover among outside financial investors with heterogeneous valuations generate a "bubble-like" resale-value component through which monetary policy affects the market price of a firm's stock. In turn, the investment and capital-structure decisions of firms that rely on equity as a source of external financing respond to exogenous (policy-induced) variation in the market price of their equity.

On the empirical front, the main challenge for estimating the q-channel is that monetary policy may affect investment and stock prices through *other* channels. For instance, a contractionary money shock may lead to a joint reduction in a firm's stock price and investment through the traditional interest-rate channel (i.e., due to higher discounting), but the reduction in the stock price is not *causing* the reduction in investment. Thus, we cannot hope to estimate the causal effect of the stock price on investment—the hallmark of the q-channel—simply from the comovement of investment and the stock price induced by monetary policy shocks.

We meet this empirical challenge by exploiting *stock turnover* as a source of cross-sectional variation in the responsiveness of stock prices to monetary shocks.[4] Our empirical strategy builds on the idea that, as long as stock turnover (and any *unobserved* firm-level characteristic that is correlated with turnover) does not affect the responsiveness to money shocks of other transmission variables that influence the outcome variable, then identified money shocks combined with heterogeneity in cross-sectional stock turnover can be used as a source of exogenous

[4] Lagos and Zhang (2020b) provide evidence that stock turnover is a strong predictor of the cross-sectional differences in the responsiveness of stock prices to monetary policy shocks.

(policy driven) cross-sectional variation in Tobin's q. We use this cross-sectional variation in the responses of stock prices to money shocks across firms with different stock turnover to identify the effects of changes in stock prices on firms' investment and capital structure decisions. Specifically, we construct an instrument for firm-level Tobin's q by interacting monetary policy shocks with a (predetermined) measure of firm-specific stock turnover. We find that such instrumented variation in Tobin's q has significant persistent effects on the equity issuance and investment decisions of firms whose balance sheets have a relatively low *liquidity ratio* (defined as the share of liquid assets in total assets). For example, for firms with below-median liquidity ratios, a 1% increase in Tobin's q causes: (i) a 0.06 pp increase in the firm's ratio of net equity issuance relative to the book value of total assets in the quarter of the monetary shock; and (ii) a response of approximately 1% higher investment rate at the two-quarter horizon. Our micro estimates imply that the q-channel accounts for about one third of the conventional estimates of the peak response of aggregate investment to monetary policy shocks.

Our work makes contact with three literatures. First, we contribute to the literature on monetary transmission by filling the empirical and theoretical void on the asset-price channel that operates through Tobin's q. Second, we contribute to the literature on the causal effects of changes in stock-market valuations on corporate investment decisions (e.g., Keynes (1936), Brainard and Tobin (1968), Tobin (1969), Tobin and Brainard (1976), Fischer and Merton (1984), Morck et al. (1990), Blanchard et al. (1993), Baker et al. (2003), Gilchrist et al. (2005), Polk and Sapienza (2008), Amihud and Levi (2022)). Our contribution to this literature is twofold. On the theory front, we develop an equilibrium model with two sectors: a productive sector where firms are managed by entrepreneurs who make investment and equity issuance decisions, and a financial sector where money and equity claims to the capital installed in the firm are traded among investors with heterogeneous valuations of the marginal product of firms' capital. Our theory highlights the roles that financial constraints (as a determinant of a firm's dependence on equity financing) and heterogeneous valuations of capital play in the transmission of monetary policy shocks to investment decisions through stock prices. On the empirical front, we propose a new instrument for changes in Tobin's q that are not caused by firm-level changes in marginal q. As mentioned above, our innovation in this regard consists of exploiting a combination of identified monetary policy shocks and the cross-sectional variation in the responsiveness of stock prices to these shocks due to differences in stock turnover. Third, our theoretical and empirical results on the response of firms' equity issuance and capital structure to fluctuations in stock prices induced by monetary shocks contribute to the corporate finance literature that studies the relationship between firms' capital structure and macroeconomic conditions in general, and stock prices in particular (e.g., Baker and Wurgler (2002), Korajczyk and Levy (2003), Hovakimian et al. (2004), Acharya et al. (2020)). Our contribution to this literature is to identify the persistent effects of monetary policy shocks on the capital structure of public firms.

2 Theory

Time is represented by a sequence of periods indexed by $t \in \{0, 1, ...\}$. Each time period is divided into two subperiods where different activities take place. There is a continuum of infinitely lived agents of two types: *investors*, each identified with a point in the set $\mathcal{I} = [0, 1]$, and *brokers*, each identified with a point in the set $\mathcal{B} = [0, 1]$. There is a continuum (with unit measure) of *entrepreneurs* (also referred to as *firms*) who live for a random number of periods. Each entrepreneur who is alive at the beginning of period t is identified with a point in the set $\mathcal{E}_t \subset \mathbb{R}_+$. A fraction $1 - \pi \in [0, 1]$ of the population of entrepreneurs in the set $\mathcal{E}_t$ dies (i.e., exits the economy) at the beginning of the second subperiod of period t. The subset of entrepreneurs who exit is a uniform random draw from the population of entrepreneurs, and each is immediately replaced by a newly born entrepreneur.

There are three commodities at each date: two consumption goods, called *good 1* and *good 2*, and a *capital* good. The consumption goods are perishable: good 1 and good 2 can only be consumed in the first and second subperiods, respectively. Capital is storable, but depreciates at rate $\delta \in [0, 1]$ between periods. Upon entering the economy, an entrepreneur is endowed with $w_0^i \in \mathbb{R}_+$ units of good 2 and $k_0 \in \mathbb{R}_+$ units of capital. We use a cumulative distribution function Ω to describe the heterogeneity in the initial endowment of (claims to) good 2 relative to capital, $\omega_0^i \equiv w_0^i/k_0$, across entrepreneurs. In the second subperiod of every period, investors and brokers are endowed with a resource called *labor (effort)* that they can use to produce good 2 one-for-one. There are two other production technologies, which can be managed only by entrepreneurs. One of these production technologies uses capital available at the beginning of period t to produce good 1 in the first subperiod of period t. Specifically, the capital stock k_t operated by an entrepreneur delivers zk_t units of good 1 at the end of the first subperiod of t, with $z \in \mathbb{R}_{++}$. The other production technology can be operated by an entrepreneur in the second subperiod of period t, and uses good 2 and the capital the entrepreneur has in place at the beginning of period t to augment the capital that the entrepreneur will have in place to produce good 1 in period $t+1$. This technology is represented by a cost function, $\mathrm{C}(x_t, k_t) \equiv x_t + \Psi(x_t/k_t) k_t$, interpreted as the cost (in terms of good 2) of producing and installing x_t units of capital for an entrepreneur whose current capital is k_t. We assume $0 < \Psi''$, and that there is a $\iota_0 \in \mathbb{R}_+$ such that $\Psi(\iota_0) = \Psi'(\iota_0) = 0$. It is convenient to define $\mathrm{c}(x_t/k_t) \equiv \mathrm{C}(x_t, k_t)/k_t$, i.e., the cost of investment per unit of installed capital. The assumptions on Ψ imply $\mathrm{c}(\iota_0) - \iota_0 = \mathrm{c}'(\iota_0) - 1 = 0 < \mathrm{c}''(\cdot)$. Once installed, capital is entrepreneur-specific, i.e., capital installed by entrepreneur i is only productive when operated by entrepreneur i.

The asset structure is as follows. In the second subperiod of every period, in order to finance the cost of investing in new capital, every entrepreneur can issue identical, durable, and perfectly divisible equity claims to the future returns from the newly created capital. Entrepreneurs are also allowed to sell equity claims on any existing capital they currently own. An equity share issued by an entrepreneur in the second subperiod of t represents ownership of one unit of capital along with the stream of *dividends* of good 1 produced by that unit of capital. When an entrepreneur dies, the outstanding equity claims they had previously issued disappear, and the underlying capital plus any financial assets, physical capital, or claims owned by the entrepreneur are distributed uniformly (lump sum) to the cohort of newly born entrepreneurs. There are two other financial instruments: a real one-period pure-discount government *bond*, and *money*. A unit of the bond issued in the second subperiod of t represents a risk-free claim to one unit of good 2 in the second subperiod of $t+1$. The stock of bonds outstanding at time t is denoted B_t, and all private agents take the sequence $\{B_t\}_{t=0}^{\infty}$ as given. Money is intrinsically useless: it is not an argument of any utility or production function, and unlike equity or bonds, money does not constitute a formal claim to any resources. The nominal money supply at the beginning of period t is denoted A_t^m, and we assume $A_{t+1}^m = \mu A_t^m$, with $\mu \in \mathbb{R}_{++}$ and $A_0^m \in \mathbb{R}_{++}$ given. The government injects or withdraws money via lump-sum transfers or taxes to investors in the second subperiod of every period. At the beginning of period $t = 0$, each investor is endowed with an equal portfolio of money.[5]

The market structure is as follows. In the second subperiod, all agents can trade good 2, equity shares, bonds, and money, in a spot Walrasian market.[6] In the first subperiod, investors

[5]We assume brokers do not hold financial assets. This assumption allows us to abstract from the broker's portfolio problem in the first subperiod, which is not essential for the questions we study in this paper. See Lagos and Zhang (2015, 2020b) for a treatment of the broker's portfolio problem in this class of models.

[6]Equity shares (i.e., the claims on installed capital and its returns) can be traded freely, but the actual

can trade equity shares and money in a random bilateral *over-the-counter (OTC) market* with brokers, while brokers can also trade equity shares and money with other brokers in a spot Walrasian *interbroker market.* We use $\alpha \in [0,1]$ to denote the probability that an individual investor is able to make contact with a broker in the OTC market. Once a broker and an investor have contacted each other, the pair negotiates the quantity of equity shares and money that the broker will trade in the interbroker market on behalf of the investor, and a fee for the broker's intermediation services. The terms of the trade between an investor and a broker in the OTC market are determined by Nash bargaining, where $\theta \in [0,1]$ is the investor's bargaining power. We assume the fee is negotiated in terms of good 2, and paid at the beginning of the following subperiod.[7] The timing is that the round of OTC trade takes place in the first subperiod and ends before equity pays out first-subperiod dividends.[8] Equity purchases in the OTC market cannot be financed by borrowing (e.g., due to anonymity and lack of commitment and enforcement). This assumption and the structure of preferences described below create the need for a medium of exchange in the OTC market.[9]

A broker's preferences are given by

$$\mathbb{E}_0^B \sum_{t=0}^{\infty} \beta^t (y_t - h_t),$$

where $\beta \in (0,1)$ is the discount factor, and y_t and h_t denote a broker's consumption of good 2, and utility cost from supplying h_t units of labor in the second subperiod of period t, respectively.[10] The expectation operator, $\mathbb{E}_0^B$, is with respect to the probability measure induced by

physical capital created and installed by a particular entrepreneur is assumed to be non tradable. The idea is that, once installed by an entrepreneur, physical capital becomes entrepreneur-specific and cannot be operated by another entrepreneur. An entrepreneur can, however, disinvest (which entails bearing the adjustment cost, Φ) to turn installed capital into good 2, which can then be traded freely in the Walrasian market. Similarly, when the entrepreneur dies, the quantity of good 2 obtained from unistalling the capital that the entrepreneur used to manage is distributed to newly born entrepreneurs (net of adjustment costs).

[7]This is the specification used in Lagos and Zhang (2020b). Lagos and Zhang (2015) instead assume the investor must pay the intermediation fee to the broker on the spot (with money or equity). The timing convention in Lagos and Zhang (2020b) simplifies the exposition without affecting the mechanisms of interest.

[8]As in previous search models of OTC markets, e.g., Duffie et al. (2005) and Lagos and Rocheteau (2009), an investor must own the equity in order to consume the dividend flow of consumption good in the OTC round.

[9]See Lagos and Zhang (2019, 2020a) for a similar model where investors can buy equity with *margin loans.*

[10]Dealers get no utility from good 1, so they have no motive for purchasing equity on their own account in the first subperiod. This assumption is easy to relax, but we adopt it because it is the standard benchmark in the search-based OTC literature, e.g., see Duffie et al. (2005), Lagos and Rocheteau (2009), Lagos et al. (2011), and Weill (2007).

the random trading process in the OTC market. An investor's preferences are given by

$$\mathbb{E}_0^I \sum_{t=0}^{\infty} \beta^t \left(\varepsilon_t c_t + y_t - h_t\right),$$

where y_t and h_t denote an investor's consumption of good 2, and utility cost from supplying h_t units of labor in the second subperiod of period t, respectively, and c_t is the investor's consumption of good 1 at the end of the first subperiod of period t. The variable ε_t denotes the realization of an idiosyncratic valuation shock for good 1 that is distributed independently over time and across investors with a differentiable cumulative distribution function G with support $[\varepsilon_L, \varepsilon_H] \subseteq [0, \infty]$, and mean $\bar{\varepsilon} \equiv \int \varepsilon dG(\varepsilon)$. An investor learns the realization ε_t at the beginning of the first subperiod of period t, immediately before the OTC trading round. The expectation operator, $\mathbb{E}_0^I$, is with respect to the probability measure induced by the investor's valuation shocks, and the trading process in the OTC market.

The preferences of an entrepreneur born in the second subperiod of t are given by

$$\sum_{j=t}^{\infty} (\beta\pi)^{(j-t)} \left(y_j + \beta\varepsilon_e c_{j+1}\right),$$

where y_j denotes consumption of good 2 in the second subperiod of period j, c_{j+1} is consumption of good 1 at the end of the first subperiod of period $j+1$, and $\varepsilon_e \in \mathbb{R}_{++}$ is the entrepreneur's valuation of good 1.

2.1 Equilibrium

Consider the determination of the terms of trade in a bilateral meeting in the OTC round of period t between a broker and an investor with valuation ε and portfolio $\boldsymbol{a}_t = (a_t^b, a_t^m, a_t^s)$, where a_t^b, a_t^m, and a_t^s denote bond, money, and equity holdings, respectively. Let $W_t(\boldsymbol{a}_t, \varpi_t)$ denote the maximum expected discounted payoff at the beginning of the second subperiod of period t of an investor who is holding portfolio $\boldsymbol{a}_t$ and has to pay a broker fee ϖ_t. Let $[\overline{\boldsymbol{a}}_t(\boldsymbol{a}_t, \varepsilon), \varpi_t(\boldsymbol{a}_t, \varepsilon)]$ represent the bargaining outcome in a bilateral trade at time t between a broker and an investor with portfolio $\boldsymbol{a}_t$ and valuation ε, where $\overline{\boldsymbol{a}}_t(\boldsymbol{a}_t, \varepsilon) \equiv \left(\overline{a}_t^b(\boldsymbol{a}_t, \varepsilon), \overline{a}_t^m(\boldsymbol{a}_t, \varepsilon), \overline{a}_t^s(\boldsymbol{a}_t, \varepsilon)\right)$ denotes the investor's post-trade portfolio. That is,

$$[\overline{\boldsymbol{a}}_t(\boldsymbol{a}_t, \varepsilon), \varpi_t(\boldsymbol{a}_t, \varepsilon)] = \arg \max_{(\overline{\boldsymbol{a}}_t, \varpi_t) \in \mathbb{R}_+^4} \Gamma(\overline{\boldsymbol{a}}_t, \boldsymbol{a}_t, \varepsilon)^{\theta} \varpi_t{}^{1-\theta} \tag{1}$$

with $\overline{\boldsymbol{a}}_t \equiv (\overline{a}_t^b, \overline{a}_t^m, \overline{a}_t^s)$,

$$\Gamma\left(\overline{\boldsymbol{a}}_t, \boldsymbol{a}_t, \varepsilon\right) \equiv \varepsilon z \overline{a}_t^s + W_t(\overline{a}_t^b, \overline{a}_t^m, \pi(1-\delta)\overline{a}_t^s, \varpi_t) - \varepsilon z a_t^s - W_t(a_t^b, a_t^m, \pi(1-\delta)a_t^s, 0),$$

and subject to

$$\begin{aligned} \overline{a}_t^m + p_t \overline{a}_t^s &\leq a_t^m + p_t a_t^s \\ 0 &\leq \Gamma\left(\overline{\boldsymbol{a}}_t, \boldsymbol{a}_t, \varepsilon\right) \\ \overline{a}_t^b &= a_t^b, \end{aligned}$$

where p_t denotes the dollar price of an equity share in the interbroker market of period t. The first and second constraints are the investor's budget, and participation constraints, respectively. The last constraint reflects the assumption that the real bond is illiquid in that it cannot be directly used as means of payment in stock-market trades.

Let $V_t\left(\boldsymbol{a}_t, \varepsilon\right)$ denote the maximum expected discounted payoff of an investor with valuation ε and portfolio $\boldsymbol{a}_t$ at the beginning of the first subperiod of period t. In the second subperiod of period t, let $\boldsymbol{\phi}_t \equiv \left(\phi_t^b, \phi_t^m, \phi_t^s\right)$, where ϕ_t^b is the real price of a newly issued government bond, ϕ_t^m, is the real price of a unit of money, and ϕ_t^s is the real price of an equity share (all in terms of good 2). At the beginning of the second subperiod the investor solves

$$W_t\left(\boldsymbol{a}_t, \varpi_t\right) = \max_{(y_t, h_t, \boldsymbol{a}_{t+1}) \in \mathbb{R}_+^5} \left[y_t - h_t + \beta \int V_{t+1}\left(\boldsymbol{a}_{t+1}, \varepsilon\right) dG(\varepsilon) \right] \tag{2}$$

$$\text{s.t. } y_t + \boldsymbol{\phi}_t \boldsymbol{a}_{t+1} \leq \boldsymbol{\phi}_t' \boldsymbol{a}_t + h_t - \varpi_t + T_t,$$

where y_t is consumption of good 2, h_t is the disutility of labor, $\boldsymbol{a}_{t+1} \equiv (a_{t+1}^b, a_{t+1}^m, a_{t+1}^s)$, $\boldsymbol{\phi}_t' \equiv (1, \phi_t^m, \phi_t^s)$, and $T_t \in \mathbb{R}$ is the real value of the lump-sum monetary transfer. The value function of an investor who enters the first subperiod of t with portfolio $\boldsymbol{a}_t$ and valuation ε is

$$\begin{aligned} V_t\left(\boldsymbol{a}_t, \varepsilon\right) &= \alpha \left\{ \varepsilon z \overline{a}_t^s\left(\boldsymbol{a}_t, \varepsilon\right) + W_t\left[\overline{\boldsymbol{a}}_t'\left(\boldsymbol{a}_t, \varepsilon\right), \varpi_t\left(\boldsymbol{a}_t, \varepsilon\right)\right] \right\} \\ &\quad + (1-\alpha)\left\{ \varepsilon z a_t^s + W_t\left[\boldsymbol{a}_t'\left(\boldsymbol{a}_t\right), 0\right] \right\}, \end{aligned} \tag{3}$$

where $\overline{\boldsymbol{a}}_t'\left(\boldsymbol{a}_t, \varepsilon\right) \equiv \left(\overline{a}_t^b\left(\boldsymbol{a}_t, \varepsilon\right), \overline{a}_t^m\left(\boldsymbol{a}_t, \varepsilon\right), \pi(1-\delta)\overline{a}_t^s\left(\boldsymbol{a}_t, \varepsilon\right)\right)$ and $\boldsymbol{a}_t'\left(\boldsymbol{a}_t\right) \equiv \left(a_t^b, a_t^m, \pi(1-\delta)a_t^s\right)$.

Let $J_t\left(\boldsymbol{b}_t\right)$ denote the maximum expected discounted payoff at the beginning of the second subperiod of period t, of an entrepreneur who currently has balance sheet $\boldsymbol{b}_t \equiv \left(a_t^b, k_t, s_t\right)$, composed of (claims to) a_t^b units of good 2, installed capital k_t, and s_t outstanding equity claims on installed capital. The value function satisfies

$$J_t(\boldsymbol{b}_t) = \max_{y_t, a_{t+1}^b, e_t, x_t} \{y_t + \beta [\varepsilon_e z(k_{t+1} - s_{t+1}) + \pi J_{t+1}(\boldsymbol{b}_{t+1})]\} \tag{4}$$

$$\begin{aligned} \text{s.t. } y_t + \mathrm{C}(x_t/k_t) k_t + \phi_t^b a_{t+1}^b &\leq \phi_t^s e_t + a_t^b && (5)\\ k_{t+1} &= (1-\delta) k_t + x_t && (6)\\ s_{t+1} &= (1-\delta) s_t + e_t && (7)\\ s_{t+1} &\in [0, k_{t+1}] && (8)\\ y_t, a_{t+1}^b &\in \mathbb{R}_+, && (9) \end{aligned}$$

where $\boldsymbol{b}_{t+1} \equiv (a_{t+1}^b, k_{t+1}, s_{t+1})$, y_t denotes consumption of good 2, x_t is the quantity of newly created capital, and e_t is the number of newly issued equity shares. Condition (5) is the entrepreneur's budget constraint (expressed in terms of good 2), while (6) and (7) are the laws of motion for the stock of installed capital and outstanding equity shares on the entrepreneur's installed capital, respectively. The condition $0 \leq s_{t+1}$ in (8) states that an entrepreneur cannot buy claims on her own dividend of good 1 issued by other agents.[11] The condition $s_{t+1} \leq k_{t+1}$ in (8) states that entrepreneurs cannot sell claims on capital that are not backed by capital owned by the entrepreneur, i.e., equity issuance must satisfy $e_t \leq x_t + (1-\delta)(k_t - s_t)$. The nonnegativity constraints in (9) rule out negative consumption of good 2, and short positions in the government bond.[12] Let the function $\boldsymbol{g}_t : \mathbb{R}_+^3 \to \mathbb{R}_+^2 \times \mathbb{R}^2$ denote the optimal decision rule implied by (4), i.e., $\boldsymbol{g}_t(\boldsymbol{b}_t) \equiv (g_t^y(\boldsymbol{b}_t), g_t^b(\boldsymbol{b}_t), g_t^e(\boldsymbol{b}_t), g_t^x(\boldsymbol{b}_t))$ gives the entrepreneur's optimal choices of second-subperiod consumption, bond holdings, equity issuance, and investment, as functions of the initial balance sheet, $\boldsymbol{b}_t$. Then, conditional on survival, the optimal path for the entrepreneur's balance sheet is described by $\boldsymbol{b}_{t+1} = \bar{\boldsymbol{g}}_t(\boldsymbol{b}_t) \equiv (\bar{g}_t^b(\boldsymbol{b}_t), \bar{g}_t^k(\boldsymbol{b}_t), \bar{g}_t^s(\boldsymbol{b}_t))$, with $\bar{g}_t^b(\boldsymbol{b}_t) \equiv g_t^b(\boldsymbol{b}_t)$, $\bar{g}_t^k(\boldsymbol{b}_t) \equiv (1-\delta) k_t + g_t^x(\boldsymbol{b}_t)$, and $\bar{g}_t^s(\boldsymbol{b}_t) \equiv (1-\delta) s_t + g_t^e(\boldsymbol{b}_t)$.

Let $j \in \{E, I\}$ denote the agent type, i.e., "E" for entrepreneurs and "I" for investors, and let $h \in \{b, m, s\}$ denote the type of financial asset, i.e., "b" for bonds, "m" for money,

[11]Equivalently, with (7) the constraint $0 \leq s_{t+1}$ can be written as $-(1-\delta) s_t \leq e_t$, i.e., the entrepreneur can buy back her own equity shares, but cannot buy back more than the quantity of shares outstanding.

[12]The formulation (4) assumes an entrepreneur does not hold money. This assumption merely simplifies the exposition. In this environment, entrepreneurs are not involved in transactions for which money is used as a medium of exchange, so we can anticipate they will never choose to carry cash given they have the option to hold interest-bearing government bonds. In our empirical work we will combine cash and "money-like" short-term financial investments (such as Treasuries) into a single asset category called *liquid assets*.

and "s" for equity shares. Then let A^h_{It} denote the quantity of financial asset h held by all investors at the beginning of period t. That is, $A^h_{It} = \int a^h_t dF_{It}(\boldsymbol{a}_t)$, where F_{It} is the cumulative distribution function over portfolios $\boldsymbol{a}_t = (a^b_t, a^m_t, a^s_t)$ held by investors at the beginning of period t. Similarly, let $\bar{F}_{Et}$ denote the joint cumulative distribution function over entrepreneur's balance sheets, $\boldsymbol{b}_t = (a^b_t, k_t, s_t)$, at the beginning of the second subperiod of period t. Let A^b_{Et} denote the quantity of bonds held by entrepreneurs at the beginning of period t. Let K_t and S_t denote the beginning-of-period t capital stock managed by all entrepreneurs, and outstanding equity claims on all installed capital, respectively. Then, we have the beginning-of-period t aggregates, $A^b_{Et} = \int a^b_t dF_{Et}(\boldsymbol{b}_t)$, $K_t = \int k_t dF_{Et}(\boldsymbol{b}_t)$, and $S_t = \int s_t dF_{Et}(\boldsymbol{b}_t)$, where F_{Et} is the cumulative distribution function over balance sheets $\boldsymbol{b}_t \equiv (a^b_t, k_t, s_t)$ held by entrepreneurs at the beginning of period t. Let $\bar{A}^m_{It}$ and $\bar{A}^s_{It}$ denote the quantities of money and shares held after the first-subperiod round of trade of period t by all the investors who are able to trade in the first subperiod. Then we have $\bar{A}^h_{It} = \alpha \int \overline{a}^h_t(\boldsymbol{a}_t, \varepsilon) dH_{It}(\boldsymbol{a}_t, \varepsilon)$ for $h \in \{m, s\}$, where H_{It} denotes the joint cumulative distribution of portfolios and valuation shocks across investors at the beginning of period t. We are now ready to define equilibrium.

Definition 1 *An equilibrium is a sequence of prices, $\{\boldsymbol{\phi}_t\}^\infty_{t=0}$, terms of trade in the first-subperiod, $\{\bar{\boldsymbol{a}}_t(\cdot), \varpi_t(\cdot)\}^\infty_{t=0}$, investor end-of-period portfolio choices, $\{\boldsymbol{a}_{t+1}\}^\infty_{t=0}$, decision rules for entrepreneurs, $\{\boldsymbol{g}_t(\cdot)\}^\infty_{t=0}$, and distributions of assets, $\{F_{It}(\cdot), F_{Et}(\cdot)\}^\infty_{t=0}$, such that: (i) the terms of trade $\{\bar{\boldsymbol{a}}_t(\cdot), \varpi_t(\cdot)\}^\infty_{t=0}$ solve (1); (ii) the portfolios $\{\boldsymbol{a}_{t+1}\}^\infty_{t=0}$ solve the individual investor's optimization problem (2), and the decision rules $\{\boldsymbol{g}_t(\cdot)\}^\infty_{t=0}$ solve (4), (iii) the paths of the distributions of assets, $\{F_{It}(\cdot), F_{Et}(\cdot)\}^\infty_{t=0}$, are consistent with the individual portfolio choices and trading decisions; and (iv) prices, $\{\boldsymbol{\phi}_t\}^\infty_{t=0}$, are such that all Walrasian markets clear, i.e., $A^b_{Et+1} + A^b_{It+1} = B_{t+1}$ (the end-of-period t Walrasian bond market clears), $A^m_{It+1} = A^m_{t+1}$ (the end-of-period t Walrasian market for money clears), $A^s_{It+1} = S_{t+1}$ (the end-of-period t Walrasian market for equity clears), $\bar{A}^m_{It} = \alpha A^m_t$ (the market for money in the first subperiod of t clears), and $\bar{A}^s_{It} = \alpha S_t$ (the market for equity in the first subperiod of t clears). An equilibrium is "monetary" if $\phi^m_t > 0$ for all t and "nonmonetary" otherwise.*

2.2 Analytical results

We focus on *stationary equilibria* in which the aggregate supply of equity and aggregate real money balances are constant over time, i.e., $S_t = S$ and $\phi^m_t A^m_t \equiv \mathrm{M}_t = \mathrm{M}$ for all t, and real equity prices are time-invariant linear functions of the dividend, i.e., $\phi^s_t = \phi^s \equiv \varphi^s z$ and

$p_t\phi_t^m = \bar{\varphi}^s z$, for all t.[13] In this section we assume $\pi = 0$ (entrepreneurs live for one period) in order to derive the main theoretical insights analytically.[14]

To characterize the equilibrium it is useful to define the *marginal stock-market valuation* in the first subperiod of t, $\varepsilon_t^* \equiv p_t\phi_t^m/z$, and the *nominal interest rate* between period t and $t+1$,

$$r_{t+1} \equiv \frac{\phi_t^m}{\beta\phi_{t+1}^m} - 1, \tag{10}$$

The marginal valuation ε_t^* is the one that makes an investor indifferent between holding equity or selling it for cash in the first subperiod.[15] In a stationary equilibrium with $\pi = 0$, $\varepsilon_t^* = \varepsilon^* \equiv \bar{\varphi}^s$ for all t. The nominal interest rate r_{t+1} is the nominal yield of a one-period risk-free nominal bond issued in the second subperiod of t and redeemed in the second subperiod of $t+1$ that is illiquid in the sense that it cannot be used to purchase stocks in the first-subperiod of $t+1$. In a stationary equilibrium, $r_{t+1} = r \equiv (\mu - \beta)/\beta$ for all t, so we regard r as the *nominal policy rate*, which can be implemented by changing the growth rate in the money supply, μ.

2.2.1 Solution to the entrepreneur's problem

For an entrepreneur who enters with initial conditions w and k in the context of a stationary equilibrium of an economy with $\pi = 0$, (4)-(9) specialize to

$$J(w, k, 0) = \max_{x,y,s_{+1}} \left[y + \beta\varepsilon_e z(k_{+1} - s_{+1})\right] \tag{11}$$

$$\begin{aligned} \text{s.t. } y + \mathrm{C}(x/k)\,k &\leq \phi^s s_{+1} + w \\ k_{+1} &= (1-\delta)\,k + x \\ s_{+1} &\in [0, k_{+1}] \\ y &\in \mathbb{R}_+. \end{aligned}$$

[13] Intuitively, ϕ_t^s and $p_t\phi_t^m$ are the ex- and cum-dividend real equity prices (in terms of good 2), respectively.

[14] We study the quantitative performance of the more general formulation with $\pi \in [0,1]$ in Section 6.

[15] In general, for $\pi \in [0,1]$, ε_t^* would be defined as $\varepsilon_t^* \equiv (p_t\phi_t^m - \pi(1-\delta)\phi_t^s)/z$. To see the role that this marginal valuation will play in the equilibrium, consider an investor with valuation ε who, in the stock market of the first subperiod of period t, is deciding whether to keep an equity share or sell it for cash. If he keeps the share, his payoff is $\varepsilon z + \pi(1-\delta)\phi_t^s$, namely his valuation of the period dividend, εz, plus the expected value (in terms of good 2) of the share of undepreciated capital in the following subperiod, $\pi(1-\delta)\phi_t^s$. If he sells it for cash, he gets payoff $p_t\phi_t^m$ (i.e., sells the share for p_t dollars, worth ϕ_t^m units of good 2 in the following subperiod). Hence, the investor keeps the share if $\varepsilon z + \pi(1-\delta)\phi_t^s > p_t\phi_t^m$, sells it for cash if $\varepsilon z + \pi(1-\delta)\phi_t^s < p_t\phi_t^m$, and is indifferent if $\varepsilon = \varepsilon_t^*$.

Let $g^x(w,k)$, $g^y(w,k)$, and $g^e(w,k)$ denote the levels of investment, consumption, and equity issuance that solve (11). Define $\iota^* \equiv g^x(w,k)/k$, $\vartheta^* \equiv g^y(w,k)/k$, $\varsigma^*_{+1} \equiv g^e(w,k)/k$, $\omega \equiv w/k$, and $\phi^s_e \equiv \beta\varepsilon_e z$. The following result characterizes $(\iota^*, \vartheta^*, \varsigma^*_{+1})$.

Proposition 1 *Let $\iota(\phi)$ denote the unique number, ι, that solves $\mathrm{C}'(\iota) = \phi$ for any $\phi \in \mathbb{R}_+$. Assume $\delta - \iota_0 \le 1 \le \phi^s$. (i) If $\phi^s_e \le \phi^s$,*

$$\begin{aligned}
\iota^* &= \iota(\phi^s) \\
\varsigma^*_{+1} &= \begin{cases} 1-\delta+\iota^* & \text{if } \phi^s_e < \phi^s \\ \left[\max\left\{0, \frac{\mathrm{C}(\iota^*)-\omega}{\phi^s}\right\}, 1-\delta+\iota^*\right] & \text{if } \phi^s_e = \phi^s. \end{cases}
\end{aligned}$$

(ii) If $\phi^s < \phi^s_e$,

$$\begin{aligned}
\iota^* &= \begin{cases} \iota(\phi^s_e) & \text{if } \mathrm{C}(\iota(\phi^s_e)) \le \omega \\ \mathrm{C}^{-1}(\omega) & \text{if } \mathrm{C}(\iota(\phi^s)) < \omega < \mathrm{C}(\iota(\phi^s_e)) \\ \iota(\phi^s) & \text{if } \omega \le \mathrm{C}(\iota(\phi^s)) \end{cases} \\
\varsigma^*_{+1} &= \begin{cases} 0 & \text{if } \mathrm{C}(\iota(\phi^s)) < \omega \\ \frac{\mathrm{C}(\iota(\phi^s))-\omega}{\phi^s} & \text{if } \omega \le \mathrm{C}(\iota(\phi^s)) \end{cases}
\end{aligned}$$

In every case, $\vartheta^* = \omega + \phi^s\varsigma^*_{+1} - \mathrm{C}(\iota^*)$.

In Proposition 1, ϕ^s_e represents the entrepreneur's marginal private value of capital, while ϕ^s represents the marginal market value of capital to the outside investors who price the entrepreneur's equity. Part (i) focuses on the case in which the market valuation of the marginal capital investment is higher than the entrepreneur's. In this case, the entrepreneur chooses the investment rate, ι^*, so that the marginal cost, $\mathrm{C}'(\iota^*)$, equals the market value of the marginal investment, ϕ^s. Moreover, because the entrepreneur's valuation is lower than the market valuation, the entrepreneur issues equity shares on any capital she owns at the beginning of the period, and finances new investment entirely by equity issuance, i.e., she chooses $\varsigma^*_{+1}k = (1-\delta+\iota^*)k$. (In the knife-edge case with $\phi^s_e = \phi^s$, the entrepreneur is indifferent between financing by equity issuance or out of her own funds, ωk.)

Part (ii) of Proposition 1 focuses on the case in which the entrepreneur's valuation of the marginal capital investment is higher than the market valuation, i.e., $\phi^s < \phi^s_e$. In this case, the investment, financing, and consumption decisions of the entrepreneur depend on her own valuation of investment, on the market valuation, and on the entrepreneur's financial wealth, represented by the ω endowment of good 2. First, if $\mathrm{C}(\iota(\phi^s_e)) \le \omega$, the entrepreneur is financially unconstrained: she chooses her first-best investment rate, $\iota(\phi_e^s)$ (the ι^* that equates the marginal cost of investment, $\mathrm{C}'(\iota^*)$, to her own marginal valuation, ϕ_e^s), finances it entirely with her own funds, i.e., $\varsigma_{+1}^* = 0$ (issues no equity), and consumes the unspent wealth, i.e., sets $\vartheta^* = \omega - \mathrm{C}(\iota(\phi_e^s))$. On the opposite extreme, if the entrepreneur's own financial wealth is very low, specifically $\omega \leq \mathrm{C}(\iota(\phi^s))$, i.e., lower than what would be needed to self-finance the level of investment that would be chosen based on outside investors' marginal valuation of investment, ϕ^s, then she chooses the investment rate $\iota(\phi^s)$ (the ι^* that equates the marginal cost of investment, $\mathrm{C}'(\iota^*)$, to the market valuation, ϕ^s), uses all of her own funds to finance part of the investment (sets $\vartheta^* = 0$), and resorts to equity issuance to finance the rest. Finally, if the entrepreneur's financial wealth is too low to self-finance her first-best investment rate but high enough to self-finance the investment rate that would be chosen based on outside investor's valuations, i.e., if $\mathrm{C}(\iota(\phi^s)) < \omega < \mathrm{C}(\iota(\phi_e^s))$, then the entrepreneur invests the maximum that can be financed with her internal funds, i.e., the investment rate ι^* that satisfies $\mathrm{C}(\iota^*) = \omega$, sets $\vartheta^* = 0$, and issues no equity.

2.2.2 Equilibrium stock prices, investment, and capital structure

For what follows, let $\iota^*(\omega)$ and $\varsigma_{+1}^*(\omega)$ denote the optimal investment and equity issuance decisions (normalized by the firm's capital stock) taken by an entrepreneur who enters with a ratio of financial wealth to physical capital equal to ω, as characterized in Proposition 1. With this notation, we can write the aggregate investment chosen by all active entrepreneurs at the end of a period as

$$X^* = \int \iota^*(\omega)\, k_0 d\Omega(\omega), \tag{12}$$

and the aggregate stock of equity shares outstanding at the beginning of a period as

$$S^* = \int \varsigma_{+1}^*(\omega)\, k_0 d\Omega(\omega). \tag{13}$$

For the remainder of this section, we assume $\delta - \iota_0 \leq 1 \leq \underline{\phi}^s$, where $\underline{\phi}^s \equiv \beta\bar{\varepsilon}z$. The following proposition characterizes the monetary equilibrium.[16] Before stating the result, it is convenient to define $\overline{\phi}^s \equiv \beta\left[\bar{\varepsilon} + \alpha\theta\left(\varepsilon_H - \bar{\varepsilon}\right)\right]z$ and $\bar{r} \equiv \alpha\theta\left(\bar{\varepsilon} - \varepsilon_L\right)/\varepsilon_L$.

[16] Proposition 3 (in Appendix A) characterizes the nonmonetary equilibrium.

Proposition 2 *Assume $r \in (0, \bar{r})$. (i) There exists a unique stationary monetary equilibrium. (ii) The equity price is*

$$\phi^s(r) = \beta\left[\bar{\varepsilon} + \eta \int_{\varepsilon_L}^{\varepsilon^*} (\varepsilon^* - \varepsilon)\, dG(\varepsilon)\right] z, \tag{14}$$

where $\eta \equiv \alpha\theta$ and $\varepsilon^* \in (\varepsilon_L, \varepsilon_H)$ is the unique solution to

$$\eta \int_{\varepsilon^*}^{\varepsilon_H} \frac{\varepsilon - \varepsilon^*}{\varepsilon^*} dG(\varepsilon) = r. \tag{15}$$

(iii) If $\phi_e^s \in (\underline{\phi}^s, \overline{\phi}^s)$, let $\hat{r} \in (0, \bar{r})$ be defined by $\phi^s(\hat{r}) = \phi_e^s$. Then: (a) If $r \in (0, \hat{r})$, then $X^* = \iota(\phi^s(r))k_0$, and $S^* = [1 - \delta + \iota(\phi^s(r))]\, k_0$. (b) If $r \in (\hat{r}, \bar{r})$, then

$$X^* = \Omega[\mathrm{C}(\iota(\phi^s(r)))]\iota(\phi^s(r))k_0 + \int_{\mathrm{C}(\iota(\phi^s(r)))}^{\mathrm{C}(\iota(\phi_e^s))} \mathrm{C}^{-1}(\omega) d\Omega(\omega)\, k_0 + \{1 - \Omega[\mathrm{C}(\iota(\phi_e^s))]\}\iota(\phi_e^s)k_0,$$

and

$$S^* = \frac{1}{\phi^s(r)} \int_0^{\mathrm{C}(\iota(\phi^s(r)))} [\mathrm{C}(\iota(\phi^s(r))) - \omega] k_0 d\Omega(\omega).$$

(iv) If $\phi_e^s < \underline{\phi}^s$, X^* and S^* are as in part (iii)(a). (v) If $\overline{\phi}^s < \phi_e^s$, X^* and S^* are as in part (iii)(b). (vi) In every case, aggregate real money balances are given by $\mathrm{M} = \frac{G(\varepsilon^*)\varepsilon^* z}{1-G(\varepsilon^*)} S^*$.

Parts (*i*), (*iii*), (*iv*), and (*v*) of Proposition 2 establish existence and uniqueness of the stationary monetary equilibrium, and describe how the sign of $\phi^s(r) - \phi_e^s$ depends on the primitives of the economy. Parts (*ii*) and (*vi*) give analytical expressions for the equilibrium equity price and real money balances, respectively. The equity price (14) can be decomposed into a term equal to the expected discounted *dividend*, i.e., $\beta\bar{\varepsilon}z$, and a term equal to the expected discounted value of a *resale option*, i.e., $\mathcal{R}(r, \eta) \equiv \beta\eta \int_{\varepsilon_L}^{\varepsilon^*(r,\eta)} [\varepsilon^*(r, \eta) - \varepsilon]\, dG(\varepsilon)z$, where $\varepsilon^*(r, \eta)$ denotes the ε^* that solves (15). The resale option represents an investor's expected gain from reselling an equity share in the first-subperiod stock market in the event that her realized valuation of the dividend is lower than the market valuation (i.e., $\varepsilon < \varepsilon^*$). The following corollary of Proposition 2 summarizes how the equity price and the firm's investment and equity issuance decisions respond to changes in the monetary policy rate, r.

Corollary 1 *In the stationary monetary equilibrium: (i) As $r \to \bar{r}$, $\mathrm{M} \to 0$, and $\phi^s \to \underline{\phi}^s$. (ii) As $r \to 0$, $\phi^s \to \overline{\phi}^s$. (iii) $d\varepsilon^*/dr < 0$ and $d\phi^s(r)/dr < 0$. (iv) $\partial\iota(\phi^s(r))/\partial r < 0 < \partial\iota(\phi^s)/\partial\phi^s$. (v) If $\phi_e^s < \phi^s$, then $\partial\varsigma_{+1}^*(\omega)/\partial r < 0 < \partial\varsigma_{+1}^*(\omega)/\partial\phi^s$. If $\phi^s < \phi_e^s$ and $\mathrm{C}(\cdot)$ is log concave, then $\partial\varsigma_{+1}^*(\omega)/\partial r < 0 < \partial\varsigma_{+1}^*(\omega)/\partial\phi^s$ for all $\omega \leq \mathrm{C}(\iota(\phi^s))$. (vi) $\partial^2\phi^s(r)/(\partial r\partial\eta) < 0$, where $\eta \equiv \alpha\theta$.*

Part (i) of Corollary 1 states that as the opportunity cost of holding money (represented by the policy rate r) approaches $\bar{r}$, the monetary equilibrium of Proposition 2 converges to the nonmonetary equilibrium (characterized in Proposition 3, Appendix A). Part (ii) is a version of the celebrated *Friedman rule*: as monetary policy drives the opportunity cost of holding money toward zero, investors' liquidity needs are satiated, which implies the equilibrium equity price is set by the highest investor valuation. Part (iii) complements parts (i) and (ii) by showing that the valuation of the marginal investor and the market price of equity are decreasing in the policy rate r.[17] Part (iv) shows that if the marginal value of the entrepreneur's investment is determined by the market price of the stock, then increases in the stock price, ϕ^s, stimulate investment, while increases in the nominal policy rate, r, discourage investment. Part (v) provides conditions such that increases in the nominal policy rate discourage equity issuance through their effect on the equity price. Part (vi) states that the magnitude of the equity-price response to changes in the policy rate is increasing in the liquidity of the stock (e.g., as measured by the parameter α, which determines the frequency of trade of the stock).[18]

3 Implications of the theory

The model presented in Section 2 consists of two blocks. The first is a *financial block* (described in part (ii) of Proposition 2) that determines the firm's equity price as function of: (a) firm parameters (such as productivity, z), (b) investor parameters (such as the distribution of idiosyncratic valuations, G), (c) the financial marketstructure where the firm's equity trades (the parameters α and θ), and (d) the nominal policy rate (the parameter r). In the theory, the nominal policy rate affects the real equity price only through what Lagos and Zhang (2020b) labeled the *turnover-liquidity transmission mechanism*, which we will refer to as the *turnover channel*, for brevity.[19] The second is an *investment block* (described in parts (iii)-(vi) of Proposition 2) that determines a firm's investment and equity issuance decisions as functions of the equity price determined in the financial block. From this investment block we learn that the invest-

[17]Lagos and Zhang (2020b) explain the mechanism in detail. Intuitively, the increase in the policy rate represents an increase in the opportunity cost of holding the monetary asset used to settle the equity trades in the first subperiod. The marginal valuation ε^* is lower under the higher opportunity cost, reflecting the fact that the investor who was indifferent between holding money and equity at the lower rate now prefers tilting the portfolio toward equity.

[18]This result is analogous to the one in part (iii) of Proposition 6 in Lagos and Zhang (2020b).

[19]Lagos and Zhang (2020b) use the longer terminology to emphasize the fact that the strength of this transmission mechanism depends on the marketstructure parameter α—a key determinant of the equity *turnover rate*, which is a standard measure of financial liquidity.

ment and equity issuance decisions of firms with certain characteristics (e.g., low ω) respond to market-driven variations in their equity prices.[20] The *q-channel* is the theoretical mechanism that transmits financial-market-driven changes in a firm's equity price to its investment and equity issuance decisions.

In the remainder of this section we discuss the implications of the theory that will guide the empirical analysis in Section 4. Section 3.1 explains the causal relationship that runs from Tobin's q to a firm's choices of investment and equity issuance, which defines the *q-channel*. Section 3.2 reviews the causal relationship that runs from the interaction between monetary policy and financial-market turnover to a firm's equity price, which defines the *turnover channel*. In Section 4 we propose a theory-based empirical identification strategy for the q-channel that relies on the observation that the turnover channel implies the turnover of a firm's stock systematically affects the responsiveness of the stock price to money shocks.

3.1 Tobin's q, investment, and capital structure: *the q-channel*

The following corollary of Proposition 1 establishes the conditions under which the marginal value of capital that the entrepreneur uses to make the optimal investment decision, which we denote q^*, is equal to *Tobin's q*, which in this model equals the stock-market price of a claim to the dividends from a unit of capital installed in the firm (i.e., ϕ^s).

Corollary 2 *The entrepreneur always chooses an investment rate, ι^*, that satisfies $\mathrm{C}'(\iota^*) = q^*$, with $q^* = \phi^s$ if $\phi^s_e \leq \phi^s$, or with*

$$q^* = \begin{cases} \phi^s_e & \text{if } \mathrm{C}(\iota(\phi^s_e)) \leq \omega \\ \mathrm{C}'(\mathrm{C}^{-1}(\omega)) & \text{if } \mathrm{C}(\iota(\phi^s)) < \omega < \mathrm{C}(\iota(\phi^s_e)) \\ \phi^s & \text{if } \omega \leq \mathrm{C}(\iota(\phi^s)) \end{cases}$$

if $\phi^s < \phi^s_e$.

In a well-known proposition, Hayashi (1982) showed that for a competitive firm with constant returns to scale in both production and installation, the marginal value of capital that the firm uses to make the optimal investment decision, which Hayashi refers to as *marginal q*, is equal to the ratio of the market value of the installed capital to the replacement cost of capital, i.e., equal to *Tobin's q*, which Hayashi refers to as *average q*. Corollary 2 is a version of this

[20] By "market-driven variations" we mean changes in the equity price that are not driven by changes in firm-level parameters. In our theory, market-driven variations may be due to changes in investor-level parameters, market-structure parameters, or policy parameters.

proposition for our model, which differs from Hayashi's more traditional neoclassical model in two ways. First, we allow for heterogeneous valuations of the fundamental marginal revenue of capital installed inside the firm: these valuations may differ across investors as well as between investors and the entrepreneur who runs the firm. Second, firms in our model face financing constraints that may affect investment decisions.

In Corollary 2 we define q^* as the marginal value of capital that the entrepreneur uses to make the firm's optimal investment decision, so the optimal choice of investment rate, ι^*, always satisfies $\textsc{c}'(\iota^*) = q^*$. Thus, q^* corresponds to what Hayashi refers to as *marginal* q in his neoclassical interpretation of Keynes and Tobin (e.g., Keynes (1936) and Tobin (1969)). In our model, the market price of k units of capital installed in a firm is $\phi^s k$ (expressed in terms of good 2), and the replacement cost of k units of capital is k (also in terms of good 2), so *Tobin's* q (what Hayashi refers to as *average* q) is equal to ϕ^s.

The main takeaway from Corollary 2 that will guide our empirical analysis in Section 4 is that, unless $\phi^s < \phi^s_e$ and $\textsc{c}(\iota(\phi^s)) < \omega$, the firm's investment and equity issuance decisions depend on the market price of equity (Tobin's q). For firms run by entrepreneurs whose valuation of marginal investment is lower than the market valuation, as in part (i) of Proposition 1, the relationship is simple: regardless of the firm's balance sheet, a higher stock price induces the firm to increase the investment rate and finance it with equity issuance. For firms run by entrepreneurs whose valuation of marginal investment is higher than the market valuation, as in part (ii) of Proposition 1, the relationship is more nuanced. On the one hand, investment and equity issuance are increasing in the equity price for firms run by entrepreneurs who are sufficiently financially constrained, in the sense that $\omega \leq \textsc{c}(\iota(\phi^s))$. On the other hand, investment and equity issuance decisions do not respond to market-driven variation in Tobin's q for firms run by entrepreneurs who are financially unconstrained, in the sense that $\textsc{c}(\iota(\phi^s)) < \omega$.

To summarize, according to the theory, firms can be classified as *equity dependent*, or as *not equity dependent*. The latter are firms that do not rely on equity issuance to finance investment (in Corollary 2, these are the firms with $\phi^s < \phi^s_e$ and $\textsc{c}(\iota(\phi^s)) < \omega$ that finance all their investment with internal funds). The equity-dependent firms are firms that finance at least some of their investment by issuing equity in the open market, and therefore their equity issuance and investment decisions are influenced by changes in Tobin's q (in Corollary 2, these are the firms with $\phi^s_e < \phi^s$, or $\phi^s < \phi^s_e$ and $\omega \leq \textsc{c}(\iota(\phi^s))$). In the empirical analysis of

Section 4, we will interpret the data through the lens of a theoretical equilibrium with $\phi^s < \phi^s_e$.[21] Accordingly, we will use a firm's *liquidity ratio* (defined as the proportion of liquid assets relative to total assets) as the empirical counterpart of ω, and will interpret a relatively low liquidity ratio as an indicator that the firm is *equity dependent*.[22]

3.2 Monetary policy, market liquidity, and Tobin's q: *the turnover channel*

Conditions (14) and (15) define the equilibrium equity price as a function of the policy rate, r, and the marketstructure parameters, $\eta \equiv \alpha\theta$. This relationship can be approximated by

$$\mathrm{q} \approx \tilde{\mathrm{q}} + \left(\alpha^{\mathrm{q}}_{\eta} + \alpha^{\mathrm{q}}_{\eta\eta}\eta + \alpha^{\mathrm{q}}_{r\eta} r\right)\eta, \tag{16}$$

where $\mathrm{q} \equiv \log \phi^s$, $\tilde{\mathrm{q}} \equiv \bar{\mathrm{q}} + \frac{\beta z}{e^{\bar{\mathrm{q}}}}\bar{\varepsilon} - 1$, $\bar{\mathrm{q}} \equiv \ln\{\beta z\left[\bar{\varepsilon} + \eta_0 I_S\left(\varepsilon^*_0\right)\right]\}$, $I_S\left(\varepsilon^*_0\right) \equiv \int_{\varepsilon_L}^{\varepsilon^*_0}\left(\varepsilon^*_0 - \varepsilon\right)dG(\varepsilon)$, $\alpha^{\mathrm{q}}_{\eta} \equiv \frac{\beta z}{e^{\bar{\mathrm{q}}}} I_S\left(\varepsilon^*_0\right)$, $\alpha^{\mathrm{q}}_{\eta\eta} \equiv -\frac{r_0}{\eta_0}\alpha^{\mathrm{q}}_{r\eta}$, $\alpha^{\mathrm{q}}_{r\eta} \equiv -\frac{\beta z}{e^{\bar{\mathrm{q}}}}\frac{G\left(\varepsilon^*_0\right)\varepsilon^*_0}{r_0+\eta_0\left[1-G\left(\varepsilon^*_0\right)\right]}$, and ε^*_0 is implicitly defined by $\eta_0\int_{\varepsilon^*_0}^{\varepsilon_H}\frac{\varepsilon-\varepsilon^*_0}{\varepsilon^*_0}dG(\varepsilon) = r_0$ for some (r_0, η_0) near (r, η). When we take the model to the data, we will associate variation in η in the theory with empirical cross-stock variation in the *turnover rate*, $\mathcal{T}$.[23] There are two main takeaways from (16). First, in line with part (*iii*) of Corollary 1, the

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[21] In Appendix B we incorporate a simple agency problem between entrepreneurs and investors to show that, in order to have an equilibrium with $\phi^s < \phi^s_e$, one need not assume parametrizations where the fundamental value of the investment is higher for entrepreneurs than for outside investors, since the agency problem makes outside equity a relatively more costly source of financing than inside equity, as proposed by the so-called *pecking-order theory* (e.g., Myers and Majluf (1984)).

[22] In general, according to the theory, this operational definition of "equity dependence" based only on ω can be too restrictive, as it may misclassify some equity-dependent firms as *not* equity dependent (e.g., firms with relatively high ω but $\phi^s_e < \phi^s$). For parametrizations that lead to equilibrium with $\phi^s < \phi^s_e$, however, the theory does imply the size of ω is a sufficient statistic for equity dependence. In the data we find that firms with relatively high liquidity ratios behave as not-equity-dependent firms, and firms with low liquidity ratios behave as equity-dependent firms (see the results in Section 4.2 and Section 4.3, in particular Figure 2 and Figure 4). Therefore, when mapping our theoretical predictions to the data, we regard either $\phi^s_e \in (\underline{\phi}^s, \overline{\phi}^s)$ with $r \in (\hat{r}, \bar{r})$, or $\overline{\phi}^s < \phi^s_e$, as the empirically relevant parametrizations, since both imply $\phi^s < \phi^s_e$ in equilibrium (Proposition 2). To see why these parametrizations are the only ones consistent with the effect of the liquidity ratio on the firms' investment and equity issuance behavior described in the data, recall that if the parametrization is either $\phi^s_e \in (\underline{\phi}^s, \overline{\phi}^s)$ with $r \in (0, \hat{r})$, or $\phi^s_e < \underline{\phi}^s$ then the theory predicts *all* firms are equity dependent, even those with very high values of ω, which is counterfactual. The parametric restriction $\phi^s_e < \overline{\phi}^s$ implies there exist some outside investors who value investment sufficiently more than the entrepreneurs. In our quantitative model (Section 6), we will assume ε is lognormally distributed, so $\varepsilon_L = 0$ and $\varepsilon_H = +\infty$, and therefore $\overline{\phi}^s = +\infty$, $\bar{r} = +\infty$, which guarantee $\phi^s_e < \overline{\phi}^s$ and $r < \bar{r}$. The condition $\hat{r} < r$ that requires a "high enough" nominal rate can be restated as a condition that is satisfied provided the entrepreneur's valuation, ε_e, is high enough for any given $r > 0$. The parametric condition $\underline{\phi}^s < \phi^s_e$ is equivalent to assuming that the *average* valuation that outside investors asign to investment is lower than the entrepreneur's valuation.

[23] The *turnover rate* of a stock is defined as the ratio between the number of outstanding shares that are traded in a given time period and the total number of outstanding shares. From Lemma 1, all financial investors with $\varepsilon < \varepsilon^*$ who have a trading opportunity in the first subperiod sell all their equity holding, so the turnover

log of Tobin's q is decreasing in the policy rate, i.e., $\frac{\partial \mathrm{q}}{\partial r} \approx \alpha^{\mathrm{q}}_{r\eta}\eta < 0$. Second, in line with part (vi) of Corollary 1, the marginal effect of the policy rate on the log of Tobin's q is stronger for equity shares that have a higher turnover rate (equivalently, higher η), i.e., $\frac{\partial^2 \mathrm{q}}{\partial \eta \partial r} \approx \alpha^{\mathrm{q}}_{r\eta} < 0$.[24] This theoretical prediction is the hallmark of the *turnover channel*—and will be the basis for our empirical identification strategy.[25]

3.3 Identification

In the theory of Section 2, monetary policy only affects investment and capital structure through its effect on the stock prices of equity-dependent firms. Thus, with data generated by the model, we could identify the *q-channel* of monetary transmission (i.e., the causal effect of monetary-policy induced changes in Tobin's q on the outcome variable) simply by regressing changes in the outcome variable on the changes in Tobin's q induced by monetary-policy shocks. However, this way of estimating the q-channel with actual data is problematic because we cannot rule out the possibility that monetary-policy shocks operate through other transmission variables that may affect both the outcome variable and Tobin's q concurrently. Next, we formalize this identification problem and propose a strategy to address it.[26]

For firm i in period t, let Y^i_t denote the *outcome variable* of interest (e.g., the firm's investment rate, or its equity issuance), which may be affected by D *transmission variables,* $\boldsymbol{v}^i_t \equiv \left(v^i_{1t}, ..., v^i_{Dt}\right) \in \mathbb{R}^D$. To make this dependence explicit, write the outcome variable as a function of the transmission variables, i.e., $Y^i_t = Y\left(\boldsymbol{v}^i_t\right)$. In our application, the first transmission variable, $v^i_{1t} \equiv \mathrm{q}^i_t$, will be a measure of firm i's Tobin's q.[27] In turn, each transmission variable $j \in \{1, ..., D\}$ is a function of the policy rate r_t and a vector of N *predetermined firm-*

rate for a firm's stock is $\mathcal{T} = \alpha G(\varepsilon^*)$, which is strictly increasing in α (and in θ). Hence, the theory implies a one-to-one relationship between η and $\mathcal{T}$. In a model similar to our *financial block*, Lagos and Zhang (2020b) show that cross-stock variation in $\mathcal{T}$ induced by cross-stock variation in G would have similar implications for the cross-sectional variation in stock prices and stock turnover as cross-stock variation in α.

[24]To gain some intuition for this result recall that in the theory, the policy rate only affects the equity price by reducing the value of the resale option, $\mathcal{R}(r, \eta)$. If η close to zero, the resale option is small, and the equity price barely responds to changes in the policy rate. Conversely, a high probability of retrading (α) and a high share of the gain from retrading (θ) make this transmission channel strong.

[25]Lagos and Zhang (2020b) document that this theoretical prediction holds at high frequency (daily) for various sortings of stocks into turnover classes. In Section 4 we reconfirm that it holds at quarterly frequency and for a different sorting of stocks.

[26]See Appendix C for more detailed derivations and proofs.

[27]Other elements of $\boldsymbol{v}^i_t$ could represent other firm-specific transmission variables, such as firm i's borrowing cost or the demand for its output, as well as marketwide transmission variables such as a baseline real interest rate, or other macro variables relevant for the firm's investment or capital-structure decisions.

level characteristics, $\boldsymbol{\kappa}^i \equiv \left(\kappa_1^i, ..., \kappa_N^i\right) \in \mathbb{R}^N$, i.e., $v_{jt}^i = v_j\left(r_t, \boldsymbol{\kappa}^i\right)$. In our application, the first characteristic, $\kappa_1^i \equiv \mathcal{T}^i$, represents the turnover rate of firm i's stock.[28]

Suppose that from period $t-1$ to period t the policy rate changes from r_{t-1} to $r_t = r_{t-1} + \varepsilon_t^m$, where ε_t^m represents an unexpected policy shock. First-order approximations to the function $v_j(\cdot)$ around the point $(\bar{r}, \bar{\boldsymbol{\kappa}}) \in \mathbb{R}^{N+1}$ (we use $\bar{\mathcal{T}}$ to denote $\bar{\kappa}_1$), and to the function $Y(\cdot)$ around the point $\bar{\boldsymbol{v}} \equiv \boldsymbol{v}(\bar{r}, \bar{\boldsymbol{\kappa}}) \in \mathbb{R}^D$ imply

$$Y_t^i - Y_{t-1}^i \approx \gamma^{\mathrm{q}}(\mathrm{q}_t^i - \mathrm{q}_{t-1}^i) + u_t^i, \tag{17}$$

where $u_t^i \equiv \sum_{j=2}^D \gamma^j (v_{jt}^i - v_{jt-1}^i) = \sum_{j=2}^D \gamma^j \alpha_r^j \varepsilon_t^m$, with $\gamma^j \equiv \partial Y(\bar{\boldsymbol{v}}) / \partial v_j$, $\alpha_r^j \equiv \partial v_j(\bar{r}, \bar{\boldsymbol{\kappa}}) / \partial r$ for $j \in \{1, ..., D\}$, and $\gamma^1 \equiv \gamma^{\mathrm{q}}$. Intuitively, the coefficient α_r^j quantifies the first-order effect of a marginal increase in the policy rate on transmission variable j, and γ^j quantifies the first-order effect of a marginal increase in transmission variable j on the outcome variable. Since we are interested in estimating γ^{q}, a natural empirical strategy suggested by the specification (17) would be to use the money shock, ε_t^m, as an instrument for $\mathrm{q}_t^i - \mathrm{q}_{t-1}^i$ to identify the policy-driven variation in the stock price. Our concern with this approach, however, is that it would be difficult to argue that the instrument ε_t^m satisfies the *exclusion restriction*, i.e., that there is no correlation between the money shock, ε_t^m, and the residual, u_t^i. Notice that since $cov\left(\varepsilon_t^m, u_t^i\right) = var\left(\varepsilon_t^m\right) \sum_{j=2}^D \gamma^j \alpha_r^j$, we have $cov\left(\varepsilon_t^m, u_t^i\right) = 0$ if and only if $\gamma^j \alpha_r^j = 0$ for all $j \in \{2, ..., D\}$. In words: the exclusion restriction is satisfied as long as the monetary shock has no effect on the outcome variable through transmission variables other than Tobin's q. That is, the identifying assumption is that for all transmission variables $j \in \{2, ..., D\}$, either the money shock has not effect on transmission variable j (i.e., $\alpha_r^j = 0$), or transmission variable j has no effect on the outcome variable (i.e., $\gamma^j = 0$). The existing literature on monetary transmission discusses many conventional channels that violate this identifying assumption.[29]

We meet this identification challenge by exploiting the cross-sectional variation in the responsiveness of stock prices to monetary shocks that is associated with cross-sectional variation in stock turnover, which we refer to as the *turnover channel*. Specifically, we will regress changes

[28]Other elements of $\boldsymbol{\kappa}^i$ could be financial variables, such as leverage, or the proportion of liquid assets relative to total assets in the firm's balance sheet, or non-financial variables such as firm i's sector, size, or age.

[29]To illustrate, suppose the outcome variable Y_t^i is a measure of firm i's investment. According to the *interest-rate channel*, for instance, an unexpected decrease in the nominal policy rate that passes through to the real interest rate would have two effects: (a) decrease the discount rate for future dividends, which increases the stock price, and (b) decrease the user cost of capital, which increases investment leading to positive correlation between ε_t^m and u_t^i.

in the outcome variable on changes in stock prices induced by monetary-policy shocks, but our identification strategy will consist of using $\varepsilon_{it}^{\mathcal{T}m} \equiv (\mathcal{T}^i - \bar{\mathcal{T}})\varepsilon_t^m$ (i.e., the *product between a firm-specific predetermined measure of stock turnover and the money shock*) as an instrument for the change in the firm's stock price. Stock turnover has a strong effect on the passthrough of the policy shock to the stock price, which implies a strong correlation between the proposed instrument and the change in the stock price.[30] Our main insight is that the relevant exclusion restriction will be satisfied as long as an individual firm's stock turnover (and any *unobserved* firm-level characteristic that is correlated with stock turnover) has no effect on the responsiveness to the monetary-policy shock of transmission variables other than Tobin's q that influence the outcome variable. This identifying assumption is weaker than the one needed for ε_t^m to be a valid instrument in the context of (17), in the sense that—as we explain below—it is not violated by the traditional transmission channels discussed in the literature.

To describe our identification strategy in more detail, we now use a *second-order* approximation to the function $v_j(\cdot)$ around the point $(\bar{r}, \bar{\boldsymbol{\kappa}}) \in \mathbb{R}^{N+1}$ for every transmission variable $j \in \{1, ..., D\}$, which implies

$$v_{jt}^i - v_{jt-1}^i \approx \mathrm{a}_t^j + \sum_{n=1}^{N} \alpha_{rn}^j \left(\kappa_n^i - \bar{\kappa}_n\right) \varepsilon_t^m, \tag{18}$$

where $\mathrm{a}_t^j \equiv \{\alpha_r^j + \alpha_{rr}^j [\varepsilon_t^m + 2(r_{t-1} - \bar{r})]\}\varepsilon_t^m$, $\alpha_{rr}^j \equiv \frac{1}{2}\frac{\partial v_j(\bar{r},\bar{\boldsymbol{\kappa}})}{\partial r \partial r}$, and $\alpha_{rn}^j \equiv \frac{\partial v_j(\bar{r},\bar{\boldsymbol{\kappa}})}{\partial \kappa_n \partial r}$ for $n \in \{1, ..., N\}$. Intuitively, the coefficient α_{rr}^j quantifies the second-order effect of a marginal increase in the policy rate on transmission variable j, and the coefficient α_{rn}^j quantifies the variation in the effect of a marginal increase in the policy rate on transmission variable j due to variation in firm-level characteristic n. We want to allow for the possibility that only the first M firm-level characteristics are observed, while the remaining characteristics are unobserved and possibly correlated with the observed characteristics. (We always treat stock turnover as an observed characteristic, so the integer M satisfies $1 \leq M \leq N$.) To this end, we express an unobserved characteristic $s \in \{M+1, ..., N\}$ as $\kappa_s^i \approx \bar{\kappa}_s + \sum_{n=1}^{M} \varkappa_{sn}(\kappa_n^i - \bar{\kappa}_n)$, where $\varkappa_{sn}$ represents the correlation between unobserved characteristic s and observed characteristic n. (Our convention is to denote $\varkappa_{s1}$ with $\varkappa_{s\mathcal{T}}$.) We can now write the policy-induced change in transmission variable j, i.e., (18), in terms of the interaction between the money shock and *observed* firm-

[30]This is essentially the *turnover-liquidity channel* documented in Lagos and Zhang (2020b).

level characteristics, i.e.,

$$v_{jt}^{i}-v_{jt-1}^{i}\approx \mathrm{a}_{t}^{j}+\sum_{n=1}^{M}\hat{\alpha}_{rn}^{j}\left(\kappa_{n}^{i}-\bar{\kappa}_{n}\right)\varepsilon_{t}^{m}, \tag{19}$$

where $\hat{\alpha}_{rn}^{j}\equiv\alpha_{rn}^{j}+\sum_{s=M+1}^{N}\alpha_{rs}^{j}\varkappa_{sn}$, for $n\in\{1,...,M\}$. Representation (19) and the first-order approximation to the function $Y(\cdot)$ around the point $\bar{\boldsymbol{v}}\equiv\boldsymbol{v}\left(\bar{r},\bar{\boldsymbol{\kappa}}\right)\in\mathbb{R}$ imply the policy-induced change in the outcome variable can be written as

$$\mathrm{Y}_{t}^{i}-\mathrm{Y}_{t-1}^{i}\approx \mathrm{b}_{t}+\gamma^{\mathrm{q}}\left(\mathrm{q}_{t}^{i}-\mathrm{q}_{t-1}^{i}\right)+\sum_{n=2}^{M}\tilde{\delta}_{rn}^{\sim\mathrm{q}}\left(\kappa_{n}^{i}-\bar{\kappa}_{n}\right)\varepsilon_{t}^{m}+\epsilon_{it}, \tag{20}$$

where $\mathrm{b}_{t}\equiv\sum_{j=2}^{D}\gamma^{j}\mathrm{a}_{t}^{j}$, $\tilde{\delta}_{rn}^{\sim\mathrm{q}}\equiv\sum_{j=2}^{D}\gamma^{j}\hat{\alpha}_{rn}^{j}$, and $\epsilon_{it}\equiv\tilde{\delta}_{r\mathcal{T}}^{\sim\mathrm{q}}\varepsilon_{it}^{\mathcal{T}m}$ (with $\tilde{\delta}_{r\mathcal{T}}^{\sim\mathrm{q}}\equiv\tilde{\delta}_{r1}^{\sim\mathrm{q}}$).

Since we are interested in estimating γ^{q}, our empirical strategy based on specification (20) is to use the money shock interacted with firm i's stock turnover, i.e., $\varepsilon_{it}^{\mathcal{T}m}$, as an instrument for $\mathrm{q}_{t}^{i}-\mathrm{q}_{t-1}^{i}$ to identify "exogenous" policy-driven variation in the stock price. Two conditions need to be satisfied for $\varepsilon_{it}^{\mathcal{T}m}$ to be a valid instrument for $\mathrm{q}_{t}^{i}-\mathrm{q}_{t-1}^{i}$ in order to estimate γ^{q} by using (20) as the basis for an IV regression. First, $\varepsilon_{it}^{\mathcal{T}m}$ must be correlated with the change in firm i's stock price, $\mathrm{q}_{t}^{i}-\mathrm{q}_{t-1}^{i}$. This correlation is negative and strong—it is the *turnover-liquidity mechanism* documented by Lagos and Zhang (2020b). Second, $\varepsilon_{it}^{\mathcal{T}m}$ must affect the outcome variable, Y_{t}^{i}, in the structural form (20) only through the transmission variable $\mathrm{q}_{t}^{i}-\mathrm{q}_{t-1}^{i}$. In other words, the instrument $\varepsilon_{it}^{\mathcal{T}m}$ must be uncorrelated with ϵ_{it}. But notice that $cov\left(\varepsilon_{it}^{\mathcal{T}m},\epsilon_{it}\right)=\tilde{\delta}_{r\mathcal{T}}^{\sim\mathrm{q}}(\mathcal{T}^{i}-\bar{\mathcal{T}})^{2}var\left(\varepsilon_{t}^{m}\right)$, so the *exclusion restriction* for $\varepsilon_{it}^{\mathcal{T}m}$ to be a valid instrument for $\mathrm{q}_{t}^{i}-\mathrm{q}_{t-1}^{i}$ is satisfied if and only if $\tilde{\delta}_{r\mathcal{T}}^{\sim\mathrm{q}}=0$, which is equivalent to

$$\sum_{j=2}^{D}\gamma^{j}\left(\alpha_{r\mathcal{T}}^{j}+\sum_{s=M+1}^{N}\alpha_{rs}^{j}\varkappa_{s\mathcal{T}}\right)=0. \tag{21}$$

Condition (21) says that $\varepsilon_{it}^{\mathcal{T}m}$ can serve as an instrument for Tobin's q if for every $j\in\{2,...,D\}$ (i.e., for every transmission variable other than Tobin's q), either $\gamma^{j}=0$, or $\alpha_{r\mathcal{T}}^{j}=\alpha_{rs}^{j}\varkappa_{s\mathcal{T}}=0$ for all $s\in\{M+1,...,N\}$.[31] In words: the exclusion restriction is satisfied as long as stock turnover (and any unobserved firm-level characteristic that is correlated with turnover)

[31] The condition $\gamma^{j}=0$ means that j does not operate as a transmission variable for the outcome of interest. The condition $\alpha_{r\mathcal{T}}^{j}=0$ means that firm i's stock turnover does not influence the marginal effect of the policy rate on transmission variable j. The condition $\alpha_{rs}^{j}\varkappa_{s\mathcal{T}}=0$ for all $s\in\{M+1,...,N\}$ means that every unobserved characteristic that is correlated with stock turnover has no influence on the marginal effect of the policy rate on transmission variable j.

has no effect on the passthrough of the monetary-policy shock to transmission variables other than Tobin's q that influence the outcome variable.[32] On theoretical grounds, this identifying assumption is weaker than the one needed for ε_t^m to be a valid instrument (as discussed in the context of (17)), in the sense that it is not violated by the traditional transmission channels discussed in the literature. For example, if transmission variable $j \in \{2, ..., D\}$ is an aggregate variable common to all firms, i.e., if $v_{jt}^i = v_{jt}$ for all i, then the response of v_{jt} to the money shock will not be affected by the predetermined firm-level characteristics of any given firm i, so in particular, $\alpha_{r\mathcal{T}}^j = \alpha_{rs}^j = 0$ for all $s \in \{M+1, ..., N\}$, so the identifying assumption $\gamma^j \hat{\alpha}_{r\mathcal{T}}^j = 0$ is automatically satisfied for transmission variable j. Thus, our identification strategy is very powerful to exclude traditional channels that operate through aggregate transmission variables that are not firm-specific.[33]

In the appendix (Section C.1) we show how our identification strategy generalizes to situations in which transmission variables affect the outcome variable (as before), *and* the outcome variable feeds back into transmission variables. The result is that in this case, a specification like (20) can identify the full effect of Tobin's q on the outcome variable. That is, the estimated coefficient on Tobin's q will capture not only the first-round effect of variation in Tobin's q on the outcome variable, but also the indirect effects (i.e., second-round, third-round... effects) associated with the variation in other transmission variables caused by the feedback from the change in the outcome variable originally triggered by the instrumented shock to Tobin's q.

While we are not aware of mainstream monetary transmission mechanisms that operate through firm-specific transmission variables whose responsiveness to monetary policy shocks

[32] In this formulation, since r_t is the only source of variation in transmission variables, $\tilde{\delta}_{r\mathcal{T}}^{\sim q} = 0$ implies $\epsilon_{it} = 0$. In the appendix (Section C) we consider a more general formulation that allows for additional random variation (across firms and over time) in transmission variables, as well as for random variation (across firms) in the mappings between unobserved and observed firm-level characteristics. In Section C.1 of the appendix we also generalize our identification strategy to the case in which there are firm-level feedbacks from the outcome variable to the transmission variables.

[33] The "textbook" version of the *interest-rate channel* described in footnote 29 is an example of a transmission mechanism that operates through aggregate transmission variables that are not firm-specific. Modern contributions in this area, e.g., Jeenas (2019) and Ottonello and Winberry (2020), emphasize that a monetary policy shock that affects the interest rate common to all firms can affect firms differently depending on firm-specific characteristics (such as an individual firm's leverage, or its share of liquid assets in total assets). In terms of the framework that we use to think about identification, the transmission mechanisms in these papers can be represented with a transmission variable j that is specific to firm i, i.e., v_{jt}^i, which measures the relevant firm-specific cost of investing (e.g., a firm-specific real interest rate, or a firm-specific shadow cost). In this context, our identifying assumption requires that the responsiveness of the relevant firm-specific cost of investment to monetary policy shocks does not depend on firm-specific stock turnover (or unobserved firm-level characteristics correlated with stock turnover).

depends on firm-level stock turnover, one could certainly contrive mechanisms mediated by firm-specific transmission variables (other than Tobin's q) whose responsiveness to money shocks depends on unobserved characteristics that are correlated with stock turnover. For example, high stock turnover may be indicative of a large degree of investor disagreement on the fundamentals of the firm, which may be more likely for firms that are in distress. Then, if for some reason a distressed firm's investment decisions were more sensitive to monetary shocks (e.g., because the firm's borrowing costs are more responsive to the shocks), this would represent a challenge to our identification strategy, potentially biasing our estimates. Our previous analysis of the identification problem, however, suggests that including in the regression interaction terms between the monetary shock and empirical proxies for the unobserved characteristics (e.g., the firm's state of "distress" in the previous example) can help to mitigate the identification concerns arising from mechanisms that operate through transmission variables (other than Tobin's q) whose responses to money shocks depend on unobserved characteristics that are correlated with stock turnover.[34]

4 Empirics

In this section we use the identification strategy described in Section 3.3 to obtain an empirical estimate of the effect of exogenous variations in Tobin's q on firms' investment and equity issuance decisions. Section 4.1 describes the data. In Section 4.2 we document that the financial turnover of a firm's stock is a significant determinant of the heterogeneous cross-firm responses of the outcome variables of interest (Tobin's q, equity issuance, and investment) to monetary shocks.[35] In Section 4.3 we estimate IV regressions based on the representation (20) (with $\mathcal{T}^i\varepsilon_t^m$ as instrument for q_t^i, and (19) as the basis for the first-stage regression). The coefficient of interest is γ^{q}, which quantifies the q-channel, i.e., the effect of an exogenous increase in

[34]For example, our baseline estimations (Section 4) include industry-time dummies that control for industry-specific responsiveness, and in robustness analysis (Appendix D) we control for heterogeneous investment responsiveness explained by observed firm-level characteristics such as age, leverage, size, and balance-sheet liquidity. These controls allay concerns about bias insofar as they proxy for the unobserved firm-level characteristics that are correlated with stock turnover and that affect the responsiveness of transmission variables (other than Tobin's q) to money shocks.

[35]Specifically, we estimate "reduced-form" OLS regressions based on the representations (175) (for Tobin's q, to estimate $\hat{\alpha}_{r\mathcal{T}}^{\mathrm{q}}$) and (178) (for equity issuance and investment, to estimate $\hat{\tilde{\delta}}_{rn}$, which equals $\gamma^{\mathrm{q}}\hat{\alpha}_{rn}^{\mathrm{q}}$ under our identifying assumptions). Our interest in the reduced-form OLS regression for Tobin's q is twofold: it revisits the results in Lagos and Zhang (2020b) (using quarterly rather than daily data), and it serves as the first-stage for our instrumental-variable (IV) approach.

Tobin's q on the outcome variable of interest (either equity issuance or investment). Our empirical analysis uses local projections in the spirit of Jordà (2005), but in a panel setting.

4.1 Data

Our empirical work uses firm-level measures of Tobin's q, equity issuance, and investment, as well as financial-market data on trade volume for individual firms' stocks, and a proxy for unexpected changes in the monetary policy rate. Our sample covers the period 1990Q1–2016Q4, and consists of the Compustat universe of publicly listed non-financial firms incorporated in the United States.[36]

For each individual common stock in the Center for Research in Security Prices (CRSP) database, we construct the daily *turnover rate* as the ratio of daily trade volume (total number of shares traded) to the number of outstanding shares. We average the daily turnover rate to obtain a quarterly series for firm i in quarter t (denoted $\mathcal{T}_t^i$), and merge it with the quarterly firm-level data from Compustat.

The key variables that we construct from Compustat are: Tobin's q, (normalized) equity issuance, and investment rate. We let q_t^i denote Tobin's q for firm i in quarter t, and define it as the book value of total assets (denoted $\bar{\mathrm{V}}_{At}^i$) plus the difference between the market value of common equity (denoted V_{Et}^i) and the book value of common equity (denoted $\bar{\mathrm{V}}_{Et}^i$), all scaled by the book value of total assets, i.e., $q_t^i \equiv 1 + (\mathrm{V}_{Et}^i - \bar{\mathrm{V}}_{Et}^i)/\bar{\mathrm{V}}_{At}^i$.[37] Our measure of *(net) equity issuance* for firm i in quarter t (denoted E_t^i) consists of all equity sales minus all equity purchases from Compustat. We normalize these quarterly net issuances by the total balance sheet size of firm i at the beginning of quarter t (i.e., $\bar{\mathrm{V}}_{At-1}^i$), and work with $\mathrm{e}_t^i \equiv \mathrm{E}_t^i/\bar{\mathrm{V}}_{At-1}^i$.[38] We define *investment* of firm i in quarter t (denoted I_t^i) as capital expenditures from Compustat, and construct the corresponding *investment rate* by dividing this measure by Compustat's measure of property, plant, and equipment (net of depreciation, depletion, and amortization) at the beginning of the quarter (denoted K_t^i).[39] Our measure of *equity dependence* will be based on

[36]Since our regression specifications include simple firm fixed effects in a dynamic panel setting, we only include firms that are in the dataset for at least 40 consecutive quarters. We discuss sample selection and other aspects of data construction in more detail in Appendix E.

[37]This is the definition of *average* q in Kaplan and Zingales (1997), except that as in Baker et al. (2003) and Cloyne et al. (2018), we do not subtract deferred taxes from the numerator (due to many missing values in our data). We follow Eberly et al. (2012) and use $\mathrm{q}_t^i \equiv \log q_t^i$ in our regressions. This specification provides a better fit given the skewness in the firm-level data, as discussed in Abel and Eberly (2002).

[38]We measure the "beginning of quarter t" values of firms' stock variables with the values reported in Compustat as of the end of quarter $t-1$.

[39]In robustness analysis, we have verified that the main results we report below are virtually unchanged if we

the *liquidity ratio* for each firm in each quarter, denoted ℓ_t^i, defined as the ratio of the firm's *cash and short-term investments*, denoted L_t^i, to the book value of total assets (both from Compustat), i.e., $\ell_t^i \equiv L_t^i/\bar{\mathrm{V}}_{At}^i$.

In order to construct unexpected changes in the nominal policy rate, we use the tick-by-tick nominal interest rate implied by the 3-month fed funds futures contract with nearest maturity after each regular monetary-policy announcement of the Federal Open Market Committee (FOMC).[40] In order to identify the impact of these monetary shocks, we follow the event-study methodology that consists of estimating the changes that occur in a 30-minute window around the time of the FOMC announcement.[41] The identification assumption is that in such a narrow window around the press release, futures rates are not affected by variables or news other than the FOMC announcement.[42] Since the firm-level data from Compustat is quarterly, we sum up the high-frequency changes in the federal funds futures rate by quarter to arrive at a quarterly series of monetary policy shocks for quarter t (denoted ε_t^m). We interpret a positive value of ε_t^m as a contractionary monetary shock, i.e., an unexpected policy-induced increase in the nominal interest rate.[43]

measure investment as capital expenditures *net* of sales of property, plant, and equipment, or if we construct the measure of the capital stock based on the perpetual inventory method. See Appendix E for more details on the construction of the variables used in the estimations. In Appendix D we also verify the robustness of the main results to incorporating firm-level measures of *size*, *age*, *leverage*, and *liquidity ratio* as additional controls.

[40]The use of a futures rate allows us to focus on the unanticipated component of the interest rate change that occurs on FOMC policy announcements dates, which we regard as monetary-policy shocks. The importance of focusing on the unanticipated component of policy announcements in order to identify the response of asset prices to monetary policy was originally pointed out by Kuttner (2001) and has been emphasized by the literature since then, e.g., Bernanke and Kuttner (2005) and Rigobon and Sack (2004).

[41]In the context of monetary policy, the event-study approach was originally used by Cook and Hahn (1989), and has been followed by a large number of papers, e.g., Bernanke and Kuttner (2005), Cochrane and Piazzesi (2002), Kuttner (2001), and Thorbecke (1997). Most recent applications follow Gürkaynak et al. (2005), who use time windows consisting of only a few minutes before and after the announcement (rather than consisting of the whole announcement day, as was common in earlier work).

[42]High-frequency movements in federal funds futures rates may encode information about future monetary policy actions (see, e.g., Nakamura and Steinsson (2018), Miranda-Agrippino and Ricco (2019), and Jarociński and Karadi (2020)). To contemplate this possibility, we consider alternative shock series in the robustness analysis that we report in Appendix D. Specifically, we redo our main estimations with a proxy for the monetary shock computed using a method proposed by Jarociński and Karadi (2020). Their approach employs a structural vector autoregression that uses high-frequency changes in federal funds futures rates alongside sign restrictions to ensure that monetary shocks generate opposite-signed surprises in futures rates and returns in the S&P500 index. The idea is that this sign restriction purges the proxy series from information effects that may generate positive high-frequency comovement between interest rates and stock returns.

[43]To construct the various measures of ε_t^m we use the dataset used by Jarociński and Karadi (2020), which is in turn based on an updated version of the dataset used by Gürkaynak et al. (2005). Since ε_t^m is possibly a noisy measure of the true monetary shocks, it should be used as an instrument in IV regressions (see, e.g., Stock and Watson (2018)). In our reduced-form specifications (Section 4.2) we treat ε_t^m as if it were an accurate measure of the true monetary shocks. In our main empirical IV specifications (Section 4.3), we instead use ε_t^m to construct

4.2 Evidence from reduced-form regressions

In this section we estimate "reduced-form" OLS regressions based on the representations (175) (for Tobin's q), and (178) (for equity issuance and investment). The main goal is to learn whether the measures of Tobin's q, equity issuance, and investment of firms with different (predetermined) stock turnover exhibit significantly different responses to monetary shocks.[44]

We estimate local-projection panel regressions of the following form:

$$y^i_{t+h} = f^i_h + d_{s,h,t+h} + \rho_h y^i_{t-1} + \beta_h \mathcal{T}^i_{t-1} + \gamma_h \mathcal{T}^i_{t-1}\varepsilon^m_t + u^i_{h,t+h}, \tag{22}$$

where $h = 0, 1, \ldots, H$ denotes the time horizon at which the effects are being estimated, and y^i_t is the outcome variable of interest for firm i in quarter t, i.e., $y^i_t \in \{\mathrm{q}^i_t, \mathrm{e}^i_t, \mathrm{x}^i_t\}$, where $\mathrm{q}^i_t \equiv \log(q^i_t)$ (log of Tobin's q), $\mathrm{e}^i_t \equiv \mathrm{E}^i_t/\bar{\mathrm{V}}^i_{At-1}$ (normalized net equity issuance), and $\mathrm{x}^i_t \equiv \log(I^i_t/K^i_t)$ (log of the investment rate).[45] The regressors are: a fixed effect for firm i at projection horizon h (denoted f^i_h), an industry-quarter dummy (2-digit SIC, quarter $t+h$) at projection horizon h (denoted $d_{s,h,t+h}$), the value of the outcome variable in the quarter prior to the shock (y^i_{t-1}), the measure of the turnover rate of firm i's stock in the quarter prior to the shock ($\mathcal{T}^i_{t-1}$), and the interaction between this lagged turnover rate and the quarterly measure of the monetary policy shock discussed above (ε^m_t). The error term in the h-quarter-horizon projection of the outcome variable of period $t+h$ for firm i is denoted $u^i_{h,t+h}$. The coefficients to be estimated are ρ_h, β_h, and γ_h. We are interested in γ_h, which measures the effect of stock turnover on the responsiveness of the outcome variable to monetary shocks at horizon h.

The baseline specification (22) uses *lagged* stock turnover to ensure it is unaffected by ε^m_t, and can therefore be regarded as a measure of the exposure of firm i's stock to the monetary shock.[46] As discussed in Section 3.3, our identification strategy relies on the cross-sectional

an instrument for changes in stock prices.

[44]The regressions involving Tobin's q are a robustness check of the empirical findings in Lagos and Zhang (2020b), who document the effect of stock turnover on the sensitivity of stock prices to money shocks at a *daily* frequency (rather than *quarterly*, as we do here). The regressions involving investment quantify the relevance of the q-monetary transmission mechanism for the real economy. The regressions involving equity issuance test of our theoretical prediction that firms respond to monetary-policy driven increases in their equity prices by issuing more equity (an instance of the "market timing" behavior studied by Baker and Wurgler (2002)).

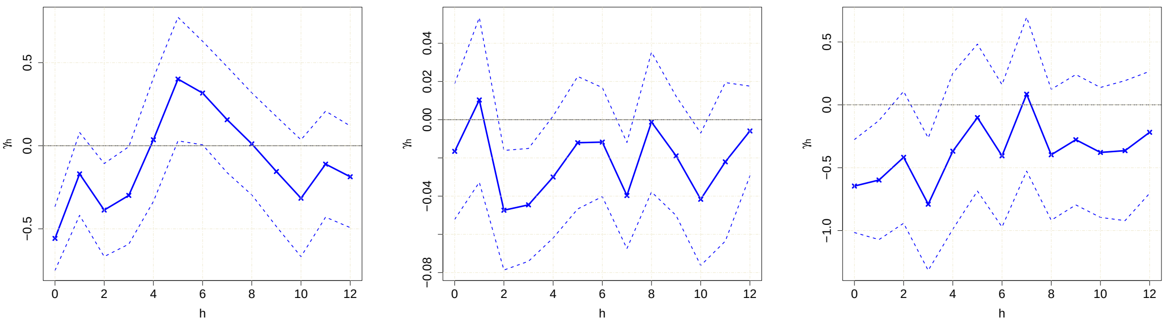
[45]We use the *log* of the investment rate since it will provide a better fit of the data given the skewness in the firm-level investment rates, as discussed in Abel and Eberly (2002). In Appendix D we verify that our main empirical findings are robust to measuring the investment rate in levels. In Appendix A (Section A.4.3) we cast our theoretical results in terms of the model counterparts of the variables that we use in our empirical estimation, i.e., the log of the investment rate ($\log \iota^*$), the log of Tobin's q ($\log \phi^s$), and the value of equity issuance relative to the firm's assets ($\phi^s s^*_{+1}$).

[46]Given persistence in stock turnover from one quarter to the next, the turnover for quarter $t-1$ proxies for

variation in the responsiveness of the outcome variable to monetary policy shocks that is induced by cross-sectional variation in firm-level stock turnover. The industry-time dummy $d_{s,h,t+h}$ is a flexible way to isolate this cross-sectional variation, so that the estimate of γ_h is driven by *within-industry*, *between-firm* variation across time.

We divide the measure of turnover, $\mathcal{T}_t^i$, by the time-series average of the standard deviation of turnover in the cross-section of firms, and we divide the measure of the monetary shock, ε_t^m, by its standard deviation between 1990Q1–2016Q4 (approximately 9.66 bp). We multiply the outcome variable y_t^i by 100, so the estimated coefficients (e.g., β_h, γ_h) associated with changes in e_t^i are intepreted in percentage points (pp), while the estimated coefficients associated with changes in q_t^i or x_t^i correspond to percentage changes. Figure 1 reports the point estimates and 95% confidence intervals for γ_h for the three outcome variables of interest: the log of Tobin's q (i.e., q_t^i), normalized equity issuance (e_t^i), and the log investment rate (x_t^i).

Figure 1: Effect of stock turnover on dynamic responses to monetary shocks (all firms)



(a) $y_t^i = \mathrm{q}_t^i$ (b) $y_t^i = \mathrm{e}_t^i$ (c) $y_t^i = \mathrm{x}_t^i$

Notes: Point estimates and 95% confidence intervals for γ_h from specification (22). Confidence intervals constructed based on two-way clustered standard errors at firm and SIC 3-digit industry-quarter levels.

The first panel of Figure 1 shows that the turnover of a firm's stock significantly predicts the response of that firm's stock price to the money shock, and that the effect persists for about three quarters. Since equity markets respond fast to shocks, the effects are strongest in the quarter of the monetary policy shock. The corresponding point estimate is approximately -0.5, which says that a firm whose stock turnover is 1 standard deviation higher than the average (across firms and over time) experiences a 0.5% stronger contraction in Tobin's q in response

turnover immediately before the FOMC announcement in quarter t. For the same reason, we lag the additional firm-level control variables in the robustness analysis of Appendix D.

to a 1 standard deviation contractionary monetary policy shock.

The middle panel of Figure 1 shows that the turnover of a firm's stock negatively predicts the change in a firm's normalized equity issuance in response to a contractionary money shock. The effect is lagged and quite persistent. The estimate is statistically significant two, three, seven and ten quarters after the shock. The estimated coefficient of approximately -0.04 at the two-quarter horizon says that a firm whose stock turnover is 1 standard deviation higher than the average (across firms and over time) experiences a 0.04 pp larger decline in net equity issuance relative to book assets two quarters after a 1 standard deviation contractionary monetary shock.

The last panel of Figure 1 shows that the turnover of a firm's stock negatively predicts the change in a firm's investment rate in response to a contractionary money shock. The effect is statistically significant in the quarter of the shock and at the one- and three-quarter horizon. The estimated coefficient of approximately -0.8 in quarter 3 says that a firm whose stock turnover is 1 standard deviation higher than the average (across firms and over time) experiences a 0.8% larger decline in its investment rate three quarters after a 1 standard deviation contractionary monetary shock.

The specification (22) is informative, but it pools firms without distinguising between their individual need for external financing. But as discussed in Section 3.1, according to the theory firms can be classified as *equity dependent*, or as *not equity dependent*. The former have low liquid assets and finance at least part of their investment by issuing equity in the open market, and therefore their equity issuance and investment decisions are influenced by policy-induced changes in their stock prices. The latter have high liquid assets and do not rely on equity issuance to finance investment, so while their stock prices may respond to monetary policy shocks, their equity issuance and investment decisions are insensitive to variation in stock prices induced by monetary policy shocks.

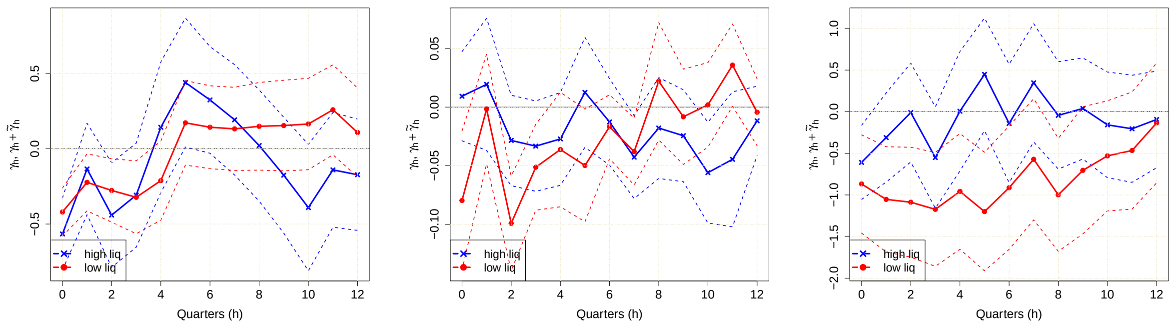
To test this theoretical prediction, we use the liquidity ratio, $\ell_t^i \equiv L_t^i/\bar{\mathrm{V}}_{At}^i$, as an indicator that the firm is equity dependent.[47] Specifically, we define the indicator $\mathbb{I}_{L,t}^i$ which equals 1 if firm i belongs in the bottom half of the liquidity ratio distribution of the cross-section of firms in quarter t, and 0 otherwise, and estimate the following generalization of (22):

$$\begin{aligned} y_{t+h}^i =& f_h^i + d_{s,h,t+h} + \alpha_h \mathbb{I}_{L,t-1}^i + \left(\rho_h + \tilde{\rho}_h \mathbb{I}_{L,t-1}^i\right) y_{t-1}^i \\ &+ \left(\beta_h + \tilde{\beta}_h \mathbb{I}_{L,t-1}^i\right) \mathcal{T}_{t-1}^i + \left(\gamma_h + \tilde{\gamma}_h \mathbb{I}_{L,t-1}^i\right) \mathcal{T}_{i,t-1} \varepsilon_t^m + u_{h,t+h}^i. \end{aligned} \tag{23}$$

[47] We regard the liquidity ratio as the empirical counterpart of ω in the theory, since it measures the availability of a broad set of liquid assets that the firm can use to finance expenditures internally.

The coefficients to be estimated are α_h, ρ_h, $\tilde{\rho}_h$, β_h, $\tilde{\beta}_h$, γ_h, and $\tilde{\gamma}_h$. We are interested in γ_h, which now measures the effect of stock turnover on the responsiveness of the outcome variable at horizon h to monetary shocks, for firms with a high liquidity ratio in the quarter prior to the shock. We are also interested in $\gamma_h + \tilde{\gamma}_h$, which measures the effect of stock turnover on the responsiveness of the outcome variable at horizon h to monetary shocks, for firms with a low liquidity ratio in the quarter prior to the shock. Figure 2 reports the point estimates and 95% confidence intervals for γ_h and $\gamma_h + \tilde{\gamma}_h$ for the three outcome variables of interest: the log of Tobin's q (i.e., q_t^i), normalized equity issuance (e_t^i), and the log investment rate (x_t^i).

Figure 2: Effect of stock turnover on dynamic responses to monetary shocks (conditional on liquidity ratio)



(a) $y_t^i = \mathrm{q}_t^i$ (b) $y_t^i = \mathrm{e}_t^i$ (c) $y_t^i = \mathrm{x}_t^i$

Notes: Point estimates and 95% confidence intervals for γ_h and $\gamma_h + \tilde{\gamma}_h$ from specification (23). Confidence intervals constructed based on two-way clustered standard errors at firm and SIC 3-digit industry-quarter levels.

The first panel in Figure 2 shows that the financial *turnover-liquidity channel* documented in Lagos and Zhang (2020b), i.e., the finding that the turnover of a firm's stock negatively predicts the change in a firm's stock price in response to a contractionary monetary policy shock, operates similarly across the stocks of firms with different pre-shock liquidity ratios. The estimated dynamic responses are close to those estimated on the pooled sample in specification (22). The effects are strongest in the quarter of the monetary policy shock (the point estimate for γ_0 is close to -0.5 in all cases), and significant up to the three-quarter horizon.

The middle panel of Figure 2 shows that, for firms with pre-shock liquidity ratios above the median, turnover does not significantly predict the response of equity issuance to money shocks up to a horizon of about 2 years.[48] On the other hand, conditional on belonging to the

[48] A significant negative effect of turnover does appear for these firms at longer horizons (e.g., at a horizon of

group with below-median liquidity ratios prior to the shock, firms with higher stock turnover exhibit significantly stronger contractions in equity issuance in response to a contractionary money shock in the quarter of the shock, and also two, three, five, and seven quarters after the shock. The point estimate of $\gamma_h + \tilde{\gamma}_h$ is roughly -0.08 on impact. This means that a firm with pre-shock liquidity ratio below the median whose stock has a turnover rate that is 1 standard deviation above the average (across all firms and over time) experiences a 0.08 pp larger decline in net equity issuance relative to book assets in response to a 1 standard deviation contractionary monetary shock. Taken together, the middle panels of Figure 1 and Figure 2 indicate that the overall negative effect of turnover on the response of equity issuance to contractionary monetary policy shocks during the first two years is driven by firms with relatively low liquid asset holdings.

The last panel of Figure 2 shows that for firms with pre-shock liquidity ratios above the median, turnover does not tend to have a significant effect on the response of the investment rate to money shocks.[49] On the other hand, conditional on belonging to the group with below-median liquidity ratios prior to the shock, firms with higher stock turnover exhibit significantly stronger contractions in investment rates in response to a contractionary money shock up to 2 years after the shock. The point estimate of $\gamma_h + \tilde{\gamma}_h$ is about -1 at the four-quarter horizon. This means that a firm with pre-shock liquidity ratio below the median whose stock has a turnover rate that is 1 standard deviation above the average (across all firms and over time) experiences a 1% larger decline in its investment rate four quarters after a 1 standard deviation contractionary monetary shock.

4.3 Evidence from IV regressions

In this section we use the identification strategy described in Section 3.3 to estimate the effect of exogenous variation in Tobin's q on firms' equity issuance and investment. Instead of the "reduced-form"specification (22) for $y_t^i \in \{\mathrm{e}_t^i, \mathrm{x}_t^i\}$ that uses the interaction term $\mathcal{T}_{t-1}^i \varepsilon_t^m$ directly as a regressor, we now adopt an IV specification that uses as a regressor the measure of the

seven and ten quarters), but this finding is not robust to some of the alternative specifications that we consider in Appendix D. The quantitative version of our model with long-lived firms that we present in Section 6 can rationalize why firms with high liquidity ratios at the time of the shock may respond with a considerable delay. The idea is that this happens whenever a firm has high liquidity ratio at the time of the shock, draws down its liquid assets over time, and eventually engages in equity financing taking advantage of the fact that the effects of the shock on stock prices have not yet dissipated.

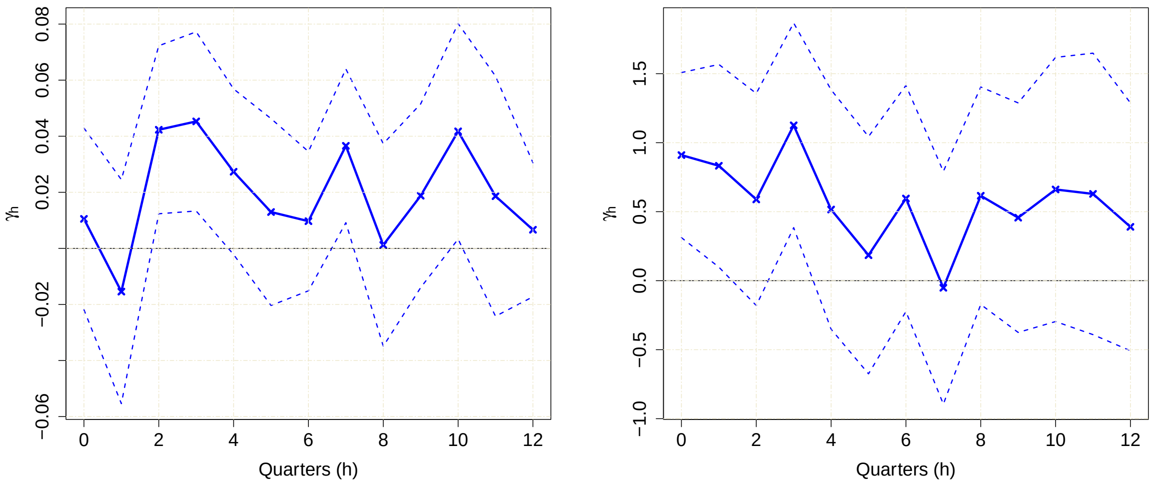
[49] The significant response on impact is absent in the baseline specification we run in Section 4.3.

firm's Tobin's q instrumented with the interaction term $\mathcal{T}^i_{t-1}\varepsilon^m_t$ (and uses (22) for $y^i_t = \mathrm{q}^i_t$ as the first stage of the IV procedure). Under the identification assumptions discussed in Section 3.3, we think of variation in q^i_t instrumented with $\mathcal{T}_{i,t-1}\varepsilon^m_t$ as the exogenous variation in (the log of) Tobin's q that is driven by monetary policy shocks. Our baseline IV specification is:

$$y^i_{t+h} = f^i_h + d_{s,h,t+h} + \rho_h y^i_{t-1} + \beta_h \mathcal{T}^i_{t-1} + \gamma_h \mathrm{q}^i_t + u^i_{h,t+h}, \tag{24}$$

where q^i_t is instrumented with $\mathcal{T}_{i,t-1}\varepsilon^m_t$. Figure 3 depicts the point estimates of γ_h and the corresponding 95% confidence intervals for $y^i_{t+h} \in \{\mathrm{e}^i_t, \mathrm{x}^i_t\}$.

Figure 3: Dynamic responses of equity issuance and investment rate to instrumented changes in Tobin's q (all firms)



(a) $y^i_t = \mathrm{e}^i_t$ (b) $y^i_t = \mathrm{x}^i_t$

Notes: Point estimates and 95% confidence intervals for γ_h from estimating specification (24). Confidence intervals constructed based on two-way clustered standard errors at firm and SIC 3-digit industry-quarter levels.

The IV estimates are in line with what one would expect based on the reduced-form OLS results reported in Section 4.2.[50] The left panel of Figure 3 shows that equity issuance responds positively to increases in Tobin's q instrumented with the turnover-liquidity mechanism (measured by the interaction term $\mathcal{T}^i_{t-1}\varepsilon^m_t$). The point estimate is statistically significant two, three, seven, and ten quarters after the shock. To get a sense of the magnitude of a response, the

[50]The estimates in panels (a) in Figures 1 and 2, do not seem to suggest that $\mathcal{T}^i_{t-1}\varepsilon^m_t$ is a weak instrument for q^i_t in the cross-section of firms. In fact, for example, when $y^i_t = \mathrm{x}^i_t$, the first stage F-statistic on the instrument is 16.0 at horizon $h = 0$.

estimate 0.04 for $h = 3$ means that a 1% increase in a firm's measure of Tobin's q causes a 0.04 pp increase in the firm's ratio of net equity issuance relative to the book value of total assets three quarters after the monetary shock. The right panel of Figure 3 shows that an increase in Tobin's q leads to an increase in the investment rate that is statistically significant in the quarter of the shock and one and three quarters after the shock. The point estimate at the three-quarter horizon is about 1.1, which means that a 1% increase in a firm's Tobin's q leads to a 1.1% increase in the firm's investment rate.

The specification (24) is the IV counterpart of (22), in that it pools firms without conditioning on their need for external financing. As discussed above (e.g., in Section 3.1 or in the discussion leading to (23)), according to the theory, policy-induced changes in Tobin's q should only affect the equity issuance and investment decisions of *equity dependent* firms, which have relatively low liquidity ratios. Thus, next we use the liquidity indicator $\mathbb{I}^i_{L,t}$ introduced in (23) to proxy for equity dependence, and estimate the following generalization of (24):

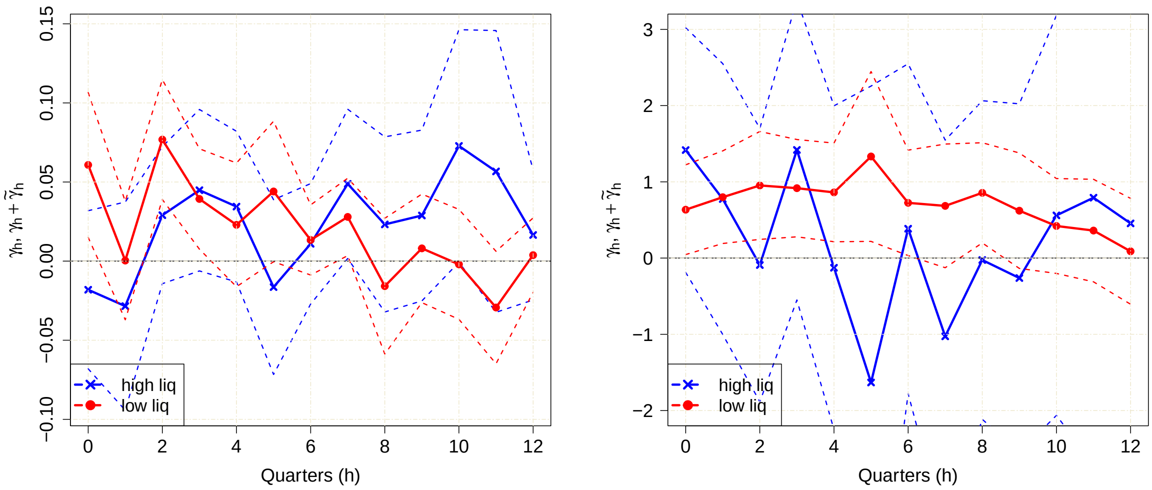
$$\begin{aligned} y^i_{t+h} =& f^i_h + d_{s,h,t+h} + \alpha_h \mathbb{I}^i_{L,t-1} + \left(\rho_h + \tilde{\rho}_h \mathbb{I}^i_{L,t-1}\right) y^i_{t-1} \\ &+ \left(\beta_h + \tilde{\beta}_h \mathbb{I}^i_{L,t-1}\right) \mathcal{T}^i_{t-1} + \left(\gamma_h + \tilde{\gamma}_h \mathbb{I}^i_{L,t-1}\right) \mathrm{q}^i_t + u^i_{h,t+h}, \end{aligned} \tag{25}$$

where q^i_t and $\mathbb{I}^i_{L,t-1}\mathrm{q}^i_t$ are instrumented with $\mathcal{T}^i_{t-1}\varepsilon^m_t$ and $\mathbb{I}^i_{L,t-1}\mathcal{T}^i_{t-1}\varepsilon^m_t$, respectively. Figure 4 depicts the point estimates of γ_h and $\gamma_h + \tilde{\gamma}_h$ and the corresponding 95% confidence intervals for $y^i_t \in \{\mathrm{e}^i_t, \mathrm{x}^i_t\}$.

The left panel of Figure 4 shows that for firms with below-median liquidity ratios, there is a positive statistically significant response of equity issuance to increases in Tobin's q in the quarter of the money shock, as well as in second, third, and seventh quarters after the shock. For example, the estimated response on impact is approximately $\gamma_0 + \tilde{\gamma}_0 = 0.06$, which means that for a firm with a liquidity ratio below the median, a 1% increase in Tobin's q causes a 0.06 pp increase in the firm's ratio of net equity issuance relative to the book value of total assets in the quarter of the monetary shock. For firms with above-median liquidity ratios, the positive relation between instrumented variation in Tobin's q and equity issuance is in general not statistically significant.

The right panel of Figure 4 shows that for firms with below-median liquidity ratios, there is a positive statistically significant response of the investment rate to increases in Tobin's q in the quarter of the money shock, and in the following six quarters after the shock. For these firms, a 1% increase in Tobin's q implies an elevated investment rate for up to six quarters

Figure 4: Dynamic responses of equity issuance and investment rate to instrumented changes in Tobin's q (conditional on liquidity ratio)



(a) $y_t^i = \mathrm{e}_t^i$ (b) $y_t^i = \mathrm{x}_t^i$

Notes: Point estimates and 95% confidence intervals for γ_h and $\gamma_h + \tilde{\gamma}_h$ from specification (25). Confidence intervals constructed based on two-way clustered standard errors at firm and SIC 3-digit industry-quarter levels.

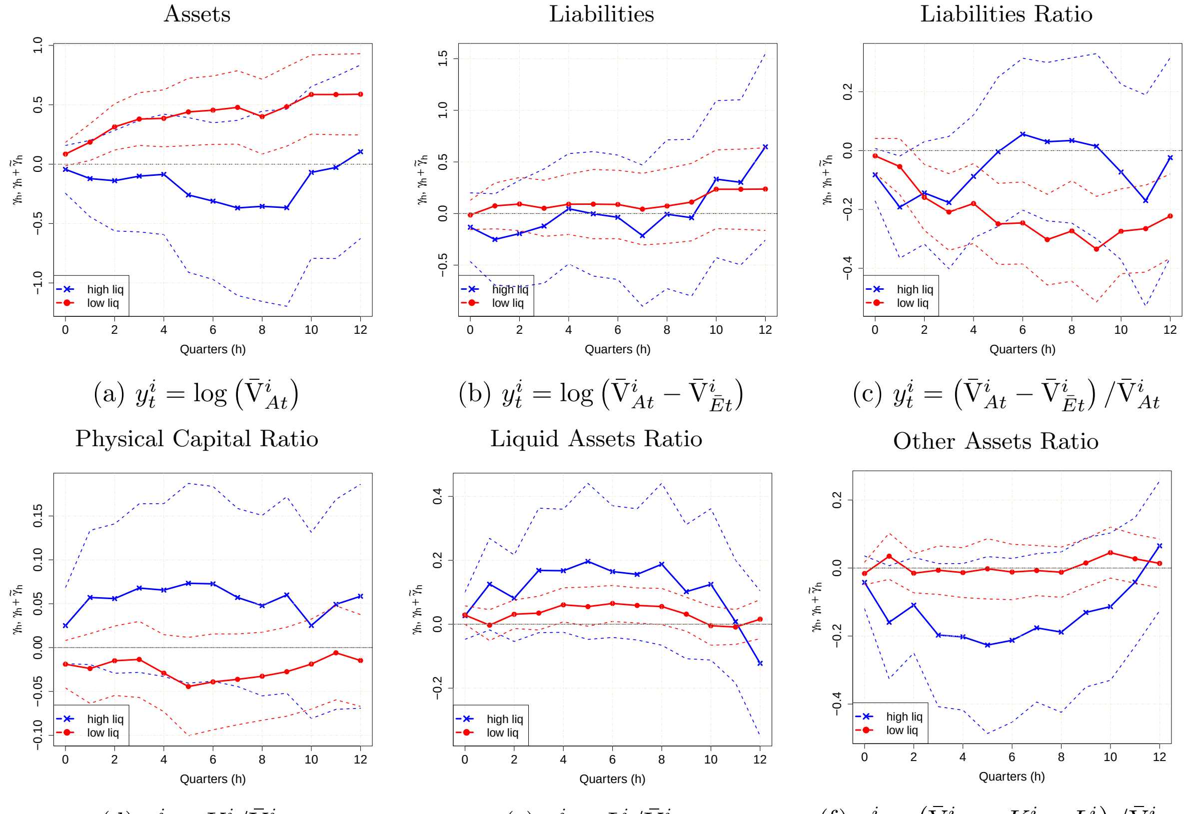
after the shock, with a response of approximately 1% higher investment rate at the two- to four-quarter horizon. In contrast, the investment rate of firms with liquidity ratios above the median exhibits no statistically significant estimates.

4.4 Asset and capital structure dynamics

In Section 4.3 we documented that exogenous increases in Tobin's q (i.e., increases in stock prices associated monetary-policy induced changes in turnover liquidity) stimulate the equity issuance and investment of firms with relatively low liquidity ratios. In this section we broaden our focus, and use the methodology of Section 4.3 to study the effect of Tobin's q on firm's capital structure and composition of assets. Figure 5 shows the dynamic responses that result from estimating specification (25) using the main balance-sheet items as outcome variables.

Panel (a) of Figure 5 shows the response of the book value of total assets, measured by $\log\left(\bar{\mathrm{V}}_{At}^i\right)$. Firms with below-median liquidity ratios respond to changes in Tobin's q by increasing their size, suggesting that the higher equity issuance documented in Figure 4 does not immediately flow out of the firms. The estimate of about 0.25 at the two-quarter horizon means

Figure 5: Effect of stock turnover on dynamic responses to monetary shocks (conditional on liquidity ratio)



(a) $y_t^i = \log\left(\bar{\mathrm{V}}_{At}^i\right)$ (b) $y_t^i = \log\left(\bar{\mathrm{V}}_{At}^i - \bar{\mathrm{V}}_{\bar{E}t}^i\right)$ (c) $y_t^i = \left(\bar{\mathrm{V}}_{At}^i - \bar{\mathrm{V}}_{\bar{E}t}^i\right)/\bar{\mathrm{V}}_{At}^i$

(d) $y_t^i = K_t^i/\bar{\mathrm{V}}_{At}^i$ (e) $y_t^i = L_t^i/\bar{\mathrm{V}}_{At}^i$ (f) $y_t^i = \left(\bar{\mathrm{V}}_{At}^i - K_t^i - L_t^i\right)/\bar{\mathrm{V}}_{At}^i$

Notes: Point estimates and 95% confidence intervals for γ_h and $\gamma_h + \tilde{\gamma}_h$ from specification (25). Confidence intervals constructed based on two-way clustered standard errors at firm and SIC 3-digit industry-quarter levels.

that a 1% increase in Tobin's q leads to a 0.25% growth in the firm's total assets. In contrast, the book value of total assets of firms with above-median liquidity ratios does not exhibit a statistically significant response to Tobin's q.

Panel (b) of Figure 5 shows the response of the book value of total liabilities, measured as $\log\left(\bar{\mathrm{V}}_{At}^i - \bar{\mathrm{V}}_{\bar{E}t}^i\right)$ (where $\bar{\mathrm{V}}_{\bar{E}t}^i$ denotes the book value of *all* equity, i.e., common and preferred). Regardless of whether they are or not equity dependent, firms do not seem to do any active managing of their total liabilities in response changes in Tobin's q.[51] This finding together with

[51]The dynamic responses of *total debt*, *long-term debt*, and *short-term debt*, both in log-levels and relative to total assets are very similar to those of our measure of *total liabilities* in panels (b) and (c) of Figure 4, so we do not report them here.

the earlier finding that low-liquidity firms tend to increase their net equity issuances, implies that these firms make persistent changes to their capital structure in response to market-driven variations in Tobin's q. This result is evident from panel (c), which shows the dynamic responses of the *liabilities ratio*, defined as the ratio of the book value of total assets to the book value of all liabilities, i.e., $\left(\bar{\mathrm{V}}^i_{At} - \bar{\mathrm{V}}^i_{\bar{E}t}\right)/\bar{\mathrm{V}}^i_{At}$. The response of the liabilities ratio is significant and persistent for firms with below-median liquidity ratios. For example, at the three-quarter horizon, the point estimate for $\gamma_h + \tilde{\gamma}_h$ is -0.2, which means that a shock that causes 1% increase in Tobin's q leads to a 0.2 pp reduction in the liabilities ratio three quarters after the shock. In sum, firms with below-median liquidity ratios tilt their capital structure toward equity financing. In contrast, the capital structure of firms with above-median liquidity ratios does not exhibit a statistically significant response to Tobin's q.

The bottom row of Figure 5 shows the dynamic changes in the composition of firms' assets. Panel (d) shows the response of the *physical capital ratio*, defined as $K^i_t/\bar{\mathrm{V}}^i_{At}$, where K^i_t denotes the book value of *net property, plant, and equipment.* Panel (e) shows the response of the *liquid assets ratio*, defined as $L^i_t/\bar{\mathrm{V}}^i_{At}$, where L^i_t denotes the book value of *cash and short-term investments.* Panel (f) shows the response of the *other assets ratio*, defined as $\left(\bar{\mathrm{V}}^i_{At} - K^i_t - L^i_t\right)/\bar{\mathrm{V}}^i_{At}$. Taken together, panels (d), (e), and (f) no exhibit significant shifts in the relative sizes of the main asset classes. This suggests that the low-liquidity firms that respond to increases in their stock prices by issuing equity, use the newly raised funds to scale up all their assets rougly in equal proportion.[52] The asset structures of firms with above-median liquidity ratios do not exhibit statistically significant responses.

5 Q-channel and monetary transmission: macro implications

In this section we quantify the relevance of the q-channel in the transmission of monetary shocks to aggregate investment. We do this in two ways. First, we report the cross-sectional distribution of estimates for the semi-elasticity of investment to money shocks transmitted

[52]The point estimates in panel (e) show a slight hump-shaped increase in the share of *cash and other liquid financial assets* for firms with below-median liquidity ratios. (The positive estimates are only marginally significant at the four-quarter and six-quarter horizons.) This response is consistent with a narrative where, in response to a financial-market shock that increases Tobin's q, low-liquidity firms raise funds by increasing their net equity issuance, initially hold the newly raised funds in the form of liquid financial assets, and then allocate these funds toward physical capital investment and other assets gradually over time—eventually returning to an asset structure that is similar to the one they had before the shock.

through the q-channel.[53] Second, we use our micro-level estimates to produce an estimate of the semi-elasticity of aggregate investment to money shocks transmitted through the q-channel.

According to specification (25), the semi-elasticity of the investment rate of firm i in quarter $t+h$ to a monetary shock in quarer t is

$$\begin{aligned} \frac{d\log(I^i_{t+h}/K^i_{t+h})}{d\varepsilon^m_t} &= \frac{d\log(I^i_{t+h}/K^i_{t+h})}{d\log\left(q^i_t\right)}\frac{d\log\left(q^i_t\right)}{d\varepsilon^m_t} \\ &= \left(\gamma_h + \tilde{\gamma}_h \mathbb{I}^i_{L,t-1}\right)\frac{d\log\left(q^i_t\right)}{d\varepsilon^m_t}, \end{aligned} \tag{26}$$

where I^i_t, K^i_t, and q^i_t denote firm i's investment, capital stock, and Tobin's q in quarter t, respectively (all as defined in Section 4.1), ε^m_t denotes the monetary policy shock in quarter t (expressed as a multiple of the average standard deviation of monetary shocks in the sample, as in Section 4), and $\mathbb{I}^i_{L,t}$ is an indicator that equals 1 if firm i has a liquidity ratio below the median, and 0 otherwise. The estimates of γ_h and $\tilde{\gamma}_h$ are reported in panel (b) of Figure 4. To obtain estimates for $d\log\left(q^i_t\right)/d\varepsilon^m_t$, we estimate the following regression:

$$\log\left(q^i_t\right) = b + f^i + \beta_0 \log\left(q^i_{t-1}\right) + \beta_1 \mathcal{T}^i_{t-1} + \beta_2 \varepsilon^m_t + \beta_3 \mathcal{T}^i_{t-1}\varepsilon^m_t + u^i_t, \tag{27}$$

where f^i is a stock fixed effect, and u^i_t is the error term for stock i in quarter t.[54] With (27), (26) can be written as

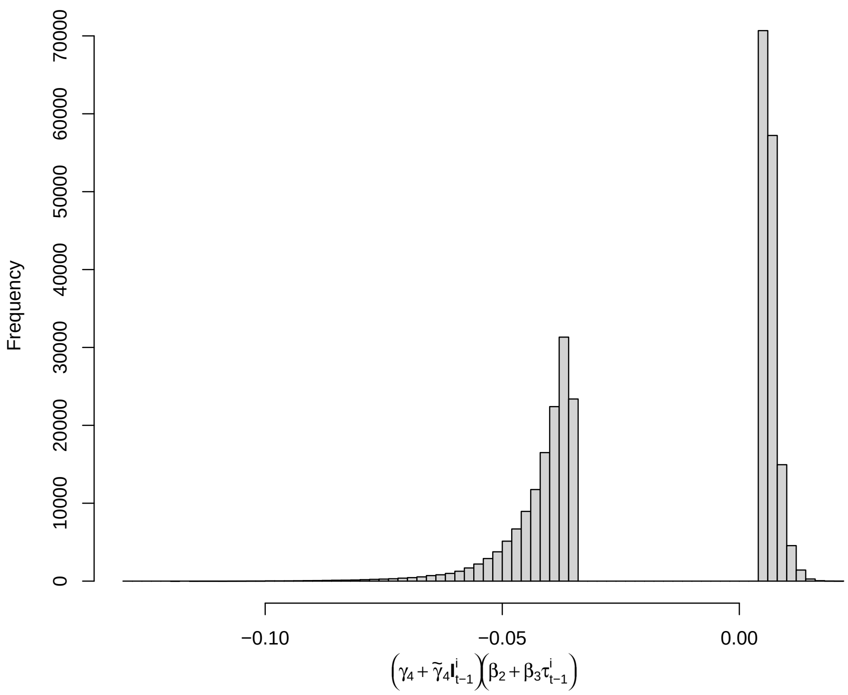
$$\frac{d\log(I^i_{t+h}/K^i_{t+h})}{d\varepsilon^m_t} = \left(\gamma_h + \tilde{\gamma}_h \mathbb{I}^i_{L,t-1}\right)\left(\beta_2 + \beta_3 \mathcal{T}^i_{t-1}\right). \tag{28}$$

Figure 6 shows the cross-sectional distribution (across all firms and quarters) of the semi-elasticities of the investment rate to the money shock at the four-quarter horizon, that is $\left\{\frac{d\log(I^i_{t+4}/K^i_{t+4})}{d\varepsilon^m_t}\right\}_{i,t}$, across firms i and quarters t in our sample.

[53] The responses across these firms are heterogeneous because their stocks have different turnover, which leads to heterogeneous stock-price responses to the same money shocks (due to the *turnover-liquidity channel*), and because their liquidity ratios are classified as either *high* or *low*, which leads to heterogeneous investment responses to the same variation in Tobin's q.

[54] This specification is similar to (20) in Lagos and Zhang (2020b), which is one of the specifications they use to estimate the turnover-liquidity channel but at a daily frequency. The estimated coefficients of interest are: $\beta_2 = -0.385837$ and $\beta_3 = -0.097444$. The first estimate means that the direct (first-order) effect of a one standard deviation surprise increase in the policy rate is to reduce a firm's stock price by about -0.39% in the quarter when the shock occurred. (Since the standard deviation of ε_t is 9.66 bp in our sample, this estimate implies a 101 bp decline in the stock price in response to a 25 bp surprise increase in the fed funds rate.) The second estimate means that a firm whose stock turnover is 1 standard deviation higher than the average (across firms and over time) experiences a 0.1% stronger contraction in Tobin's q in response to a 1 standard deviation contractionary monetary policy shock.

Figure 6: Distribution (across all firms and quarters) of semi-elasticity of investment rate at horizon $h = 4$ to a 1 bp surprise in the fed funds rate (computed as in the right side of (28))



Next, we assess the quantiative relevance of the q-channel for aggregate investment, $\bar{I}_t = \sum_{i \in \mathbb{F}} I^i_t$, where I^i_t is the level of investment of firm i in quarter t, and $\mathbb{F}$ denotes the set of firms in our sample.[55] We are interested in using the distribution of firm-level estimates of the semi-elasticity of the investment rate to money shocks, $d \log(I^i_{t+h}/K^i_{t+h})/d\varepsilon^m_t$ (from (26)) to obtain an estimate of the *aggregate* semi-elasticity of investment to money shocks, i.e., $d \log\left(\bar{I}_{t+h}\right)/d\varepsilon^m_t$. If, as is typically the case empirically, we have $d \log\left(I^i_{t+h-s}\right)/d\varepsilon^m_t \leq 0$ for $s \in \{1, ..., h\}$ and $i \in \mathbb{F}$, then

$$\frac{d \log\left(\bar{I}_{t+h}\right)}{d\varepsilon^m_t} \leq \sum_{i \in \mathbb{F}} \frac{I^i_{t+h}}{\bar{I}_{t+h}} \frac{d \log(I^i_{t+h}/K^i_{t+h})}{d\varepsilon^m_t}. \tag{29}$$

Thus, we can use the right side of (29), i.e. the average cross-sectional semi-elasticity of investment rates to money shocks transmitted through the q-channel (weighted by firm's investment shares), as an (upper-bound) estimate for the (negative) semi-elasticity of aggregate investment

[55]We will also provide estimates for the case where $\mathbb{F}$ is the set of all firms, not just publicly traded firms.

to money shocks transmitted through the q-channel.[56]

Based on the estimates reported in Figure 6, our estimate for $d\log\left(\bar{I}_{t+4}\right)/d\varepsilon_t^m$ equals -0.002831, which means that a one standard deviation surprise increase in the policy rate changes aggregate investment of Compustat firms by -0.2831% four quarters after the shock. The standard deviation of ε_t^m is 9.66 bp in our sample, so this estimate implies a 0.73% decline in investment in response to a 25 bp *surprise* increase in the fed funds rate. Since it is customary to express this semi-elasticity in terms of changes in the policy rate (instead of *surprise* changes in the policy rate), we note that on average, in our sample, for every 3 bp change in the policy rate, about 1 bp is a surprise change (as measured by the change in the fed funds futures rate).[57] Hence, our estimate for $d\log\left(\bar{I}_{t+4}\right)/d\varepsilon_t^m$ based on (29) implies a 0.25% decline in investment of Compustat firms in response to a 25 bp increase in the fed funds rate. The share of aggregate nonresidential investment by publicly traded firms in the United States is about 0.45, so our estimate implies a 0.11% decline in *aggregate* investment in response to a 25 bp increase in the fed funds rate operating exclusively through the q-channel.[58] As way of comparison, Christiano et al. (2005) report a peak response in aggregate investment of about 0.4% to a 25 bp decline in the policy rate.[59] To summarize: our micro estimates imply that the q-channel accounts for about one third of the conventional estimate of the peak response of aggregate investment to monetary policy shocks.

6 Quantitative analysis

In this section we assess the ability of the theory to match the dynamic responses of investment through the q-channel documented in Section 4. To this end, we generalize the model of Section 2 along three dimensions.

First, we introduce a monetary policy shock in the form of an unexpected change in the path of the nominal policy rate, r_t (defined in (10)). Specifically, we assume that following

[56]For a derivation of (29), see Lemma 5 (Appendix A).

[57]We obtain this estimate by regressing quarterly changes in the fed funds rate on our series of surprise changes in the fed funds rate, $\{\varepsilon_t^m\}$. With both expressed in basis points, the estimated coefficient is 2.98, so a 25 bp increase in the fed funds rate is associated to a 8.39 bp *surprise* increase in the fed funds rate.

[58]This last estimate assumes that the q-channel is inoperative for non-publicly traded firms. However, it will be an understimate to the extent that equity stakes on non-publicly traded firms are sometimes traded—albeit privately, in over-the-counter style markets rather than in public organized exchanges.

[59]Figure 1 in Christiano et al. (2005), for example, shows that a 60 bp decrease in the policy rate is associated with a 1% increase in aggregate investment eight quarters after the shock, which is the peak response according to their estimation.

the unexpected policy shock $\varepsilon^m \in \mathbb{R}$, the policy rate follows an autoregressive path, $r_{t+1} = \bar{r} + \rho_n (r_t - \bar{r})$, with $\rho_n \in (0,1)$ and $r_0 = \bar{r} + \varepsilon^m$, where $\bar{r} \in \mathbb{R}_+$ is the steady-state policy rate.

Second, we introduce a stochastic fixed cost of equity issuance. Specifically, an entrepreneur with capital stock k_t who issues equity in the second subperiod of period t (i.e., who chooses $e_t^i > 0$) bears a disutility cost $\xi_t k_t$, where $\xi_t \in \mathbb{R}_+$ is the realization of a uniform random variable independently distributed across entrepreneurs and over time, with support $[0, \bar{\xi}]$.[60]

Third, we assume that in addition to producing $z \in \mathbb{R}_+$ units of good 1 at the end of the first subperiod, each unit of installed capital also delivers $\tilde{z} \in \mathbb{R}_+$ units of good 2 in the second subperiod. Each equity share represents ownership of a unit of capital along with the stream of dividends of good 1 and good 2 produced by that unit of capital. In addition, we assume that instead of paying out the $\tilde{z}s_t$ units of good 2 to the shareholders, the entrepreneur invests this dividend to augment the capital stock, and issues $\tilde{e}_t = \frac{\tilde{z}s_t}{\phi_t^s}$ equity claims on the newly created capital to the shareholders (without bearing the fixed cost of issuance).[61]

6.1 Calibration

We let a model period correspond to a quarter, and set $\beta = 0.995$, $\delta = 0.025$, $1 - \pi = 0.017$ (the exit rate targeted by Begenau and Salomao (2019)), and $\alpha = \theta = 1$ (corresponding to a frictionless stock market that abstracts from micro-level pricing frictions induced by search bargaining). The distribution of financial investors' valuations of the good 1 dividend, G, is assumed to be lognormal, i.e., $\log \varepsilon_t \sim \mathcal{N}(\mu_\varepsilon, \sigma_\varepsilon)$, with $\mu_\varepsilon = -\sigma_\varepsilon^2/2$. The value of σ_ε is chosen so that the stock-price response to the money shock in the model matches the price response to the money shock of stocks with 10% highest turnover in our sample. The monetary policy

[60] The practical motivation for introducing the equity issuance cost is that it delivers a nontrivial distribution of liquid asset holdings, and at the same time makes the model flexible enough to match the empirical frequency of equity issuance (the fraction of firms that issue equity in any given quarter).

[61] Conceptually, this assumption captures the idea that firms can also finance investment with retained earnings, which economizes on equity issuance costs. The practical motivation for the assumption is that it allows a more flexible mapping between capital accumulation and the size of the fixed cost of equity issuance. If we did not allow firms to finance investment through retained earnings, then a fixed cost that is high enough to match the (relatively low) empirical frequency of equity issuance, also tends to imply an average investment rate that is too low relative to our empirical target. Notice that shareholders are indifferent between receiving $\tilde{z}s_t$ units of good 2, or $\tilde{e}_t$ equity shares each worth ϕ_t^s units of good 2. And since the shadow value of good 2 is higher for entrepreneurs than for shareholders (because entrepreneurs have a higher valuation of the dividend of good 1 that results from investment of good 2 than shareholders), an entrepreneur always prefers retaining the earnings $\tilde{z}s_t$ of good 2 and issuing equity shares worth $\tilde{e}_t\phi_t^s$ units of good 2, rather paying out the $\tilde{z}s_t$ units of good 2 to investors as dividend. Thus, the capital structure assumption implicit in our treatment of the capital return of good 2 is compatible with the agents' incentives.

parameters are $\rho_n = 0.5$, $\bar{r} = 0.04/4$, and we choose the size of the policy shock, ε^m, so as to induce a 1% increase in stock prices (conditional on other parameter values). We assume all entrepreneurs enter with a given ratio of (claims to) good 2 to capital, $\omega_0 \equiv w_0/k_0 \in \mathbb{R}_{++}$, and set $\omega_0 = 2/3$, which is consistent with an average ratio of cash to assets of approximately 0.40 for firms upon entering the Compustat sample (e.g., Begenau and Palazzo (2020)).[62] For any investment rate $\iota \in \mathbb{R}_+$, we assume the adjustment cost is $\Psi(\iota) = \frac{\psi}{2}\iota^2$. We calibrate the values of ε_e, z, $\tilde{z}$, $\bar{\xi}$, and ψ so that the stationary equilibrium of our model matches the following five moments from the sample of Compustat firms used in our empirical analysis of Section 4: (*i*) median liquidity ratio, (*ii*) average investment rate for firms with below-median liquidity ratio, (*iii*) average investment rate for firms with above-median liquidity ratio, (*iv*) unconditional frequency of equity issuance across firms and time, (*v*) average ratio of equity issuance relative to total assets conditional on equity issuance.[63] Table 1 summarizes the calibration targets and the resulting parameter values.

6.2 Results

In this section we compare the theoretical and empirical impulse responses of investment to a money shock that induces a 1% increase in stock prices. To obtain the model counterparts of the impulse responses estimated in Section 4.3, we calculate the average dynamic responses of log investment for a large sample of firms drawn from the invariant distribution of the model.[64]

[62]The entrepreneur's problem is homogeneous of degree 1 in capital, so we only need to specify the ratio of good 2 to capital of entrants. Also, although we assume all entrepreneurs are identical upon entry, two idiosyncratic shocks, i.e., the fixed cost of equity issuance, and the exit shock) lead to *ex post* heterogeneity in entrepreneurs' balance sheets.

[63]We follow the standard practice in the corporate finance literature of classifying a firm i as "issuing equity" if the ratio of net equity issuance to assets, e_t^i, exceeds a specified threshold. One rationale for this practice is that, as pointed out in McKeon (2015), the timing of the proceeds from stock sales reported in firms' financial statements may reflect employees' decisions to exercise stock options rather than a managerial decision to sell stock, which is the relevant decision for our purposes. Since firm-initiated equity issuances tend to be large and infrequent, McKeon (2015) proposes using an issuance threshold as a reliable way to identify equity issuances that contain a firm-initiated component. Leary and Roberts (2005), for example, use a cutoff of 5% when working with annual Compustat data. We adopt a cutoff of 1% for our quarterly analysis.

[64]In the theory, monetary policy only affects investment through its effect on the equity prices of equity-dependent firms. So we don't face the identification problem discussed in Section 3.3 when working with model-generated data. The procedure to compute the impulse responses in the quantitative model is as follows. (1) Compute the stationary equilibrium, which involves computing the invariant distribution of liquid assets and outstanding equity (per unit of capital) across firms. (2) Draw a random sample of 20,000 firms from the stationary distribution, and label them as "low-liquidity" or "high-liquidity" depending on whether their ratio of liquid assets to capital is below or above the median of the stationary distribution. (3) Simulate the equilibrium path for each of these firms by drawing thirteen realizations of the fixed equity issuance shock. (4) Redo step (3) (for the same sample of firms, and conditional on the same realizations of equity issuance shocks), but instead

Table 1: Calibrated parameter values and calibration targets

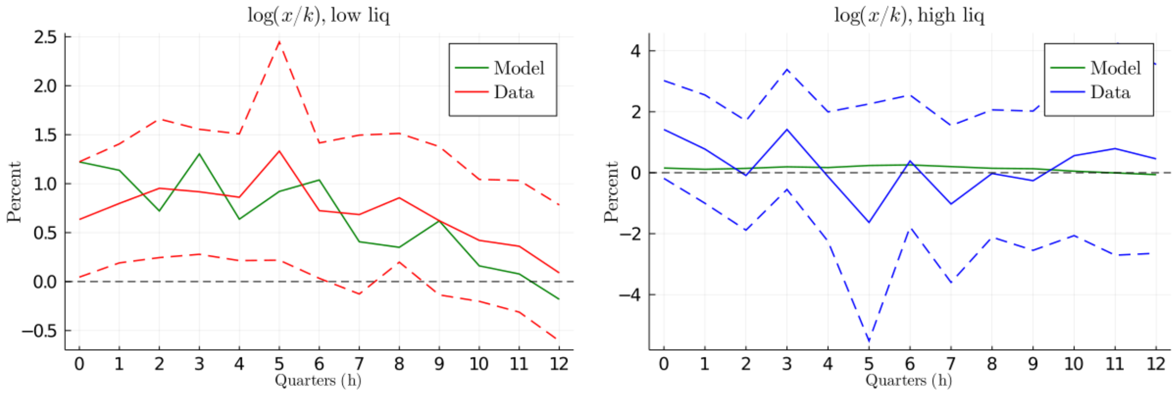
Parameter	Value	Target / Source
Externally calibrated		
β	0.995	2% annual real rate
$\bar{r}$	0.04/4	4% annualized nominal rate
δ	0.025	Conventional
$1-\pi$	0.017	Compustat exit (Begenau and Salomao, 2019)
σ_{ε}	2.56	Top 10% turnover ϕ_t^i response to MP
$(\alpha, \theta, \mu_{\varepsilon})$	$\left(1, 1, -\frac{\sigma_{\varepsilon}^2}{2}\right)$	Normalization (Lagos and Zhang, 2020b)
ω_0	2/3	Average cash-to-assets at IPO (Begenau and Palazzo, 2020)
Internally calibrated		
z	0.031	$\text{med}\left(\ell_t^i\right) = 7.96\%$ (**Model: 7.81%**)
$\tilde{z}$	0.038	$\text{avg}(I_t^i/K_t^i)\vert_{\mathbb{I}_{L,t-1}=1} = 2.74\%$ (**2.80%**)
ε_e	4.21	$\text{avg}(I_t^i/K_t^i)\vert_{\mathbb{I}_{L,t-1}=0} = 3.69\%$ (**3.68%**)
$\bar{\xi}$	0.244	$\text{freq}(\text{e}_t^i > 0.01) = 0.0714$ (**0.0719**)
ψ	27.80	$\text{avg}(\text{e}_t^i)\vert_{\text{e}_t^i>0.01} = 9.57\%$ (**9.73%**)

Figure 7 depicts the theoretical impulse responses of log investment rates alongside the corresponding point estimates and confidence intervals presented in panel (b) of Figure 4. In the theory, firms with liquidity ratios below the median of the invariant distribution increase their investment rates by roughly 1% on average in response to a monetary shock that increases Tobin's q by 1%. The path of the average response of low-liquidity firms is very similar in the model and the data. The average theoretical response for firms with liquidity ratios above the median of the invariant distribution is considerably smaller, consistent with our finding no evidence of the q-channel affecting the investment of high-liquidity firms in the data.[65]

of keeping the policy rate constant at the steady-state level as in step (3), assume it follows the autoregressive process described in the text (assuming firms have perfect foresight of the policy rate following the unexpected shock ε^m in the first of the thirteen periods). (5) For each firm and each of the thirteen periods, compute the difference between log investment in step (4) and log investment in step (3). The reported impulse responses for high- and low-liquidity firms is the average of these log differences across all sampled high- and low-liquidity firms, respectively.

[65]The average investment response of high-liquidity firms in this model with long-lived entrepreneurs is not exactly zero, e.g., as it was in the simpler model with two-period lived entrepreneurs of Section 2.2. This happens for two reasons. First, in any period when there is a reduction in the policy rate, some firms with above-median liquidity ratios are getting low enough draws of the equity issuance cost, ξ_t, and take advantage of the beneficial conditions to issue equity. Second, because the monetary shock is persistent and its effect on stock prices lasts for several periods, firms with liquidity ratios that are higher than the median but still relatively low, anticipate they will be issuing equity soon, which combined with the investment-smoothing motive introduced by the convex adjustment cost, induces them to increase investment financed with their own liquid asset holdings from the time

Figure 7: Comparison of investment rate responses from model and data estimates



(a) Low liquidity ratio firms (b) High liquidity ratio firms

Notes: Data refers to point estimates and 95% confidence intervals for γ_{h,h_q} and $\gamma_{h,h_q} + \tilde{\gamma}_{h,h_q}$ from specification (25) with $y_t^i = \mathrm{x}_t^i$ as the outcome variable. *Model* response is computed as the average firm-level impulse response of log investment rates, averaged over a large panel of firms drawn from the stationary distribution of the model. High and low liquidity ratios are defined as above or below the cross-sectional median cash-to-assets ratio in both model and the data.

7 Conclusion

Over 50 years ago, Tobin (1969) outlined a "general equilibrium approach to monetary theory" proposing that the principal way in which financial policies and events affect the economy is by changing the valuation of physical assets relative to their replacement cost—a variable he denoted "q." Since then, Tobin's q has played a key role in the theory of Investment, but—despite being its *raison d'être*—the role of Tobin's q in the transmission of monetary shocks only subsists in undergraduate textbook narratives of a long list of plausible monetary transmission mechanisms.

In this paper we have taken two steps toward (re-)establishing Tobin's q as a major conduit between monetary policy and the real economy. First, we have developed an empirical identification strategy for the q-channel, and have used it to quantify its relevance in the transmission of monetary policy to the capital structure and investment decisions of the corporate sector in the United States. Second, we have developed a theoretical model that clarifies the roles that financial constraints (as a determinant of a firm's dependence on equity financing for investment), the stock market (as a mechanism where outside investors determine the market price of

of the money shock (even though they may not yet be accessing the equity market).

equity claims on firms), and money (as a means of payment in financial trades among outside investors) play in the transmission of monetary policy shocks through stock prices. We hope the identification strategy and the theoretical mechanisms that we have described here will be useful to study the effects of other financial or policy shocks on the economy.

References

Abel, A. B. and J. C. Eberly (2002): "Investment and q with fixed costs: An empirical analysis," Working paper, University of Pennsylvania.

Acharya, V. V., S. Byoun, and Z. Xu (2020): "The Sensitivity of Cash Savings to the Cost of Capital," Working Paper 27517, NBER.

Amihud, Y. and S. Levi (2022): "The Effect of Stock Liquidity on the Firm's Investment and Production," *Review of Financial Studies*.

Baker, M., J. C. Stein, and J. Wurgler (2003): "When Does the Market Matter? Stock Prices and the Investment of Equity-Dependent firms," *Quarterly Journal of Economics*, 118, 969–1005.

Baker, M. and J. Wurgler (2002): "Market Timing and Capital Structure," *Journal of Finance*, 57, 1–32.

Begenau, J. and B. Palazzo (2020): "Firm Selection and Corporate Cash Holdings," *Journal of Financial Economics*, forthcoming.

Begenau, J. and J. Salomao (2019): "Firm Financing Over the Business Cycle," *Review of Financial Studies*, 32, 1235–1274.

Bernanke, B. and M. Gertler (1989): "Agency Costs, Net Worth, and Business Fluctuations," *American Economic Review*, 79, 14–31.

Bernanke, B. S. and K. N. Kuttner (2005): "What Explains the Stock Market's Reaction to Federal Reserve Policy?" *Journal of Finance*, 60, 1221–1257.

Blanchard, O., C. Rhee, and L. Summers (1993): "The Stock Market, Profit, and Investment," *Quarterly Journal of Economics*, 108, 115–136.

Boivin, J., M. T. Kiley, and F. S. Mishkin (2010): "How has the Monetary Transmission Mechanism Evolved Over Time?" in *Handbook of Monetary Economics*, Elsevier, vol. 3, 369–422.

Brainard, W. C. and J. Tobin (1968): "Pitfalls in Financial Model Building," *American Economic Review*, 58, 99–122.

Christiano, L. J., M. Eichenbaum, and C. L. Evans (2005): "Nominal Rigidities and the Dynamic Effects of a Shock to Monetary Policy," *Journal of Political Economy*, 113, 1–45.

Cloyne, J., C. Ferreira, M. Froemel, and P. Surico (2018): "Monetary Policy, Corporate Finance and Investment," NBER Working Paper No. 25366.

Cochrane, J. H. and M. Piazzesi (2002): "The Fed and Interest Rates: a High-Frequency Identification," *American Economic Review*, 92, 90–95.

Cook, T. and T. Hahn (1989): "The Effect of Changes in the Federal Funds Rate Target on Market Interest Rates in the 1970s," *Journal of Monetary Economics*, 24, 331–351.

Duffie, D., N. Gârleanu, and L. H. Pedersen (2005): "Over-the-Counter Markets," *Econometrica*, 73, 1815–1847.

Eberly, J., S. Rebelo, and N. Vincent (2012): "What explains the lagged-investment effect?" *Journal of Monetary Economics*, 59, 370–380.

Fischer, S. and R. C. Merton (1984): "Macroeconomics and Finance: The Role of the Stock Market," in *Carnegie-Rochester Conference Series on Public Policy*, Elsevier, vol. 21, 57–108.

Gilchrist, S., C. P. Himmelberg, and G. Huberman (2005): "Do Stock Price Bubbles Influence Corporate Investment?" *Journal of Monetary Economics*, 52, 805–827.

Gürkaynak, R. S., B. Sack, and E. T. Swansonc (2005): "Do Actions Speak Louder Than Words? The Response of Asset Prices to Monetary Policy Actions and Statements," *International Journal of Central Banking*, 1, 55–94.

Hayashi, F. (1982): "Tobin's Marginal q and Average q: A Neoclassical Interpretation," *Econometrica*, 213–224.

Hovakimian, A., G. Hovakimian, and H. Tehranian (2004): "Determinants of Target Capital Structure: The Case of Dual Debt and Equity Issues," *Journal of Financial Economics*, 71, 517–540.

Jarociński, M. and P. Karadi (2020): "Deconstructing Monetary Policy Surprises – The Role of Information Shocks," *American Economic Journal: Macroeconomics*, 12, 1–43.

Jeenas, P. (2019): "Firm Balance Sheet Liquidity, Monetary Policy Shocks, and Investment Dynamics," Working paper, Universitat Pompeu Fabra.

Jordà, O. (2005): "Estimation and inference of impulse responses by local projections," *American Economic Review*, 95, 161–182.

Jorgenson, D. W. (1963): "Capital Theory and Investment Behavior," *American Economic Review*, 53, 247–259.

Kaplan, S. N. and L. Zingales (1997): "Do Investment-Cash Flow Sensitivities Provide Useful Measures of Financing Constraints?*," *Quarterly Journal of Economics*, 112, 169–215.

Keynes, J. M. (1936): *The General Theory of Employment, Interest and Money*, Macmillan, 14th edition, 1973.

Kiyotaki, N. and J. Moore (1997): "Credit Cycles," *Journal of Political Economy*, 105, 211–248.

Korajczyk, R. A. and A. Levy (2003): "Capital Structure Choice: Macroeconomic Conditions and Financial Constraints," *Journal of Financial Economics*, 68, 75–109.

Kuttner, K. N. (2001): "Monetary Policy Surprises and Interest Rates: Evidence from the Fed Funds Futures Market," *Journal of Monetary Economics*, 47, 523–544.

Lagos, R. and G. Rocheteau (2009): "Liquidity in Asset Markets With Search Frictions," *Econometrica*, 77, 403–426.

Lagos, R., G. Rocheteau, and P.-O. Weill (2011): "Crises and Liquidity in Over-the-Counter Markets," *Journal of Economic Theory*, 146, 2169–2205.

Lagos, R. and S. Zhang (2015): "Monetary Exchange in Over-the-Counter Markets: A Theory of Speculative Bubbles, the Fed Model, and Self-Fulfilling Liquidity Crises," Working Paper 21528, NBER.

——— (2019): "The Limits of onetary Economics: On Money as a Medium of Exchange in Near-Cashless Credit Economies," Working Paper 25803, NBER.

——— (2020a): "The Limits of onetary Economics: On Money as a Latent Medium of Exchange," Working Paper 26756, NBER.

——— (2020b): "Turnover Liquidity and the Transmission of Monetary Policy," *American Economic Review*, 110, 1635–1672.

Leary, M. T. and M. R. Roberts (2005): "Do Firms Rebalance Their Capital Structures?" *The Journal of Finance*, 60, 2575–2619.

McKeon, S. B. (2015): "Employee Option Exercise and Equity Issuance Motives," Working Paper.

Miranda-Agrippino, S. and G. Ricco (2019): "The Transmission of Monetary Policy Shocks," Working Paper.

Mishkin, F. S. (1995): "Symposium on the Monetary Transmission Mechanism," *Journal of Economic Perspectives*, 9, 3–10.

——— (1996): "The Channels of Monetary Transmission: Lessons for Monetary Policy," Working Paper 5464, NBER.

——— (2001): "The Transmission Mechanism and the Role of Asset Prices in Monetary Policy," Working Paper 8617, NBER.

Morck, R., A. Shleifer, R. W. Vishny, M. Shapiro, and J. M. Poterba (1990): "The Stock Market and Investment: Is the Market a Sideshow?" *Brookings Papers on Economic Activity*, 1990, 157–215.

Myers, S. C. and N. S. Majluf (1984): "Corporate Financing and Investment Decisions When Firms Have Information that Investors Do Not Have," *Journal of Financial Economics*, 13, 187–221.

Nakamura, E. and J. Steinsson (2018): "High Frequency Identification of Monetary Non-Neutrality: The Information Effect," *Quarterly Journal of Economics*, 133, 1283–1330.

Ottonello, P. and T. Winberry (2020): "Financial Heterogeneity and the Investment Channel of Monetary Policy," *Econometrica*, 88, 2473–2502.

Polk, C. and P. Sapienza (2008): "The Stock Market and Corporate Investment: A Test of Catering Theory," *Review of Financial Studies*, 22, 187–217.

Rigobon, R. and B. Sack (2004): "The Impact of Monetary Policy on Asset Prices," *Journal of Monetary Economics*, 51, 1553–1575.

Stock, J. H. and M. W. Watson (2018): "Identification and estimation of dynamic causal effects in macroeconomics using external instruments," *The Economic Journal*, 128, 917–948.

Thorbecke, W. (1997): "On Stock Market Returns and Monetary Policy," *Journal of Finance*, 52, 635–654.

Tobin, J. (1969): "A General Equilibrium Approach to Monetary Theory," *Journal of Money, Credit and Banking*, 1, 15–29.

Tobin, J. and W. C. Brainard (1976): "Asset Markets and the Cost of Capital," Tech. rep., Cowles Foundation for Research in Economics, Yale University.

Weill, P.-O. (2007): "Leaning Against the Wind," *Review of Economic Studies*, 74, 1329–1354.

A Proofs

A.1 Investor's portfolio and bargaining problems

Lemma 1 *Let*

$$\varepsilon_t^* \equiv \frac{p_t\phi_t^m - \pi(1-\delta)\phi_t^s}{z} \tag{30}$$

and define the correspondence $\chi : \mathbb{R}^2 \rightrightarrows [0,1]$ *as*

$$\chi\left(\varepsilon_t^*, \varepsilon\right) \begin{cases} = 1 & \text{if } \varepsilon_t^* < \varepsilon \\ \in [0,1] & \text{if } \varepsilon_t^* = \varepsilon \\ = 0 & \text{if } \varepsilon < \varepsilon_t^*. \end{cases}$$

Consider a bilateral meeting in the first subperiod of period t between a dealer and an investor with portfolio $\boldsymbol{a}_t$ and valuation ε. The investor's post-trade portfolio,

$$\bar{\boldsymbol{a}}\left(\boldsymbol{a}_t, \varepsilon\right) \equiv \left(\overline{a}_t^b\left(\boldsymbol{a}_t, \varepsilon\right), \overline{a}_t^m\left(\boldsymbol{a}_t, \varepsilon\right), \overline{a}_t^s\left(\boldsymbol{a}_t, \varepsilon\right)\right),$$

is given by

$$\begin{aligned} \overline{a}_t^b\left(\boldsymbol{a}_t, \varepsilon\right) &= a_t^b \\ \overline{a}_t^m\left(\boldsymbol{a}_t, \varepsilon\right) &= \left[1 - \chi\left(\varepsilon_t^*, \varepsilon\right)\right]\left(a_t^m + p_t a_t^s\right) \\ \overline{a}_t^s\left(\boldsymbol{a}_t, \varepsilon\right) &= a_t^s + \frac{1}{p_t}\left[a_t^m - \overline{a}_t^m\left(\boldsymbol{a}_t, \varepsilon\right)\right], \end{aligned}$$

and the intermediation fee charged by the dealer is

$$\varpi_t\left(\boldsymbol{a}_t, \varepsilon\right) = (1-\theta)\left(\varepsilon_t^* - \varepsilon\right) z \frac{1}{p_t}\left[\overline{a}_t^m\left(\boldsymbol{a}_t, \varepsilon\right) - a_t^m\right].$$

Proof. The value function (2) can be written as

$$\begin{aligned} W_t\left(\boldsymbol{a}_t, \varpi_t\right) &= \boldsymbol{\phi}_t' \boldsymbol{a}_t - \varpi_t + \bar{W}_t \\ &= a_t^b + \phi_t^m a_t^m + \phi_t^s a_t^s - \varpi_t + \bar{W}_t, \end{aligned} \tag{31}$$

where

$$\bar{W}_t \equiv T_t + \max_{\boldsymbol{a}_{t+1} \in \mathbb{R}_+^3}\left[-\boldsymbol{\phi}_t \boldsymbol{a}_{t+1} + \beta \int V_{t+1}\left(\boldsymbol{a}_{t+1}, \varepsilon\right) dG(\varepsilon)\right]. \tag{32}$$

With (31) we can write

$$\begin{aligned} \Gamma\left(\overline{\boldsymbol{a}}_t, \boldsymbol{a}_t, \varepsilon\right) = {} & \overline{a}_t^b + \phi_t^m \overline{a}_t^m + \left(\varepsilon z + \pi(1-\delta)\phi_t^s\right)\overline{a}_t^s \\ & - \left[a_t^b + \phi_t^m a_t^m + \left(\varepsilon z + \phi_t^s \pi(1-\delta)\right) a_t^s\right] - \varpi_t, \end{aligned}$$

so the solution to (1) is

$$\begin{aligned}
\overline{a}_t^b(\boldsymbol{a}_t,\varepsilon) &= a_t^b \\
\overline{a}_t^s(\boldsymbol{a}_t,\varepsilon) &= a_t^s+\frac{1}{p_t}[a_t^m-\overline{a}_t^m(\boldsymbol{a}_t,\varepsilon)] \\
\varpi_t(\boldsymbol{a}_t,\varepsilon) &= (1-\theta)(\varepsilon_t^*-\varepsilon)z\frac{1}{p_t}[\overline{a}_t^m(\boldsymbol{a}_t,\varepsilon)-a_t^m] \\
\overline{a}_t^m(\boldsymbol{a}_t,\varepsilon) &= \arg\max_{0\le\overline{a}_t^m\le p_t a_t^s+a_t^m}\left[(\varepsilon_t^*-\varepsilon)z\frac{1}{p_t}(\overline{a}_t^m-a_t^m)\right].
\end{aligned}$$

This concludes the proof. ■

Lemma 2 *Let* $\left(a_{t+1}^b,a_{t+1}^m,a_{t+1}^s\right)$ *denote the portfolio chosen by an investor in the second subperiod of period* t*. This portfolio must satisfy the following first-order necessary and sufficient conditions:*

$$\phi_t^b\ge\beta,\ \textit{with "= " if } a_{t+1}^b>0 \tag{33}$$

$$\phi_t^m\ge\beta\left[\phi_{t+1}^m+\alpha\theta\int_{\varepsilon_{t+1}^*}^{\varepsilon_H}\left(\varepsilon-\varepsilon_{t+1}^*\right)zdG(\varepsilon)\frac{1}{p_{t+1}}\right],\ \textit{with "= " if } a_{t+1}^m>0 \tag{34}$$

$$\phi_t^s\ge\beta\left[\bar{\varepsilon}z+\pi(1-\delta)\phi_{t+1}^s+\alpha\theta\int_{\varepsilon_L}^{\varepsilon_{t+1}^*}\left(\varepsilon_{t+1}^*-\varepsilon\right)zdG(\varepsilon)\right],\ \textit{with "= " if } a_{t+1}^s>0. \tag{35}$$

Proof. With (31) and the bargaining outcome described in the statement of Lemma 1, (3) can be written as

$$\begin{aligned}
V_t(\boldsymbol{a}_t,\varepsilon) &= a_t^b+(\varepsilon z+\pi(1-\delta)\phi_t^s)a_t^s+\phi_t^m a_t^m+\bar{W}_t \\
&\quad+\alpha\theta(\varepsilon-\varepsilon_t^*)z\frac{1}{p_t}[a_t^m-\overline{a}_t^m(\boldsymbol{a}_t,\varepsilon)].
\end{aligned}$$

Hence, using the expression for $\overline{a}_{t+1}^m(\boldsymbol{a}_{t+1},\varepsilon)$ from Lemma 1,

$$\begin{aligned}
\int V_{t+1}(\boldsymbol{a}_{t+1},\varepsilon)\,dG(\varepsilon) &= a_{t+1}^b+\left[\bar{\varepsilon}z+\pi(1-\delta)\phi_{t+1}^s+\alpha\theta\int_{\varepsilon_L}^{\varepsilon_{t+1}^*}\left(\varepsilon_{t+1}^*-\varepsilon\right)zdG(\varepsilon)\right]a_{t+1}^s \\
&\quad+\left[\phi_{t+1}^m+\alpha\theta\frac{1}{p_{t+1}}\int_{\varepsilon_{t+1}^*}^{\varepsilon_H}\left(\varepsilon-\varepsilon_{t+1}^*\right)zdG(\varepsilon)\right]a_{t+1}^m+\bar{W}_{t+1}
\end{aligned}$$

.

Thus, the necessary and sufficient first-order conditions corresponding to the maximization problem in (32) are as in the statement of the lemma. ■

A.2 Stock-market clearing

Lemma 3 *In period t, the first-subperiod market-clearing condition for equity is*

$$[1 - G(\varepsilon_t^*)] \frac{1}{p_t} A_t^m = G(\varepsilon_t^*) S_t. \quad (36)$$

Proof. Recall that $\bar{A}_{It}^s = \alpha \int \bar{a}_t^s(\boldsymbol{a}_t, \varepsilon) dH_{It}(\boldsymbol{a}_t, \varepsilon)$, so using the bargaining outcomes in Lemma 1, we have

$$\bar{A}_{It}^s = \alpha [1 - G(\varepsilon_t^*)] \left(S_t + \frac{1}{p_t} A_t^m \right).$$

With this expression, the market-clearing condition for equity in the first subperiod of period t, i.e., $\bar{A}_{It}^s = \alpha S_t$, can be written as (36). ■

A.3 Equilibrium characterization: stock prices and real money balances

The following result characterizes the equilibrium paths $\{\mathrm{M}_t\}_{t=0}^{\infty}$ and $\{\phi_t^s\}_{t=0}^{\infty}$ taking as given the path for the outstanding aggregate quantity of stocks, $\{S_t\}_{t=0}^{\infty}$.

Corollary 3 *In equilibrium, aggregate real money balances, $\{\mathrm{M}_t\}_{t=0}^{\infty}$, and the real price of equity shares, $\{\phi_t^s\}_{t=0}^{\infty}$, satisfy the following conditions:*

$$\mathrm{M}_t \geq \frac{\beta}{\mu} \left[1 + \alpha\theta \int_{\varepsilon_{t+1}^*}^{\varepsilon_H} \frac{\varepsilon - \varepsilon_{t+1}^*}{\varepsilon_{t+1}^* z + \pi(1-\delta)\phi_{t+1}^s} z dG(\varepsilon) \right] \mathrm{M}_{t+1}, \text{ with "} = \text{" if } \mathrm{M}_{t+1} > 0 \quad (37)$$

$$\phi_t^s = \beta \left[\bar{\varepsilon} z + \pi(1-\delta)\phi_{t+1}^s + \alpha\theta \int_{\varepsilon_L}^{\varepsilon_{t+1}^*} \left(\varepsilon_{t+1}^* - \varepsilon \right) z dG(\varepsilon) \right], \quad (38)$$

where for all $t \geq 0$, ε_t^* satisfies

$$\frac{1 - G(\varepsilon_t^*)}{\varepsilon_t^* z + \pi(1-\delta)\phi_t^s} \mathrm{M}_t = G(\varepsilon_t^*) S_t. \quad (39)$$

Proof. Conditions (37), (38), and (39) follow from (34), (35), and (36), respectively, using $\mathrm{M}_t \equiv \phi_t^m A_t^m$, $A_{t+1}^m / A_t^m = \mu$, and (30). ■

The following result characterizes the equilibrium paths $\{\mathrm{M}_t\}_{t=0}^{\infty}$ and $\{\phi_t^s\}_{t=0}^{\infty}$ taking as given the path for the outstanding aggregate quantity of stocks, $\{S_t\}_{t=0}^{\infty}$—in the context of a stationary equilibrium.

Corollary 4 *In a stationary equilibrium,* $S_t = S$*,* $\varepsilon_t^* = \varepsilon^*$*,* $\phi_t^s = \varphi^s z$*, and* $\mathrm{M}_t = \mathrm{M}$ *for all* t*, and* $(\varepsilon^*, \varphi^s, \mathrm{M})$ *satisfy the following conditions:*

$$r \geq \alpha\theta \int_{\varepsilon^*}^{\varepsilon_H} \frac{\varepsilon - \varepsilon^*}{\varepsilon^* + \pi(1-\delta)\varphi^s} dG(\varepsilon), \text{ with "= " if } \mathrm{M} > 0 \quad (40)$$

$$\varphi^s = \frac{\beta}{1 - \beta\pi(1-\delta)} \left[\bar{\varepsilon} + \alpha\theta \int_{\varepsilon_L}^{\varepsilon^*} (\varepsilon^* - \varepsilon)\, dG(\varepsilon) \right], \quad (41)$$

where ε^* *satisfies*

$$\frac{1 - G(\varepsilon^*)}{[\varepsilon^* + \pi(1-\delta)\varphi^s]\, z} \mathrm{M} = G(\varepsilon^*)\, S. \quad (42)$$

Proof. Conditions (37)-(39) follow immediately from (40)-(42) imposing the stationarity conditions described in the statement. ■

Lemma 4 *Let* $S > 0$ *be given. Then:*

(i) There always exists a solution to (40)-(42) in which money is not valued, i.e., $\mathrm{M} = 0$*,* $\varepsilon^* = \varepsilon_L$*, and* $\varphi^s = \frac{\beta}{1-\beta\pi(1-\delta)}\bar{\varepsilon}$*.*

(ii) Let

$$\bar{r} \equiv \frac{\alpha\theta\,(\bar{\varepsilon} - \varepsilon_L)}{\varepsilon_L + \frac{\beta\pi(1-\delta)}{1-\beta\pi(1-\delta)}\bar{\varepsilon}}.$$

If $r \in (0, \bar{r})$ *there exists a unique solution to (40)-(42) with* $\mathrm{M} > 0$*, i.e.,*

$$\mathrm{M} = \frac{G(\varepsilon^*)\,[\varepsilon^* + \pi(1-\delta)\varphi^s]\, z}{1 - G(\varepsilon^*)} S \quad (43)$$

$$\varphi^s = \frac{\beta}{1 - \beta\pi(1-\delta)} \left[\bar{\varepsilon} + \alpha\theta \int_{\varepsilon_L}^{\varepsilon^*} (\varepsilon^* - \varepsilon)\, dG(\varepsilon) \right], \quad (44)$$

where $\varepsilon^* \in (\varepsilon_L, \varepsilon_H]$ *is the unique solution to*

$$\frac{\alpha\theta \int_{\varepsilon^*}^{\varepsilon_H} (\varepsilon - \varepsilon^*)\, dG(\varepsilon)}{\varepsilon^* + \frac{\beta\pi(1-\delta)}{1-\beta\pi(1-\delta)} \left[\bar{\varepsilon} + \alpha\theta \int_{\varepsilon_L}^{\varepsilon^*} (\varepsilon^* - \varepsilon)\, dG(\varepsilon) \right]} = r. \quad (45)$$

Moreover:

(a) As $r \to \bar{r}$*,* $\varepsilon^* \to \varepsilon_L$*,* $\mathrm{M} \to 0$*, and* $\varphi^s \to \frac{\beta}{1-\beta\pi(1-\delta)}\bar{\varepsilon}$*.*

(b) As $r \to 0$*,* $\varepsilon^* \to \varepsilon_H$ *and* $\varphi^s \to \frac{\beta}{1-\beta\pi(1-\delta)}\left[\bar{\varepsilon} + \alpha\theta\,(\varepsilon_H - \bar{\varepsilon})\right]$*.*

(c) $\frac{\partial \varepsilon^*}{\partial r} < 0$*,* $\frac{\partial \mathrm{M}}{\partial r} < 0$*, and* $\frac{\partial \varphi^s}{\partial r} < 0$*.*

Proof. To establish part (i), simply set $\mathrm{M} = 0$ in (40)-(42). To establish part (ii), proceed as follows. Assume $\mathrm{M} > 0$; then (40) holds with equality, and using (41) to substitute φ^s from (40) gives $T(\varepsilon^*; r) = 0$, where

$$T(\varepsilon^*; r) \equiv \frac{\alpha\theta \int_{\varepsilon^*}^{\varepsilon_H} (\varepsilon - \varepsilon^*) \, dG(\varepsilon)}{\varepsilon^* + \frac{\beta\pi(1-\delta)}{1-\beta\pi(1-\delta)} \left[\bar{\varepsilon} + \alpha\theta \int_{\varepsilon_L}^{\varepsilon^*} (\varepsilon^* - \varepsilon) \, dG(\varepsilon)\right]} - r.$$

First, notice that

$$\frac{\partial T(\varepsilon^*; r)}{\partial \varepsilon^*} = -\frac{[1-G(\varepsilon^*)]\left\{\varepsilon^* + \frac{\beta\pi(1-\delta)}{1-\beta\pi(1-\delta)}\left[\bar{\varepsilon} + \alpha\theta \int_{\varepsilon_L}^{\varepsilon^*} (\varepsilon^* - \varepsilon) dG(\varepsilon)\right]\right\} + \left[\int_{\varepsilon^*}^{\varepsilon_H} (\varepsilon - \varepsilon^*) dG(\varepsilon)\right]\left[1 + \frac{\beta\pi(1-\delta)}{1-\beta\pi(1-\delta)} \alpha\theta G(\varepsilon^*)\right]}{\frac{1}{\alpha\theta}\left\{\varepsilon^* + \frac{\beta\pi(1-\delta)}{1-\beta\pi(1-\delta)}\left[\bar{\varepsilon} + \alpha\theta \int_{\varepsilon_L}^{\varepsilon^*} (\varepsilon^* - \varepsilon) dG(\varepsilon)\right]\right\}^2} < 0.$$

Assume $r \in (0, \bar{r})$. Then

$$T(\varepsilon_H; r) = -r < 0 < T(\varepsilon_L; r) = \bar{r} - r. \tag{46}$$

Since T is a continuous function of ε^*, $\partial T(\varepsilon^*; r) / \partial \varepsilon^* < 0$ and (46) imply that for any $r \in (0, \bar{r})$ there exists a unique ε^* that solves $T(\varepsilon^*; r) = 0$ on the interval $(\varepsilon_L, \varepsilon_H)$. Given the ε^* that solves $T(\varepsilon^*; r) = 0$, M and ϕ_t^s are given by (43) and (44), respectively.

Part $(ii)(a)$ is immediate from (43) and (44), and the observation that $T(\varepsilon_L; \bar{r}) = 0$. Part $(ii)(b)$ is immediate from (44), and the observation that $T(\varepsilon_H; 0) = 0$. Part $(ii)(c)$, follows from

$$\begin{aligned} \frac{\partial \mathrm{M}}{\partial r} &= \frac{G'(\varepsilon^*)}{[1 - G(\varepsilon^*)]^2} S \frac{\partial \varepsilon^*}{\partial r} + \frac{G(\varepsilon^*)}{1 - G(\varepsilon^*)} \frac{\partial S}{\partial r} \\ \frac{\partial \varphi^s}{\partial r} &= \alpha\theta \frac{\beta}{1 - \beta\pi(1-\delta)} G(\varepsilon^*) \frac{\partial \varepsilon^*}{\partial r} \end{aligned}$$

together with the fact that

$$\frac{\partial \varepsilon^*}{\partial r} = \frac{1}{\frac{\partial T(\varepsilon^*; r)}{\partial \varepsilon^*}}$$

and $\partial T(\varepsilon^*; r) / \partial \varepsilon^* < 0$. ■

A.4 Economy with $\pi = 0$

A.4.1 Entrepreneur's choice of investment and capital structure

Proof of Proposition 1. The Lagrangian for the optimization problem of the one-period-lived entrepreneur at entry, i.e., (11), is

$$\begin{aligned} \mathcal{L} = \; & y + \phi_e^s [(1-\delta) k + x - s_{+1}] \\ & + \xi [\phi^s s_{+1} + w - y - \mathrm{C}(x/k) k] \\ & + \zeta_L^e s_{+1} + \zeta_H^e [(1-\delta) k + x - s_{+1}] + \zeta_L^c y, \end{aligned}$$

where ξ, ζ_L^e, ζ_H^e, and ζ_L^c are the Lagrange multipliers on the entrepreneur's budget constraint, nonnegativity constraint on equity issuance, upper bound on equity issuance, and nonnegativity constraint on consumption, respectively.

The first-order conditions are

$$0 = 1 - \xi + \zeta_L^c \tag{47}$$

$$0 = \phi_e^s - \xi \mathrm{C}'(x/k) + \zeta_H^e \tag{48}$$

$$0 = -\phi_e^s + \xi \phi^s + \zeta_L^e - \zeta_H^e \tag{49}$$

$$0 = \xi \left[\phi^s s_{+1} + w - y - \mathrm{C}(x/k) k\right] \tag{50}$$

$$0 = \zeta_L^c y \tag{51}$$

$$0 = \zeta_L^e s_{+1} \tag{52}$$

$$0 = \zeta_H^e \left[(1-\delta) k + x - s_{+1}\right]. \tag{53}$$

Conditions (47)-(49) are the first-order conditions with respect to y, x, and s_{+1}, respectively. Condition (47) implies $\xi = 1 + \zeta_L^c > 0$, so (50) implies

$$0 = \phi^s s_{+1} + w - y - \mathrm{C}(x/k) k. \tag{54}$$

There are potentially eight cases depending on whether the multipliers $(\zeta_L^c, \zeta_L^e, \zeta_H^e)$ are positive or equal to zero. We consider each in turn. Recall ι_0 is the investment rate that satisfies $\mathrm{C}'(\iota_0) = 1$, so $\mathrm{C}'' > 0$ and the assumptions $\delta - \iota_0 \leq 1 \leq \phi^s$ in the statement of the proposition imply

$$\delta - 1 \leq \iota_0 \leq \iota(\phi^s). \tag{55}$$

Case 1: $\zeta_L^e = \zeta_H^e = 0 < \zeta_L^c$. In this case condition (51) implies

$$y = 0,$$

condition (54) implies

$$\phi^s s_{+1} = \mathrm{C}(x/k) k - w, \tag{56}$$

and conditions (48) and (49) imply

$$\mathrm{C}'(x/k) = \phi^s.$$

For this case to be a solution we need three conditions to be satisfied. First, $0 < \zeta_L^c$, which by (47) is equivalent to $\xi > 1$, which by (49) is equivalent to

$$\phi^s < \phi_e^s.$$

Second, since the solution must satisfy the constraints $0 \le s_{+1} \le (1-\delta)\,k + x$, (56) implies we must have

$$\Xi\left(\iota\left(\phi^s\right)\right) \le \omega \le \mathrm{C}\left(\iota\left(\phi^s\right)\right),$$

where

$$\Xi\left(\iota\right) \equiv \mathrm{C}\left(\iota\right) - \mathrm{C}'\left(\iota\right)\left(1-\delta+\iota\right). \tag{57}$$

Notice $\Xi\left(\iota_0\right) = \delta - 1 \le 0$ and $\Xi'\left(\iota\right) = -\,\mathrm{C}''\left(\iota\right)\left(1-\delta+\iota\right) \le 0$ for all $\iota \ge \iota_0$, so (55) implies the condition $\Xi\left(\iota\left(\phi^s\right)\right) \le \omega$ is satisfied for any $\omega \ge 0$.

Case 2: $\zeta_L^c = \zeta_H^e = 0 < \zeta_L^e$. In this case (47) implies $\xi = 1$, (48) implies

$$\mathrm{C}'\left(x/k\right) = \phi_e^s,$$

(49) implies

$$\zeta_L^e = \phi_e^s - \phi^s, \tag{58}$$

(52) implies

$$s_{+1} = 0,$$

and (54) implies

$$y = w - \mathrm{C}(\iota(\phi_e^s))k. \tag{59}$$

For this case to be a solution we need three conditions to be satisfied. First, $0 < \zeta_L^e$, which by (58) is equivalent to

$$\phi^s < \phi_e^s. \tag{60}$$

Second, $0 \le y$, which by (59) is equivalent to

$$\mathrm{C}(\iota(\phi_e^s))k \le w.$$

Third, $0 \le k_{+1} - s_{+1}$, is equivalent to

$$0 \le 1 - \delta + \iota(\phi_e^s).$$

This condition is implied by (55) and (60).

Case 3: $\zeta_L^c = \zeta_L^e = 0 < \zeta_H^e$. In this case (47) implies $\xi = 1$, (49) implies

$$\zeta_H^e = \phi^s - \phi_e^s, \tag{61}$$

and this together with (48) implies

$$\mathrm{C}'(x/k) = \phi^s.$$

Then condition (53) implies

$$s_{+1} = [1 - \delta + \iota(\phi^s)]\, k \tag{62}$$

and (54) implies

$$y = \{\phi^s [1 - \delta + \iota(\phi^s)] + \omega - \mathrm{C}(\iota(\phi^s))\}\, k. \tag{63}$$

For this case to be a solution we need three conditions to be satisfied. First, $0 < \zeta_H^e$, which by (61) is equivalent to

$$\phi_e^s < \phi^s.$$

Second, $0 \leq s_{+1}$, which by (62) is equivalent to

$$0 \leq 1 - \delta + \iota(\phi^s).$$

This condition is implied by (55). Third, $0 \leq y$, which by (63) is equivalent to

$$\Xi(\iota(\phi^s)) \leq \omega, \tag{64}$$

where $\Xi(\cdot)$ is as defined in (57). Notice $\Xi(\iota_0) = \delta - 1 \leq 0$ and $\Xi'(\iota) = -\mathrm{C}''(\iota)(1 - \delta + \iota) \leq 0$ for all $\iota \geq \iota_0$, so (55) implies (64) is satisfied for any $\omega \geq 0$.

Case 4: $\zeta_H^e = 0 < \min(\zeta_L^c, \zeta_L^e)$. In this case (51) implies

$$y = 0,$$

(52) implies

$$s_{+1} = 0, \tag{65}$$

and hence (54) implies

$$x/k = \mathrm{C}^{-1}(\omega). \tag{66}$$

Conditions (47) and (48) imply

$$\zeta_L^c = \frac{\phi_e^s - \mathrm{C}'(\mathrm{C}^{-1}(\omega))}{\mathrm{C}'(\mathrm{C}^{-1}(\omega))}, \tag{67}$$

and conditions (48) and (49) imply

$$\zeta_L^e = \frac{\mathrm{C}'(\mathrm{C}^{-1}(\omega)) - \phi^s}{\mathrm{C}'(\mathrm{C}^{-1}(\omega))}\phi_e^s. \tag{68}$$

For this case to be a solution we need three conditions to be satisfied. First, $0 < \zeta_L^c$, which by (67) is equivalent to

$$\mathrm{C}'(\mathrm{C}^{-1}(\omega)) < \phi_e^s \Leftrightarrow \mathrm{C}^{-1}(\omega) < \iota(\phi_e^s) \tag{69}$$

Second, $0 < \zeta_L^e$, which by (68) is equivalent to

$$\phi^s < \mathrm{C}'(\mathrm{C}^{-1}(\omega)) \Leftrightarrow \iota(\phi^s) < \mathrm{C}^{-1}(\omega). \tag{70}$$

Notice that conditions (69) and (70) can both be satisfied only if

$$\phi^s < \phi_e^s.$$

The third condition that needs to be satisfied for this case to be a solution is $0 \leq k_{+1} - s_{+1}$, which (using $k_{+1} = (1-\delta)k + x$, (65), and (66)) is equivalent to

$$0 \leq 1 - \delta + \mathrm{C}^{-1}(\omega). \tag{71}$$

From (70), we know that $\mathrm{C}(\iota(\phi^s)) < \omega$, which together with (55) implies

$$\iota_0 = \mathrm{C}(\iota_0) \leq \mathrm{C}(\iota(\phi^s)) < \omega.$$

Hence, $\iota_0 < \mathrm{C}^{-1}(\omega)$, which implies condition (71) is satisfied.

Case 5: $\zeta_L^e = 0 < \min(\zeta_L^c, \zeta_H^e)$. In this case (51) implies

$$y = 0,$$

and conditions (48) and (49) imply

$$\mathrm{C}'(x/k) = \phi^s.$$

Then (53) implies

$$s_{+1} = [1 - \delta + \iota(\phi^s)]\,k. \tag{72}$$

For this case to be a solution we need four conditions to be satisfied. First, $0 \leq s_{+1}$, which with (72) is equivalent to

$$0 \leq 1 - \delta + \iota(\phi^s).$$

This condition is implied by (55). Second, (50) and (53) require that

$$\omega = \Xi(\iota(\phi^s)) \tag{73}$$

with $\Xi(\cdot)$ as defined in (57). As argued in Case 3, the assumptions in the statement of the lemma imply $\Xi(\iota(\phi^s)) \leq 0$. Since $\omega \geq 0$, (73) implies this case is only possible if $\omega = 0$ and $\iota(\phi^s) = \iota_0$. Third, $0 < \zeta_L^c$ requires that $1 < \xi$. Fourth, $0 < \zeta_H^e$ requires that $\zeta_H^e = \xi\phi^s - \phi_e^s > 0$. There exist values of ξ that satisfy both these conditions.

Case 6: $\zeta_L^c = 0 < \min(\zeta_L^e, \zeta_H^e)$. In this case (52) implies

$$s_{+1} = 0$$

and then (53) implies

$$x/k = \delta - 1,$$

and condition (50) implies

$$y = [\omega - \text{C}(\delta - 1)]\, k. \tag{74}$$

Conditions (48) and (49) imply

$$\zeta_L^e = \text{C}'(\delta - 1) - \phi^s \tag{75}$$

$$\zeta_H^e = \text{C}'(\delta - 1) - \phi_e^s. \tag{76}$$

For this case to be a solution, we need three conditions to hold: $0 \leq y$, $0 < \zeta_H^e$, and $0 < \zeta_L^e$. With (75), the condition $0 < \zeta_L^e$ is equivalent to

$$\phi^s < \text{C}'(\delta - 1). \tag{77}$$

Notice that (55) implies

$$\text{C}'(\delta - 1) \leq \text{C}'(\iota_0) \leq \text{C}'(\iota(\phi^s)) = \phi^s, \tag{78}$$

which contradicts (77), so this case cannot be a solution.

Case 7: $0 < \min(\zeta_L^c, \zeta_L^e, \zeta_H^e)$. In this case (51)-(53) imply

$$y = 0$$

$$s_{+1} = 0$$

$$x/k = \delta - 1.$$

For this to be a solution, we need the following conditions to hold

$$\begin{aligned} w &= \mathrm{C}(\delta - 1)\,k \\ 1 &< \xi \\ \zeta_L^e &= \xi\left[\mathrm{C}'(\delta-1) - \phi^s\right] > 0 && (79) \\ \zeta_H^e &= \xi\left[\mathrm{C}'(\delta-1)\right] - \phi_e^s > 0. && (80) \end{aligned}$$

The first is implied by (50), the second by the condition $0 < \zeta_L^c$, and the third and fourth by the conditions (48) and (49), and the requirement that $0 < \min(\zeta_L^e, \zeta_H^e)$. Notice (55) implies (78), which contradicts (79), so this case cannot be a solution.

Case 8: $\zeta_L^c = \zeta_L^e = \zeta_H^e = 0$. In this case conditions (48) and (49) imply

$$\mathrm{C}'(x/k) = \phi_e^s = \phi^s,$$

condition (54) implies

$$y = \phi^s s_{+1} + \left[\omega - \mathrm{C}(\iota(\phi^s))\right]k,$$

and s_{+1} is any number that satisfies that satisfies

$$\max\left\{0, \frac{\mathrm{C}(\iota(\phi^s)) - \omega}{\phi^s}k\right\} \leq s_{+1} \leq \left[1 - \delta + \iota(\phi^s)\right]k.$$

Cases 1, 2, and 4, are summarized in part (*ii*) of the statement of the lemma, while part (*i*) summarizes cases 3, 5, and 8. This concludes the proof. ■

A.4.2 Equilibrium characterization

The following proposition characterizes the nonmonetary equilibrium.

Proposition 3 *A nonmonetary equilibrium exists for any parametrization. In the nonmonetary equilibrium, money has no value, i.e.,* $\mathrm{M} = 0$*, and the price of an equity share is* $\underline{\phi}^s$*. Moreover: (i) If* $\phi_e^s < \underline{\phi}^s$*, then* $X^* = \iota(\underline{\phi}^s)k_0$*, and* $S^* = \left[1 - \delta + \iota(\underline{\phi}^s)\right]k_0$*. (ii) If* $\underline{\phi}^s < \phi_e^s$*, then*

$$\frac{X^*}{k_0} = \Omega[\mathrm{C}(\iota(\underline{\phi}^s))]\iota(\underline{\phi}^s) + \int_{\mathrm{C}(\iota(\underline{\phi}^s))}^{\mathrm{C}(\iota(\phi_e^s))} \mathrm{C}^{-1}(\omega)d\Omega(\omega) + \{1 - \Omega[\mathrm{C}(\iota(\phi_e^s))]\}\iota(\phi_e^s),$$

and

$$\frac{S^*}{k_0} = \frac{1}{\underline{\phi}^s}\int_0^{\mathrm{C}(\iota(\underline{\phi}^s))} [\mathrm{C}(\iota(\underline{\phi}^s)) - \omega]d\Omega(\omega).$$

Proof of Proposition 3. In a stationary nonmonetary equilibrium, we know from Lemma 4 that $\mathrm{M} = 0$, $\varepsilon^* = \varepsilon_L$, and $\phi^s = \varphi^s z$, with $\varphi^s = \frac{\beta}{1-\beta\pi(1-\delta)}\bar{\varepsilon}$. In this case $\pi = 0$, so $\phi^s = \beta\bar{\varepsilon}z \equiv \underline{\phi}^s$. The expressions for X^* and S^* in parts (i) and (ii) follow from (12) and (13), and the expressions in parts (i) and (ii) of Proposition 1. ■

Proof of Proposition 2. The existence and uniqueness claim in part (i) follows from the fact that there exists a unique ε^* that satisfies (15), as established in Lemma 4. Parts (ii) and (vi) also follow from Lemma 4. To establish parts (iii), (iv), and (v) we again rely on Lemma 4, which shows that $\varphi^s(r)$ is continuous, with $\frac{\partial \varphi^s(r)}{\partial r} < 0$, $\phi^s(0) = \overline{\phi}^s$, and $\phi^s(\bar{r}) = \underline{\phi}^s$. From this it follows that for every $\phi_e^s \in (\underline{\phi}^s, \overline{\phi}^s)$ there exists a unique $\hat{r} \in (0, \bar{r})$ that satisfies $\phi^s(\hat{r}) = \phi_e^s$, with $\phi^s(r) > \phi_e^s$ for all $r \in (0, \hat{r})$, and $\phi^s(r) < \phi_e^s$ for all $r \in (\hat{r}, \bar{r})$. Given this, the expressions for X^* and S^* then follow from (12), (13), and Proposition 1. ■

Proof of Corollary 1. (i) As $r \to \bar{r}$, (15) implies $\varepsilon^* \to \varepsilon_L$, so (14) implies $\phi^s \to \underline{\phi}^s$, and part (vi) of Proposition 2 implies $\mathrm{M} \to 0$. (ii) As $r \to 0$, (15) implies $\varepsilon^* \to \varepsilon_H$, so (14) implies $\phi^s \to \overline{\phi}^s$. (iii) Condition (15) implies

$$\frac{\partial \varepsilon^*}{\partial r} = -\frac{\varepsilon^*}{r + \alpha\theta\left[1 - G(\varepsilon^*)\right]} < 0,$$

and condition (14) implies

$$\frac{\partial \phi^s(r)}{\partial r} = \beta\eta G(\varepsilon^*) z \frac{\partial \varepsilon^*}{\partial r} < 0. \tag{81}$$

(iv) Since $\iota(\phi^s(r))$ is the ι that satisfies $\mathrm{C}'(\iota) = \phi^s(r)$, we have

$$\frac{\partial \iota(\phi^s(r))}{\partial r} = \frac{1}{\mathrm{C}''(\iota)} \frac{\partial \phi^s(r)}{\partial r} < 0 < \frac{\partial \iota(\phi^s)}{\partial \phi^s} = \frac{1}{\mathrm{C}''(\iota)}. \tag{82}$$

(v) From part (i) of Proposition 1, if $\phi_e^s < \phi^s$, then

$$\varsigma_{+1}^*(\omega) = 1 - \delta + \iota(\phi^s(r)).$$

Hence,

$$\frac{\partial \varsigma_{+1}^*(\omega)}{\partial r} = \frac{\partial \iota(\phi^s(r))}{\partial r} < 0 < \frac{\partial \varsigma_{+1}^*(\omega)}{\partial \phi^s} = \frac{\partial \iota(\phi^s)}{\partial \phi^s},$$

where $\frac{\partial \iota(\phi^s(r))}{\partial r}$ and $\frac{\partial \iota(\phi^s)}{\partial \phi^s}$ are as in (82). From part (ii) of Proposition 1, if $\phi^s < \phi_e^s$ and $\omega \leq \mathrm{C}(\iota(\phi^s(r)))$, then

$$\varsigma_{+1}^*(\omega) = \frac{\mathrm{C}(\iota(\phi^s(r))) - \omega}{\phi^s(r)}.$$

Hence,

$$\frac{\partial \varsigma_{+1}^{*}(\omega)}{\partial \phi^{s}} = \frac{\mathrm{C}'(\iota(\phi^{s}))\frac{\partial \iota(\phi^{s})}{\partial \phi^{s}}\phi^{s} - \mathrm{C}(\iota(\phi^{s}(r))) + \omega}{(\phi^{s})^{2}}$$
$$= \frac{\left\{[\mathrm{C}'(\iota(\phi^{s}))]^{2} - \mathrm{C}''(\iota(\phi^{s}))\,\mathrm{C}(\iota(\phi^{s}))\right\}\frac{1}{\mathrm{C}''(\iota(\phi^{s}))} + \omega}{(\phi^{s})^{2}}.$$

If C is log concave, then $0 < \mathrm{C}'\mathrm{C}' - \mathrm{C}''\mathrm{C}$, and therefore $0 < \frac{\partial \varsigma_{+1}^{*}(\omega)}{\partial \phi^{s}}$ for all $\omega \geq 0$, and therefore,

$$\frac{\partial \varsigma_{+1}^{*}(\omega)}{\partial r} = \frac{\partial \varsigma_{+1}^{*}(\omega)}{\partial \phi^{s}}\frac{\partial \phi^{s}}{\partial r} < 0,$$

since $\frac{\partial \phi^{s}}{\partial r}$ is given by (81). (*vi*) Write the equilibrium conditions (14) and (15) as

$$\phi^{s} = \beta\left[\bar{\varepsilon} + \eta\int_{\varepsilon_{L}}^{\varepsilon^{*}}(\varepsilon^{*} - \varepsilon)\,dG(\varepsilon)\right]z \tag{83}$$
$$r\varepsilon^{*} = \eta\int_{\varepsilon^{*}}^{\varepsilon_{H}}(\varepsilon - \varepsilon^{*})\,dG(\varepsilon), \tag{84}$$

where $\eta \equiv \alpha\theta$. These conditions determine the pair $\phi^{s} = \varrho_{\phi}(r,\eta)$ and $\varepsilon^{*} = \varrho_{\varepsilon}(r,\eta)$. Let (r,η) be given, consider a parametrization (r_0,η_0) with $|r - r_0|$ and $|\eta - \eta_0|$ small, and let $\phi_0^{s} \equiv \varrho_{\phi}(r_0,\eta_0)$, and $\varepsilon_0^{*} \equiv \varrho_{\varepsilon}(r_0,\eta_0)$. Let

$$I_B(\varepsilon^{*}) \equiv \int_{\varepsilon^{*}}^{\varepsilon_{H}}(\varepsilon - \varepsilon^{*})\,dG(\varepsilon)$$
$$I_S(\varepsilon^{*}) \equiv \int_{\varepsilon_{L}}^{\varepsilon^{*}}(\varepsilon^{*} - \varepsilon)\,dG(\varepsilon).$$

Then

$$I_j(\varepsilon^{*}) \approx I_j(\varepsilon_0^{*}) + I_j'(\varepsilon_0^{*})(\varepsilon^{*} - \varepsilon_0^{*})$$

for $j \in \{B,S\}$, so

$$I_B(\varepsilon^{*}) \approx I_B(\varepsilon_0^{*}) - [1 - G(\varepsilon_0^{*})](\varepsilon^{*} - \varepsilon_0^{*}) \tag{85}$$
$$I_S(\varepsilon^{*}) \approx I_S(\varepsilon_0^{*}) + G(\varepsilon_0^{*})(\varepsilon^{*} - \varepsilon_0^{*}). \tag{86}$$

With (85) and (86), we can approximate (83) and (84) with

$$\frac{1}{\beta z}\phi^{s} \approx \bar{\varepsilon} + \eta\left[I_S(\varepsilon_0^{*}) + G(\varepsilon_0^{*})(\varepsilon^{*} - \varepsilon_0^{*})\right] \tag{87}$$
$$\varepsilon^{*} \approx \frac{\eta\left\{I_B(\varepsilon_0^{*}) + [1 - G(\varepsilon_0^{*})]\varepsilon_0^{*}\right\}}{r + \eta[1 - G(\varepsilon_0^{*})]} \equiv \hat{\varrho}_{\varepsilon}(r,\eta). \tag{88}$$

Approximate

$$\hat{\varrho}_{\varepsilon}(r,\eta) \approx \varepsilon_0^* + \frac{\partial \hat{\varrho}_{\varepsilon}(r_0,\eta_0)}{\partial r}(r - r_0) + \frac{\partial \hat{\varrho}_{\varepsilon}(r_0,\eta_0)}{\partial \eta}(\eta - \eta_0)$$

and use (88) to write

$$\varepsilon^* \approx \varepsilon_0^* + \frac{\partial \hat{\varrho}_{\varepsilon}(r_0,\eta_0)}{\partial r}(r - r_0) + \frac{\partial \hat{\varrho}_{\varepsilon}(r_0,\eta_0)}{\partial \eta}(\eta - \eta_0), \tag{89}$$

where

$$\frac{\partial \hat{\varrho}_{\varepsilon}(r_0,\eta_0)}{\partial r} = -\frac{\varepsilon_0^*}{r_0 + \eta_0\left[1 - G\left(\varepsilon_0^*\right)\right]} \tag{90}$$

$$\frac{\partial \hat{\varrho}_{\varepsilon}(r_0,\eta_0)}{\partial \eta} = \frac{r_0 \varepsilon_0^*}{\left\{r_0 + \eta_0\left[1 - G\left(\varepsilon_0^*\right)\right]\right\}\eta_0}. \tag{91}$$

By substituting (90) and (91) into (89), we get

$$\varepsilon^* \approx \left[1 + \frac{r_0 \eta - \eta_0 r}{\eta_0\left\{r_0 + \eta_0\left[1 - G\left(\varepsilon_0^*\right)\right]\right\}}\right]\varepsilon_0^*. \tag{92}$$

We can use (92) to approximate (87) by

$$\phi^s \approx \beta z\left\{\bar{\varepsilon} + \eta\left[I_S\left(\varepsilon_0^*\right) + \frac{G\left(\varepsilon_0^*\right)\varepsilon_0^*\left(r_0\eta - \eta_0 r\right)}{\eta_0\left\{r_0 + \eta_0\left[1 - G\left(\varepsilon_0^*\right)\right]\right\}}\right]\right\}. \tag{93}$$

Next, we use (93) to obtain

$$\frac{\partial^2 \phi^s}{\partial r \partial \eta} = \frac{\partial}{\partial \eta}\left[\frac{\partial \phi^s}{dr}\right] \approx \frac{\partial}{\partial \eta}\left[-\beta z \eta \frac{G\left(\varepsilon_0^*\right)\varepsilon_0^*}{r_0 + \eta_0\left[1 - G\left(\varepsilon_0^*\right)\right]}\right] = -\beta z \frac{G\left(\varepsilon_0^*\right)\varepsilon_0^*}{r_0 + \eta_0\left[1 - G\left(\varepsilon_0^*\right)\right]} < 0.$$

This concludes the proof. ■

Proof of Corollary 2. The Lagrangian for (11) can be written as

$$\begin{aligned}
\mathcal{L} = \; & y + \phi_e^s(k_{+1} - s_{+1}) \\
& + \hat{q}\left[(1-\delta)k + x - k_{+1}\right] \\
& + \xi\left[\phi^s s_{+1} + w - y - \mathrm{C}(x/k)\,k\right] \\
& + \zeta_L^e s_{+1} + \zeta_H^e\left(k_{+1} - s_{+1}\right) + \zeta_L^c y,
\end{aligned}$$

where ξ, ζ_L^e, ζ_H^e, and ζ_L^c are the Lagrange multipliers on the entrepreneur's budget constraint, nonnegativity constraint on equity issuance, upper bound on equity issuance, and nonnegativity constraint on consumption, respectively. The Lagrange multiplier $\hat{q}$ is associated to the law of motion of the capital stock, and is interpreted as the shadow price of a margial unit of capital to the entrepreneur. The first-order conditions with respect to y, x, s_{+1}, and k_{+1} are, respectively,

$$0 = 1 - \xi + \zeta_L^c \tag{94}$$

$$0 = \hat{q} - \xi \text{C}'(x/k) \tag{95}$$

$$0 = -\phi_e^s + \xi\phi^s + \zeta_L^e - \zeta_H^e \tag{96}$$

$$0 = \phi_e^s - \hat{q} + \zeta_H^e. \tag{97}$$

Condition (97) implies the shadow price of capital to the entrepreneur, $\hat{q}$, is at least as large as the discounted value that she assigns to the return on capital, ϕ_e^s, but could exceed it if the entrepreneur is facing a binding financing constraint, i.e., in the form of a binding upper bound on equity issuance ($0 < \zeta_H^e$). If we use (97) to substitute $\hat{q}$ in (95), then (94)-(96) become identical to (47)-(49) in the proof of Proposition 1. For what follows, it is convenient to define

$$q \equiv \frac{\hat{q}}{\xi}. \tag{98}$$

Intuitively, ξ is the shadow price to the entrepreneur of a unit of good 2 (in terms of second-subperiod marginal utility). Since the entrepreneur's utility for good 2 is linear, this shadow price equals 1 in an interior solution. But it will exceed 1 if the entrepreneur is financially constrained in the sense that it would like to be able to borrow good 2 to invest but is unable to do so. This "binding financial constraint" manifests itself with $0 < \zeta_L^c$, i.e., a situation in which the nonnegativity constraint on consumption binds. In sum, the q defined in (98) is the *return* (gross of adjustment costs) to the entrepreneur from investing an additional unit good 2 into capital. When investing an additional unit of good 2, the entrepreneur pays utility cost ξ to get payoff $\hat{q}$. Condition (95) then says that at an optimum, $\text{C}'(x/k) = q$, i.e., the marginal (technological) cost of investing, $\text{C}'(x/k)$, must equal the marginal return to investing, q. Next, we derive the value of q corresponding to every case in Proposition 1.

Case 1. This case corresponds to the lowest endowment range (i.e., $\omega \leq \text{C}(\iota(\phi^s)))$ in part (*ii*) of Proposition 1. In this case the Lagrange multipliers are:

$$\begin{aligned} \zeta_L^e &= \zeta_H^e = 0 < \zeta_L^c = \frac{\phi_e^s}{\phi^s} - 1 = \xi - 1 \\ \hat{q} &= \phi_e^s \\ q &= \phi^s, \end{aligned}$$

and the optimal investment rate, ι^*, satisfies

$$\text{C}'(\iota^*) = \phi^s.$$

Case 2. This case corresponds to the highest endowment range (i.e., $\text{C}(\iota(\phi_e^s)) \leq \omega$) in part (ii) of Proposition 1. In this case the Lagrange multipliers are:

$$\begin{aligned}
\zeta_L^c &= \zeta_H^e = 0 = \xi - 1 < \phi_e^s - \phi^s = \zeta_L^e \\
\hat{q} &= \phi_e^s \\
q &= \phi_e^s,
\end{aligned}$$

and the optimal investment rate, ι^*, satisfies

$$\text{C}'(\iota^*) = \phi_e^s.$$

Case 4. This case corresponds to the intermediate endowment range (i.e., $\text{C}(\iota(\phi^s)) < \omega < \text{C}(\iota(\phi_e^s)))$ in part (ii) of Proposition 1. In this case the Lagrange multipliers are:

$$\begin{aligned}
0 &= \zeta_H^e \\
0 &< \zeta_L^c = \xi - 1 = \frac{\phi_e^s}{\text{C}'(\iota^*)} - 1 \\
0 &< \zeta_L^e = \left[1 - \frac{\phi^s}{\text{C}'(\iota^*)}\right] \phi_e^s \\
\hat{q} &= \phi_e^s \\
q &= \text{C}'(\iota^*),
\end{aligned}$$

and the optimal investment rate, ι^*, satisfies

$$\text{C}(\iota^*) = \omega.$$

Case 3. This case corresponds to the case with $\phi_e^s < \phi^s$ in part (i) of Proposition 1. In this case the Lagrange multipliers are:

$$\begin{aligned}
\zeta_L^c &= \zeta_L^e = 0 < \zeta_H^e = \phi^s - \phi_e^s \\
\xi &= 1 \\
q &= \hat{q} = \phi^s,
\end{aligned}$$

and the optimal investment rate, ι^*, satisfies

$$\text{C}'(\iota^*) = \phi^s.$$

Case 5. This case corresponds to the case with $\vartheta^* = 0 < \phi^s - \phi_e^s$ in part (i) of Proposition 1. In this case the Lagrange multipliers are:

$$\begin{aligned} 0 &< \xi - 1 = \zeta_L^c \\ 0 &< \hat{q} - \phi_e^s = \zeta_H^e \\ q &= \phi^s, \end{aligned}$$

and the optimal investment rate, ι^*, satisfies

$$\text{C}'(\iota^*) = \phi^s.$$

Case 8. This case corresponds to the case with $\phi^s = \phi_e^s$ in part (i) of Proposition 1. In this case the Lagrange multipliers are:

$$\begin{aligned} 0 &= \zeta_L^c = \zeta_L^e = \zeta_H^e = \xi - 1 \\ q &= \hat{q} = \phi^s = \phi_e^s, \end{aligned}$$

and the optimal investment rate, ι^*, satisfies

$$\text{C}'(\iota^*) = \phi^s.$$

By collecting all cases we obtain the expressions in the statement. ■

Corollary 5 *The value function (11) can be written as*

$$J(w, k, 0) = [\vartheta^* + \phi_e^s(1 - \delta + \iota^* - \varsigma_{+1}^*)]k,$$

with $(\iota^*, \vartheta^*, \varsigma_{+1}^*)$ *as given in Proposition 1.*

(i) If $\phi_e^s \leq \phi^s$,

$$\frac{J(w, k, 0)}{k} = \phi^s[1 - \delta + \iota(\phi^s)] + \omega - \text{C}(\iota(\phi^s)).$$

(ii) If $\phi^s < \phi_e^s$,

$$\frac{J(w, k, 0)}{k} = \begin{cases} \phi_e^s[1 - \delta + \iota(\phi_e^s)] + \omega - \text{C}(\iota(\phi_e^s)) & \text{if } \text{C}(\iota(\phi_e^s)) \leq \omega \\ \phi_e^s[1 - \delta + \text{C}^{-1}(\omega)] & \text{if } \text{C}(\iota(\phi^s)) < \omega < \text{C}(\iota(\phi_e^s)) \\ \phi_e^s[1 - \delta + \iota(\phi^s) - \frac{\text{C}(\iota(\phi^s)) - \omega}{\phi^s}] & \text{if } \omega \leq \text{C}(\iota(\phi^s)). \end{cases}$$

In every case, the value function can be written as $\mathcal{J}(\omega)k$, *where* $\mathcal{J}(\omega) \equiv J(\omega k, k, 0)/k$.

A.4.3 Theoretical basis for the empirical analysis

Define $\mathrm{x}^* \equiv \log \iota^*$, $\mathrm{q} \equiv \log \phi^s$, and $\mathrm{e}^* \equiv \phi^s \varsigma^*_{+1}$, i.e., the log of the investment rate, the log of Tobin's q, and the value of equity issuance, respectively. By Proposition 1, in an equilibrium with $\phi^s < \phi^s_e$, a firm's investment and equity issuance decisions are:

$$\mathrm{x}^* = \begin{cases} \bar{\iota}(\omega) & \text{if } \mathrm{C}(\iota(e^{\mathrm{q}})) < \omega \\ \log(\mathrm{C}'^{-1}(e^{\mathrm{q}})) & \text{if } \omega \leq \mathrm{C}(\iota(e^{\mathrm{q}})) \end{cases} \quad \text{and} \quad \mathrm{e}^* = \begin{cases} 0 & \text{if } \mathrm{C}(\iota(e^{\mathrm{q}})) < \omega \\ \mathrm{C}(\mathrm{x}^*) - \omega & \text{if } \omega \leq \mathrm{C}(\iota(e^{\mathrm{q}})), \end{cases}$$

where $\bar{\iota}(\omega) \equiv \log \iota(\phi^s_e)\mathbb{I}_{\{\mathrm{C}(\iota(\phi^s_e)) \leq \omega\}} + \log(\mathrm{C}^{-1}(\omega))\mathbb{I}_{\{\mathrm{C}(\iota(e^{\mathrm{q}})) < \omega < \mathrm{C}(\iota(\phi^s_e))\}}$. Clearly, a firm with $\mathrm{C}(\iota(e^{\mathrm{q}})) < \omega$ has $\frac{\partial \mathrm{x}^*}{\partial \mathrm{q}} = \frac{\partial \mathrm{e}^*}{\partial \mathrm{q}} = 0$, which is our theoretical justification for classifying firms with a high proportion of liquid financial wealth as not being equity dependent. Conversely, for a firm with $\omega \leq \mathrm{C}(\iota(e^{\mathrm{q}}))$, and for some $(\bar{\mathrm{q}}, \bar{\omega})$ near (q, ω) (that also satisfies $\bar{\omega} \leq \mathrm{C}(\iota(e^{\bar{\mathrm{q}}}))$), we have

$$\mathrm{x}^* \approx \bar{\mathrm{x}}^* + \gamma^{\mathrm{q}}_{\mathrm{x}}\mathrm{q} \tag{99}$$

$$\mathrm{e}^* \approx \bar{\mathrm{e}}^* + \gamma^{\mathrm{q}}_{\mathrm{e}}\mathrm{q}, \tag{100}$$

where $\bar{\mathrm{x}}^* \equiv \log(\mathrm{C}'^{-1}(e^{\bar{\mathrm{q}}})) - \gamma^{\mathrm{q}}_{\mathrm{x}}\bar{\mathrm{q}}$, $\bar{\mathrm{e}}^* \equiv \mathrm{C}(\iota(e^{\bar{\mathrm{q}}})) - \bar{\omega} - \gamma^{\mathrm{q}}_{\mathrm{e}}\bar{\mathrm{q}}$, $\gamma^{\mathrm{q}}_{\mathrm{x}} \equiv \left.\frac{\partial \mathrm{x}^*}{\partial \mathrm{q}}\right|_{(\mathrm{q},\omega)=(\bar{\mathrm{q}},\bar{\omega})} = \frac{e^{\mathrm{q}-\mathrm{x}^*}}{\mathrm{C}''(\mathrm{x}^*)} > 0$, and $\gamma^{\mathrm{q}}_{\mathrm{e}} \equiv \left.\frac{\partial \mathrm{e}^*}{\partial \mathrm{q}}\right|_{(\mathrm{q},\omega)=(\bar{\mathrm{q}},\bar{\omega})} = \mathrm{C}'(\mathrm{x}^*)\gamma^{\mathrm{q}}_{\mathrm{x}} > 0$. Thus, a firm with $\omega \leq \mathrm{C}(\iota(e^{\mathrm{q}}))$ has $\frac{\partial \mathrm{x}^*}{\partial \mathrm{q}} \approx \gamma^{\mathrm{q}}_{\mathrm{x}} > 0$, and $\frac{\partial \mathrm{e}^*}{\partial \mathrm{q}} \approx \gamma^{\mathrm{q}}_{\mathrm{e}} > 0$, which is our theoretical justification for classifying firms with a low proportion of liquid financial wealth as being equity dependent. To conclude, note that equity-dependent firms, i.e., firms with $\frac{\partial \mathrm{x}^*}{\partial \mathrm{q}} > 0$ and $\frac{\partial \mathrm{e}^*}{\partial \mathrm{q}} > 0$, are firms for which the q-channel is operative, in the sense that their investment and equity issuance decisions are stimulated by exogenous increases in Tobin's q. One of the goals of our empirical work in Section 4 will be to estimate coefficients like $\gamma^{\mathrm{q}}_{\mathrm{x}}$ and $\gamma^{\mathrm{q}}_{\mathrm{e}}$ in order to gauge the strength of the q-channel for corporate investment and equity issuance decisions.

A.5 Aggregate implications of microeconomic estimates

Lemma 5 *Suppose* $d\log\left(I^i_{t+h-s}\right)/d\varepsilon^m_t \leq 0$ *for* $s \in \{1, ..., h\}$ *and* $i \in \mathbb{F}$. *Then (29) holds.*

Proof. First, notice that

$$\frac{d\log\left(\bar{I}_{t+h}\right)}{d\varepsilon^m_t} = \sum_{i\in\mathbb{F}} \frac{I^i_{t+h}}{\bar{I}_{t+h}} \frac{d\log\left(I^i_{t+h}\right)}{d\varepsilon^m_t}, \tag{101}$$

and

$$
\begin{aligned}
\frac{d\log(I^i_{t+h}/K^i_{t+h})}{d\varepsilon^m_t} &= \frac{K^i_{t+h}}{I^i_{t+h}}\frac{d(I^i_{t+h}/K^i_{t+h})}{d\varepsilon^m_t} \\
&= \frac{K^i_{t+h}}{I^i_{t+h}}\frac{\frac{dI^i_{t+h}}{d\varepsilon^m_t}K^i_{t+h} - I^i_{t+h}\frac{dK^i_{t+h}}{d\varepsilon^m_t}}{\left(K^i_{t+h}\right)^2} \\
&= \frac{1}{I^i_{t+h}}\left(\frac{dI^i_{t+h}}{d\varepsilon^m_t} - \frac{I^i_{t+h}}{K^i_{t+h}}\frac{dK^i_{t+h}}{d\varepsilon^m_t}\right) \\
&= \frac{d\log\left(I^i_{t+h}\right)}{d\varepsilon^m_t} - \frac{1}{K^i_{t+h}}\frac{dK^i_{t+h}}{d\varepsilon^m_t}. \qquad (102)
\end{aligned}
$$

The law of motion for the capital stock is

$$
K^i_{t+h} = (1-\delta_K)^h K^i_t + \sum_{s=1}^{h}(1-\delta_K)^{s-1} I^i_{t+h-s},
$$

where $\delta_K \in [0,1]$ is the depreciation rate of capital, so we can write (102) as

$$
\frac{d\log\left(I^i_{t+h}\right)}{d\varepsilon^m_t} = \frac{d\log(I^i_{t+h}/K^i_{t+h})}{d\varepsilon^m_t} + \sum_{s=1}^{h}\zeta^i_{t,h,s}\frac{d\log\left(I^i_{t+h-s}\right)}{d\varepsilon^m_t}, \qquad (103)
$$

where

$$
\zeta^i_{t,h,s} \equiv \frac{(1-\delta_K)^{s-1} I^i_{t+h-s}}{K^i_{t+h}}.
$$

Then (101) and (103) imply

$$
\frac{d\log\left(\bar{I}_{t+h}\right)}{d\varepsilon^m_t} = \sum_{i\in\mathbb{F}}\frac{I^i_{t+h}}{\bar{I}_{t+h}}\frac{d\log(I^i_{t+h}/K^i_{t+h})}{d\varepsilon^m_t} + \sum_{s=1}^{h}\sum_{i\in\mathbb{F}}\zeta^i_{t,h,s}\frac{I^i_{t+h}}{\bar{I}_{t+h}}\frac{d\log\left(I^i_{t+h-s}\right)}{d\varepsilon^m_t}. \qquad (104)
$$

Since $\zeta^i_{t,h,s}\frac{I^i_{t+h}}{\bar{I}_{t+h}} \geq 0$, then (104) and $\frac{d\log\left(I^i_{t+h-s}\right)}{d\varepsilon^m_t} \leq 0$ for $s \in \{1,...,h\}$ and $i \in \mathbb{F}$ imply (29). ■

B Adverse selection

In this section we formalize a simple agency problem between entrepreneurs and investors to show that in order to have an equilibrium with $\phi^s < \phi^s_e$, one need not assume that the fundamental value of the dividend of good 1 is higher for entrepreneurs than for outside investors.

Consider a generalization of the model of Section 2.2 in which the productivity of a unit of capital created in the second subperiod of period t is a random variable $Z_{t+1} \in \{0, z\}$. A fraction $1-\lambda$ of the entrepreneurs draw productivity $Z_{t+1}=0$, while the remaining draw $Z_{t+1}=z>0$.[66] The timing of information is that an entrepreneur makes the investment and equity issuance decisions at the end of period t, having observed the realization of Z_{t+1}, while outside investors learn this realization at the beginning of period $t+1$ (before the round of stock-market trades in the first subperiod). We maintain the assumption of competitive trade in the second subperiod, so the stock price in the second subperiod of period t (i.e., at the time the investment in physical capital is made and equity claims on these units of capital are issued) is determined in a competitive market in which all shares trade at the same price.[67]

As in Section 2.2, we focus on stationary equilibria, and maintain the assumption $\pi=0$ (entrepreneurs live for one period). In addition, to simplify the exposition, in this section we assume $\delta=1$ (capital only lasts one period), and $\iota_0=0$. Under these conditions, the entrepreneur's problem (analogous to (11)) is:

$$\max_{x,y,s_{+1}} \left[y+\beta\varepsilon_e Z(x-s_{+1})\right] \tag{105}$$

$$\begin{aligned} \text{s.t. } y+\mathrm{C}\left(x/k\right)k &\le \phi^s s_{+1}+w & (106)\\ 0 &\le s_{+1}\le x & (107)\\ 0 &\le y. & (108)\end{aligned}$$

Let $\tilde{g}^x\left(Z,w,k\right)$, $\tilde{g}^y\left(Z,w,k\right)$, and $\tilde{g}^e\left(Z,w,k\right)$ denote the levels of investment, consumption of good 2, and equity issuance that solve (105)-(108) for an entrepreneur with productivity realization $Z\in\{0,z\}$. Define $\iota^*\equiv g^x\left(Z,w,k\right)/k$, $\vartheta^*\equiv g^y\left(Z,w,k\right)/k$, $\varsigma^*_{+1}\equiv g^e\left(Z,w,k\right)/k$, and $\omega\equiv w/k$. The following result, analogous to Proposition 1, characterizes $(\iota^*,\vartheta^*,\varsigma^*_{+1})$ as a function of the entrepreneur's marginal valuation, $\phi^s_e\equiv\beta\varepsilon_e Z$, and the market valuation, ϕ^s.

Lemma 6 *Consider the economy with adverse selection, and assume $\pi=1-\delta=\iota_0=0$. Let $\iota(\phi)$ denote the unique number, ι, that solves $\mathrm{C}'\left(\iota\right)=\phi$ for any $\phi\in\mathbb{R}_+$.*

(i) If $\max\left(\phi^s_e,\phi^s\right)<1$, then $\iota^*=\varsigma^*_{+1}=0$.

[66] The model of Section 2.2 corresponds to the special case with $\lambda=1$. For simplicity, we assume this random variable is independent across entrepreneurs and uncorrelated with the entrepreneur's characteristics (e.g., her capital, k, and claims to good 2, w).

[67] This would be a natural market outcome in a context in which investors know the probability distribution over Z_{t+1} but have no way of obtaining entrepreneur-specific information. One could instead set the model up as a signalling game in which entrepreneurs play the role of senders and investors play the role of receivers.

(ii) If $1 \leq \max(\phi_e^s, \phi^s)$ *and* $\phi_e^s \leq \phi^s$*, then*

$$\begin{aligned}
\iota^* &= \iota(\phi^s) \\
\varsigma_{+1}^* &= \begin{cases} \iota^* & \text{if } \phi_e^s < \phi^s \\ \left[\max\left\{0, \frac{\mathrm{C}(\iota^*)-\omega}{\phi^s}\right\}, \iota^*\right] & \text{if } \phi_e^s = \phi^s. \end{cases}
\end{aligned}$$

(iii) If $1 \leq \phi^s < \phi_e^s$*, then*

$$\begin{aligned}
\iota^* &= \begin{cases} \iota(\phi_e^s) & \text{if } \mathrm{C}(\iota(\phi_e^s)) \leq \omega \\ \mathrm{C}^{-1}(\omega) & \text{if } \mathrm{C}(\iota(\phi^s)) < \omega < \mathrm{C}(\iota(\phi_e^s)) \\ \iota(\phi^s) & \text{if } \omega \leq \mathrm{C}(\iota(\phi^s)) \end{cases} \\
\varsigma_{+1}^* &= \begin{cases} 0 & \text{if } \mathrm{C}(\iota(\phi^s)) < \omega \\ \frac{\mathrm{C}(\iota(\phi^s))-\omega}{\phi^s} & \text{if } \omega \leq \mathrm{C}(\iota(\phi^s)). \end{cases}
\end{aligned}$$

(iv) If $\phi^s < 1 \leq \phi_e^s$*, then*

$$\begin{aligned}
\iota^* &= \begin{cases} \iota(\phi_e^s) & \text{if } \mathrm{C}(\iota(\phi_e^s)) \leq \omega \\ \mathrm{C}^{-1}(\omega) & \omega < \mathrm{C}(\iota(\phi_e^s)) \end{cases} \\
\varsigma_{+1}^* &= 0.
\end{aligned}$$

(v) In every case, $\vartheta^* = \omega + \phi^s \varsigma_{+1}^* - \mathrm{C}(\iota^*)$.

Proof. Since the constraint (106) will bind at an optimum, the problem (105)-(108) implies

$$(\iota^*, \varsigma_{+1}^*) = \arg\max_{\iota, \varsigma_{+1}} \left[\phi_e^s \iota - \mathrm{C}(\iota) + (\phi^s - \phi_e^s)\varsigma_{+1}\right] \tag{109}$$

$$\text{s.t. } \max\left(0, \frac{\mathrm{C}(\iota) - \omega}{\phi^s}\right) \leq \varsigma_{+1} \leq x \tag{110}$$

and

$$\vartheta^* = \omega + \phi^s \varsigma_{+1}^* - \mathrm{C}(\iota^*). \tag{111}$$

The Lagrangian for (109)-(110) is

$$\mathcal{L} = \phi_e^s \iota - \mathrm{C}(\iota) + (\phi^s - \phi_e^s)\varsigma_{+1} + \zeta_L^e \varsigma_{+1} + \zeta_H^e(\iota - \varsigma_{+1}) + \zeta_L^c\left[\omega + \phi^s \varsigma_{+1} - \mathrm{C}(\iota)\right],$$

where ζ_L^e, ζ_H^e, and ζ_L^c are the Lagrange multipliers on the nonnegativity constraint on equity issuance, the upper bound on equity issuance, and the nonnegativity constraint on consumption of good 2, respectively. The first-order conditions are

$$
\begin{aligned}
0 &= \phi_e^s - (1+\zeta_L^c)\,\mathrm{C}'(\iota) + \zeta_H^e && (112)\\
0 &= (1+\zeta_L^c)\,\phi^s - \phi_e^s + \zeta_L^e - \zeta_H^e && (113)\\
0 &= \zeta_L^e \varsigma_{+1} && (114)\\
0 &= \zeta_H^e(\iota - \varsigma_{+1}) && (115)\\
0 &= \zeta_L^c\left[\omega + \phi^s\varsigma_{+1} - \mathrm{C}(\iota)\right]. && (116)
\end{aligned}
$$

There are eight cases depending on whether the multipliers $(\zeta_L^e, \zeta_H^e, \zeta_L^c)$ are positive or equal to zero. We consider each in turn. In every case, we suppose $0 < \min(\phi^s, \phi_e^s)$.

Case 1: $\zeta_L^e = \zeta_H^e = 0 < \zeta_L^c$. In this case (112)-(116) imply the optimum is characterized by

$$
\begin{aligned}
\mathrm{C}'(\iota^*) &= \phi^s < \phi_e^s && (117)\\
\varsigma_{+1}^* &= \frac{\mathrm{C}(\iota^*) - \omega}{\phi^s}. && (118)
\end{aligned}
$$

Recall that $\mathrm{C}'(0) = 1$ and $\mathrm{C}'' > 0$, so $1 \leq \mathrm{C}'(\iota)$ for all $\iota \geq 0$ (with "=" only if $\iota = 0$). Hence for (117) to hold it is necessary that

$$1 \leq \phi^s < \phi_e^s. \tag{119}$$

Also, for (117)-(118) to be a solution it must satisfy $0 \leq \varsigma_{+1}^* \leq \iota^*$, or equivalently,

$$\mathrm{C}(\iota^*) - \mathrm{C}'(\iota^*)\,\iota^* \leq \omega \leq \mathrm{C}(\iota^*). \tag{120}$$

The second inequality in (120) is equivalent to

$$\omega \leq \mathrm{C}(\iota(\phi^s)). \tag{121}$$

Next, we show that the first inequality in (120) is redundant. Since C is strictly convex, we have

$$\mathrm{C}(\iota) \geq \mathrm{C}(\iota^*) + \mathrm{C}'(\iota^*)(\iota - \iota^*), \tag{122}$$

with "=" only if $\iota = \iota^*$. Since $\mathrm{C}(0) = 0$, evaluating (122) at $\iota = 0$ implies

$$\mathrm{C}(\iota^*) - \mathrm{C}'(\iota^*)\,\iota^* \leq 0, \tag{123}$$

so the first inequality in (120) is satisfied for all $\omega \in \mathbb{R}_+$.

Case 2: $\zeta_L^c = \zeta_H^e = 0 < \zeta_L^e$. In this case (112)-(116) imply the optimum is characterized by

$$\phi^s < \phi_e^s = \text{C}'(\iota^*) \tag{124}$$

$$\varsigma_{+1}^* = 0. \tag{125}$$

Recall that $\text{C}'(0) = 1$ and $\text{C}'' > 0$, so $1 \leq \text{C}'(\iota)$ for all $\iota \geq 0$ (with "=" only if $\iota = 0$). Hence for (124) to hold it is necessary not only that $\phi^s < \phi_e^s$, but also that $1 \leq \phi_e^s$, which together can be written as

$$\max(1, \phi^s) \leq \phi_e^s, \text{ with " < " if } \max(1, \phi^s) = \phi^s. \tag{126}$$

Also, for (124)-(125) to be a solution, it must satisfy

$$\frac{\text{C}(\iota^*) - \omega}{\phi^s} \leq \varsigma_{+1}^* \leq \iota^*,$$

or equivalently,

$$\frac{\text{C}(\iota^*) - \omega}{\phi^s} \leq 0 \leq \iota^*. \tag{127}$$

The first inequality in (127) is equivalent to

$$\text{C}(\iota(\phi_e^s)) \leq \omega, \tag{128}$$

and the second inequality in (127) is implied by (126).

Case 3: $\zeta_L^c = \zeta_L^e = 0 < \zeta_H^e$. In this case (112)-(116) imply the optimum is characterized by

$$\phi_e^s < \phi^s = \text{C}'(\iota^*) \tag{129}$$

$$\varsigma_{+1}^* = \iota^*. \tag{130}$$

Recall that $\text{C}'(0) = 1$ and $\text{C}'' > 0$, so $1 \leq \text{C}'(\iota)$ for all $\iota \geq 0$ (with "=" only if $\iota = 0$). Hence for (129) to hold it is necessary not only that $\phi_e^s < \phi^s$, but also that $1 \leq \phi^s$, which together can be written as

$$\max(1, \phi_e^s) \leq \phi^s, \text{ with " < " if } \max(1, \phi_e^s) = \phi_e^s. \tag{131}$$

Also, for (129)-(130) to be a solution, it must satisfy

$$0 \leq \varsigma_{+1}^* \text{ and } 0 \leq \omega + \phi^s \varsigma_{+1}^* - \text{C}(\iota^*),$$

which using (129) and (130) are equivalent to

$$0 \leq \iota^* \text{ and } 0 \leq \omega + \text{C}'(\iota^*)\,\iota^* - \text{C}(\iota^*). \tag{132}$$

The first inequality in (132) is redundant since it follows from (129), (131), $\text{c}'(0) = 1$, and $\text{c}'' > 0$, which imply

$$\text{c}'(0) - \phi^s = 1 - \phi^s \leq 0 = \text{c}'(\iota^*) - \phi^s.$$

The second inequality in (132) is satisfied for all $\omega \in \mathbb{R}_+$ since the maintained assumptions $\text{c}(0) = 0 < \text{c}''$ imply (123).

Case 4: $\zeta_H^e = 0 < \min(\zeta_L^c, \zeta_L^e)$. In this case (112)-(116) imply the optimum satisfies

$$\omega = \text{c}(\iota^*) \tag{133}$$

$$\varsigma_{+1}^* = 0. \tag{134}$$

For (133)-(134) to be a solution, it must also satisfy $\varsigma_{+1}^* \leq \iota^*$ and

$$\phi^s < \text{c}'(\iota^*) < \phi_e^s. \tag{135}$$

With (134), the condition $\varsigma_{+1}^* \leq \iota^*$ is equivalent to $0 \leq \iota^*$, which is implied by (133) for any $\omega \in \mathbb{R}_+$, since $\text{c}(0) = 0 < \text{c}'$. For (135) to hold it is necessary that

$$\max(1, \phi^s) < \phi_e^s. \tag{136}$$

Under assumption (136) we have $\phi_e^s = \text{c}'(\iota(\phi_e^s))$, and can write the second inequality in (135) as

$$\text{c}'(\iota^*) < \text{c}'(\iota(\phi_e^s)),$$

which is equivalent to

$$\omega < \text{c}(\iota(\phi_e^s)). \tag{137}$$

If $\phi^s < 1$, then the first inequality in (135) holds for all $\omega \in \mathbb{R}_+$. Conversely, if $1 \leq \phi^s$, then we can write $\phi^s = \text{c}'(\iota(\phi^s))$ and the first inequality in (135) can be written as

$$\text{c}'(\iota(\phi^s)) < \text{c}'(\iota^*),$$

which is equivalent to

$$\text{c}(\iota(\phi^s)) < \omega \qquad \text{if } 1 \leq \phi^s. \tag{138}$$

Conditions (137) and (13) can be written jointly as

$$\omega \in \begin{cases} (\text{c}(\iota(\phi^s)), \text{c}(\iota(\phi_e^s))) & \text{if } 1 \leq \phi^s \\ (-\infty, \text{c}(\iota(\phi_e^s))) & \text{if } \phi^s < 1. \end{cases} \tag{139}$$

Case 5: $\zeta_L^e = 0 < \min(\zeta_L^c, \zeta_H^e)$. In this case (112)-(116) imply

$$\begin{aligned} \mathrm{C}'(\iota^*) &= \phi^s & (140) \\ \varsigma_{+1}^* &= \iota^*. & (141) \end{aligned}$$

For (140)-(141) to be a solution, it must also satisfy

$$\omega = \mathrm{C}(\iota^*) - \mathrm{C}'(\iota^*)\,\iota^* \tag{142}$$

and $0 \leq \iota^*$. Also, $1 \leq \phi^s$ is necessary for (140) to hold (since $\mathrm{C}'(\iota) \geq 1$ for all $\iota \geq 0$). The maintained assumptions $\mathrm{C}(0) = 0 < \mathrm{C}''$ imply (123), so (142) can only hold if $\iota^* = 0$ and

$$\omega = 0. \tag{143}$$

Together with (140), $\iota^* = 0$ implies we must also have

$$\phi^s = 1, \tag{144}$$

while ϕ_e^s can take any nonnegative value. To summarize, if (143) and (144) hold, the solution for this case is

$$\varsigma_{+1}^* = \iota^* = 0. \tag{145}$$

Case 6: $\zeta_L^c = 0 < \min(\zeta_L^e, \zeta_H^e)$. In this case (112)-(116) imply the optimum is characterized by

$$\varsigma_{+1}^* = \iota^* = 0 \tag{146}$$

provided

$$\max(\phi_e^s, \phi^s) < 1. \tag{147}$$

Case 7: $0 < \min(\zeta_L^c, \zeta_L^e, \zeta_H^e)$. In this case (112)-(116) imply the optimum is characterized by

$$\varsigma_{+1}^* = \iota^* = 0 \tag{148}$$

provided

$$\omega = 0 \tag{149}$$

and

$$\phi^s < 1 \tag{150}$$

while ϕ_e^s can take any nonnegative value.

Case 8: $\zeta_L^c = \zeta_L^e = \zeta_H^e = 0$. In this case (112)-(116) imply the optimum is characterized by

$$\begin{aligned} \mathrm{C}'(\iota^*) &= \phi^s & (151)\\ \varsigma_{+1}^* &\in \left[\max\left\{0, \frac{\mathrm{C}(\iota^*) - \omega}{\phi^s}\right\}, \iota^*\right] & (152)\end{aligned}$$

provided

$$1 \leq \phi_e^s = \phi^s. \tag{153}$$

Part (i) in the statement of the lemma corresponds to Case 6 and Case 7. Part (ii) corresponds to Case 3 and Case 8. Part (iii) corresponds to Case 1, Case 2, and Case 4. Part (iv) corresponds to Case 2 and Case 4. Part (v) is the same as (111). ■

In this model, the outside investor's Euler equations for money and equity analogous to (34) and (35) are

$$\phi_t^m \geq \beta\left[\phi_{t+1}^m + \alpha\theta\int_{\varepsilon_{t+1}^*}^{\varepsilon_H}\left(\varepsilon - \varepsilon_{t+1}^*\right) z dG(\varepsilon)\frac{1}{p_{t+1}}\right], \text{ with " = " if } a_{t+1}^m > 0 \tag{154}$$

$$\phi_t^s \geq \beta\Lambda\left[\bar{\varepsilon}z + \alpha\theta\int_{\varepsilon_L}^{\varepsilon_{t+1}^*}\left(\varepsilon_{t+1}^* - \varepsilon\right) z dG(\varepsilon)\right], \text{ with " = " if } a_{t+1}^s > 0, \tag{155}$$

where $\Lambda \in [0,1]$ is the investor's belief that a traded equity share represents a claim to a *productive* unit of capital. The stock-market clearing condition in the first subperiod (analogous to (39)) is

$$\frac{1 - G(\varepsilon^*)}{\varepsilon^* z}\mathrm{M}_t = G(\varepsilon^*)\Lambda S_t.$$

As in Section 2.2, we focus on *stationary equilibria* in which the aggregate supply of equity and aggregate real money balances are constant over time, i.e., $S_t = S$ and $\phi_t^m A_t^m \equiv \mathrm{M}_t = \mathrm{M}$ for all t, and real equity prices are time-invariant linear functions of the (*expected*) dividend, i.e., $\phi_t^s = \phi^s \equiv \varphi^s z$ for all t. Thus (again imposing $\pi = 0$), the stationary-equilibrium conditions in Corollary 4 become

$$
\begin{aligned}
r &\geq \alpha\theta \int_{\varepsilon^*}^{\varepsilon_H} \frac{\varepsilon-\varepsilon^*}{\varepsilon^*} dG(\varepsilon), \text{ with `` = '' if } \mathrm{M}>0 && (156)\\
\varphi^s &= \beta\Lambda\left[\bar{\varepsilon}+\alpha\theta\int_{\varepsilon_L}^{\varepsilon^*}\left(\varepsilon^*-\varepsilon\right)dG(\varepsilon)\right] && (157)\\
\mathrm{M} &= \frac{G\left(\varepsilon^*\right)}{1-G\left(\varepsilon^*\right)}\varepsilon^* z\Lambda S. && (158)
\end{aligned}
$$

The equilibrium conditions (156)-(158) for the economy with adverse selection are a simple generalization of conditions (40)-(42) (with $\pi=0$) for the economy without adverse selection (both sets of conditions coincide if $\Lambda=1$). The following result is analogous to Lemma 4, but for an economy with $\pi=0$ and adverse selection.

Lemma 7 *Let $S>0$ and $\Lambda\in[0,1]$ be given. Then:*

(i) There always exists a solution to (156)-(158) in which money is not valued, i.e., $\mathrm{M}=0$, $\varepsilon^*=\varepsilon_L$, *and* $\varphi^s=\Lambda\beta\bar{\varepsilon}$.

(ii) Let $\bar{r}\equiv\alpha\theta\left(\bar{\varepsilon}-\varepsilon_L\right)/\varepsilon_L$. *If* $r\in(0,\bar{r})$, *there exists a unique solution to (156)-(158) with* $\mathrm{M}>0$, *i.e.,*

$$
\begin{aligned}
\mathrm{M} &= \frac{G\left(\varepsilon^*\right)\varepsilon^* z}{1-G\left(\varepsilon^*\right)}\Lambda S\\
\varphi^s &= \Lambda\beta\left[\bar{\varepsilon}+\alpha\theta\int_{\varepsilon_L}^{\varepsilon^*}\left(\varepsilon^*-\varepsilon\right)dG(\varepsilon)\right], && (159)
\end{aligned}
$$

where $\varepsilon^*\in(\varepsilon_L,\varepsilon_H]$ *is the unique solution to*

$$
\alpha\theta\int_{\varepsilon^*}^{\varepsilon_H}\frac{\varepsilon-\varepsilon^*}{\varepsilon^*}dG(\varepsilon)=r. \tag{160}
$$

Moreover:

(a) As $r\to\bar{r}$, $\varepsilon^*\to\varepsilon_L$, $\mathrm{M}\to 0$, *and* $\varphi^s\to\Lambda\beta\bar{\varepsilon}$.

(b) As $r\to 0$, $\varepsilon^*\to\varepsilon_H$ *and* $\varphi^s\to\Lambda\beta\left[\bar{\varepsilon}+\alpha\theta\left(\varepsilon_H-\bar{\varepsilon}\right)\right]$.

(c) $\frac{\partial\varepsilon^*}{\partial r}<0$, $\frac{\partial \mathrm{M}}{\partial r}<0$, *and* $\frac{\partial\varphi^s}{\partial r}<0$.

(d) $\frac{\partial \mathrm{M}}{\partial\Lambda}>0$, *and* $\frac{\partial\varphi^s}{\partial\Lambda}>0$.

Proof. Immediate from the equilibrium conditions (156)-(158) by following steps similar to those in the proof of Lemma 4. ■

Part (i), and parts (ii), (a), (b), and (c), of Lemma 7 are results analogous to their counterparts in Lemma 4. Part (ii) (d) shows how real money balances and the equity price change with the investor's belief about the proportion of outstanding shares that are claims to the productive capital.

For what follows, let $\iota_Z^*(\omega)$ and $\varsigma_Z^*(\omega)$ denote the optimal investment and equity issuance decisions (normalized by the firm's capital stock) of an entrepreneur with productivity realization $Z \in \{0, z\}$ and a balance sheet with financial wealth per unit of own capital equal to ω. We can write the aggregate investment chosen at the end of a period by all entrepreneurs with productivity $Z \in \{0, z\}$, as

$$X_Z^* = \lambda_Z \int \iota_Z^*(\omega) k_0 d\Omega(\omega),$$

and the aggregate stock of equity claims on the capital of entrepreneurs with productivity $Z \in \{0, z\}$ outstanding at the beginning of a period as

$$S_Z^* = \lambda_Z \int \varsigma_Z^*(\omega) k_0 d\Omega(\omega),$$

where $\lambda_Z \equiv \lambda \mathbb{I}_{\{Z=z\}} + (1-\lambda)\mathbb{I}_{\{Z=0\}}$ for $Z \in \{0, z\}$.

The following lemma characterizes the behavior of the entrepreneurs' optimal investment and equity issuance decisions as a function of the market belief, Λ, for a given policy rate, r. To state the result it is convenient to make explicit the dependence of the equity price on the belief, Λ, and the nominal rate, r, by defining the price function $\phi^s(\Lambda, r) \equiv \varphi^s z$, where φ^s is given in Lemma 7, i.e.,

$$\phi^s(\Lambda, r) = \begin{cases} \Lambda\beta\left[\bar{\varepsilon} + \alpha\theta \int_{\varepsilon_L}^{\varepsilon^*} (\varepsilon^* - \varepsilon)\, dG(\varepsilon)\right] z, \text{ with } \varepsilon^* \text{ given by (160)} & \text{if } 0 \le r \le \bar{r} \\ \Lambda\beta\bar{\varepsilon}z & \text{if } \bar{r} < r. \end{cases} \quad (161)$$

Lemma 8 *Assume* $1 < \min\{\phi_z^s, \phi^s(1, \bar{r})\}$, *where* $\phi_Z^s \equiv \beta\varepsilon_e Z$ *for* $Z \in \{0, z\}$. *For any* $r \in \mathbb{R}_+$, *let* $\Lambda' \in (0,1)$ *be the number that satisfies* $\phi^s(\Lambda', r) = 1$.

(i) If $\phi^s(1, r) < \phi_z^s$, *then:*

(a) If $\Lambda' \le \Lambda$,

$$\frac{X_z^*}{k_0} = \lambda\left[\Omega\left[\mathrm{C}\left(\iota\left(\phi^s(\Lambda, r)\right)\right)\right]\iota(\phi^s(\Lambda, r)) + \int_{\mathrm{C}(\iota(\phi^s(\Lambda,r)))}^{\mathrm{C}(\iota(\phi_z^s))} \mathrm{C}^{-1}(\omega) d\Omega(\omega) + \{1 - \Omega\left[\mathrm{C}(\iota(\phi_z^s))\right]\}\iota(\phi_z^s)\right]$$

$$\frac{S_z^*}{k_0} = \lambda \int_0^{\mathrm{C}(\iota(\phi^s(\Lambda,r)))} \frac{\mathrm{C}(\iota(\phi^s(\Lambda, r))) - \omega}{\phi^s(\Lambda, r)} d\Omega(\omega)$$

and

$$X_0^* = S_0^* = (1-\lambda)\,\iota\left(\phi^s(\Lambda, r)\right) k_0.$$

(b) If $\Lambda < \Lambda'$,

$$\begin{aligned} X_z^* &= \lambda \left[\int_0^{\text{C}(\iota(\phi_z^s))} \text{C}^{-1}(\omega) d\Omega(\omega) + \{1 - \Omega[\text{C}(\iota(\phi_z^s))]\} \iota(\phi_z^s) \right] k_0 \\ S_z^* &= 0 \end{aligned}$$

and

$$X_0^* = S_0^* = 0.$$

(ii) If $\phi_z^s < \phi^s(1, r)$, let $\Lambda'' \in (\Lambda', 1)$ be the number that satisfies $\phi^s(\Lambda'', r) = \phi_z^s$. Then:

(a) If $\Lambda'' \leq \Lambda$, $X_z^* = \lambda \iota(\phi^s(\Lambda, r)) k_0$, $X_0^* = S_0^* = (1 - \lambda) \iota(\phi^s(\Lambda, r)) k_0$, and

$$S_z^* \begin{cases} = X_z^* & \text{if } \Lambda'' < \Lambda \\ \in \left[\lambda \int_0^{\text{C}(\iota(\phi^s(\Lambda, r)))} \frac{\text{C}(\iota(\phi^s(\Lambda, r))) - \omega}{\phi^s} k_0 d\Omega(\omega), X_z^* \right] & \text{if } \Lambda = \Lambda''. \end{cases}$$

(b) If $\Lambda' \leq \Lambda < \Lambda''$, X_Z^* and S_Z^* for $Z \in \{0, z\}$ are as in part (i)(a).

(c) If $\Lambda < \Lambda'$, X_Z^* and S_Z^* for $Z \in \{0, z\}$ are as in part (i)(b).

Proof. (i) (a) The expressions for $x_z^*(\omega)$ and $s_z^*(\omega)$ used to compute X_z^* and S_z^* are from part (iii) of Lemma 6, and the expressions for $x_0^*(\omega)$ and $s_0^*(\omega)$ used to compute X_0^* and S_0^* are from part (ii) of Lemma 6.

(i) (b) The expressions for $x_z^*(\omega)$ and $s_z^*(\omega)$ used to compute X_z^* and S_z^* are from part (iv) of Lemma 6, and the expressions for $x_0^*(\omega)$ and $s_0^*(\omega)$ used to compute X_0^* and S_0^* are from part (i) of Lemma 6.

(ii) (a) The expressions for $x_Z^*(\omega)$ and $s_Z^*(\omega)$ used to compute X_Z^* and S_Z^* for $Z \in \{0, z\}$ are from part (ii) of Lemma 6.

(ii) (b) The expressions for $x_z^*(\omega)$ and $s_z^*(\omega)$ used to compute X_z^* and S_z^* are from part (iii) of Lemma 6, and the expressions for $x_0^*(\omega)$ and $s_0^*(\omega)$ used to compute X_0^* and S_0^* are from part (ii) of Lemma 6.

(ii) (c) The expressions for $x_z^*(\omega)$ and $s_z^*(\omega)$ used to compute X_z^* and S_z^* are from part (iv) of Lemma 6, and the expressions for $x_0^*(\omega)$ and $s_0^*(\omega)$ used to compute X_0^* and S_0^* are from part (i) of Lemma 6. ■

The assumption $1 < \min\{\phi_z^s, \phi^s(1, \bar{r})\}$, or equivalently, $1 < \min\{\varepsilon_e, \bar{\varepsilon}\} \beta z$, in the statement of Lemma 8 ensures that, in the absence of adverse selection, entrepreneurs and outside investors would want to invest a positive amount under any monetary policy (i.e., even in the nonmonetary equilibrium that obtains for $r > \bar{r}$).[68]

To be part of an equilibrium, an investor's belief, Λ, that a traded equity share represents a claim to *productive* unit of capital that yields dividend $z > 0$ (as opposed to a claim to an unproductive unit of capital that yields zero dividend) must satisfy

$$\Lambda = \Upsilon(\Lambda) \in [0,1],$$

where

$$\Upsilon(\Lambda) \equiv \frac{S_z^*}{S_0^* + S_z^*}, \tag{162}$$

with S_Z^* for $Z \in \{0, z\}$ as described in Lemma 8. Next, we provide a more explicit characterization of the mapping Υ.

Lemma 9 *Let $\phi^s(\Lambda, r)$ be given by (161), define $\phi_Z^s \equiv \beta \varepsilon_e Z$ for $Z \in \{0, z\}$, and assume $1 < \min\{\phi_z^s, \phi^s(1, \bar{r})\}$. For any $r \in \mathbb{R}_+$, let $\Lambda'(r) \in (0,1)$ be the number that satisfies $\phi^s(\Lambda', r) = 1$, and for any $(\Lambda, r) \in [\Lambda'(r), 1] \times \mathbb{R}_+$ define*

$$\Theta(\Lambda, r) \equiv \int_0^{\mathrm{C}(\iota(\phi^s(\Lambda,r)))} \frac{\mathrm{C}(\iota(\phi^s(\Lambda, r))) - \omega}{\phi^s(\Lambda, r)\, \iota(\phi^s(\Lambda, r))} d\Omega(\omega).$$

(i) If $\phi^s(1, r) < \phi_z^s$,

$$\Upsilon(\Lambda) = \begin{cases} \frac{\lambda}{\lambda + (1-\lambda)\frac{1}{\Theta(\Lambda,r)}} & \text{for } \Lambda'(r) < \Lambda \\ 0 & \text{for } \Lambda = \Lambda'(r), \end{cases}$$

and equity is not issued if $\Lambda < \Lambda'(r)$.

(ii) If $\phi_z^s < \phi^s(1, r)$, let $\Lambda''(r) \in (\Lambda'(r), 1)$ be the number that satisfies $\phi^s(\Lambda'', r) = \phi_z^s$. Then:

$$\Upsilon(\Lambda) \begin{cases} = \lambda & \text{for } \Lambda''(r) < \Lambda \\ \in \left[\frac{\lambda}{\lambda + (1-\lambda)\frac{1}{\Theta(\Lambda,r)}}, \lambda\right] & \text{for } \Lambda''(r) = \Lambda \\ = \frac{\lambda}{\lambda + (1-\lambda)\frac{1}{\Theta(\Lambda,r)}} & \text{for } \Lambda'(r) < \Lambda < \Lambda''(r) \\ = 0 & \text{for } \Lambda = \Lambda'(r), \end{cases}$$

and equity is not issued if $\Lambda < \Lambda'(r)$.

[68] The condition $1 < \phi_z^s$ says that the entrepreneur with the high productivity realization has an incentive to invest because the entrepreneur's private return from investing a marginal unit of capital is higher than the price of capital (in terms of good 2, which equals 1). The condition $1 < \phi^s(1, \bar{r})$ says that in the absence of adverse selection, an outside investor's discounted expected marginal return from investment in productive capital under no equity trade, i.e., $\beta \bar{\varepsilon} z$, is higher than the price of capital (in terms of good 2, which equals 1).

Proof. The expression for $\Upsilon(\Lambda)$ for the case with $\Lambda'(r) < \Lambda$ in part (i) follows from (162) and parts $(i)(a)$ and $(i)(b)$ of Lemma 8. The expression for $\Upsilon(\Lambda)$ in part (ii) for the cases with $\Lambda'(r) < \Lambda$ follow from (162) and parts $(ii)(a)$, $(ii)(b)$, and $(ii)(c)$ of Lemma 8. To show that $\Upsilon(\Lambda'(r)) = 0$, both for $\Lambda'(r) \leq \Lambda$ in part (i), and for $\Lambda \in [\Lambda'(r), \Lambda''(r))$ in part (ii), proceed as follows. Write $\Theta(\Lambda, r)$ as

$$\Theta(\Lambda, r) = \frac{\int_0^{\mathrm{C}(\iota(\phi^s(\Lambda,r)))} [\mathrm{C}(\iota(\phi^s(\Lambda, r))) - \omega]\, d\Omega(\omega)}{\phi^s(\Lambda, r)\, \iota(\phi^s(\Lambda, r))},$$

notice that

$$\lim_{\Lambda \downarrow \Lambda'(r)} \phi^s(\Lambda, r) - 1 = \lim_{\Lambda \downarrow \Lambda'(r)} \iota(\phi^s(\Lambda, r)) = \lim_{\Lambda \downarrow \Lambda'(r)} \mathrm{C}(\iota(\phi^s(\Lambda, r))) = 0, \tag{163}$$

so

$$\lim_{\Lambda \downarrow \Lambda'(r)} \int_0^{\mathrm{C}(\iota(\phi^s(\Lambda,r)))} [\mathrm{C}(\iota(\phi^s(\Lambda, r))) - \omega]\, d\Omega(\omega) = \lim_{\Lambda \downarrow \Lambda'(r)} \phi^s(\Lambda, r)\, \iota(\phi^s(\Lambda, r)) = 0.$$

By L'Hôpital's rule,

$$\begin{aligned}
\lim_{\Lambda \downarrow \Lambda'(r)} \Theta(\Lambda, r) &= \lim_{\Lambda \downarrow \Lambda'(r)} \frac{\frac{\partial}{\partial \Lambda} \int_0^{\mathrm{C}(\iota(\phi^s(\Lambda,r)))} [\mathrm{C}(\iota(\phi^s(\Lambda, r))) - \omega]\, d\Omega(\omega)}{\frac{\partial}{\partial \Lambda} [\phi^s(\Lambda, r)\, \iota(\phi^s(\Lambda, r))]} \\
&= \lim_{\Lambda \downarrow \Lambda'(r)} \frac{\int_0^{\mathrm{C}(\iota(\phi^s(\Lambda,r)))} \mathrm{C}'(\iota(\phi^s(\Lambda, r))) \frac{\partial \iota(\phi^s(\Lambda,r))}{\partial \phi^s(\Lambda,r)} \frac{\partial \phi^s(\Lambda,r)}{\partial \Lambda} d\Omega(\omega)}{\iota(\phi^s(\Lambda, r)) \frac{\partial \phi^s(\Lambda,r)}{\partial \Lambda} + \phi^s(\Lambda, r) \frac{\partial \iota(\phi^s(\Lambda,r))}{\partial \phi^s(\Lambda,r)} \frac{\partial \phi^s(\Lambda,r)}{\partial \Lambda}} \\
&= \frac{\lim_{\Lambda \downarrow \Lambda'(r)} \left[\mathrm{C}'(\iota(\phi^s(\Lambda, r))) \frac{\partial \iota(\phi^s(\Lambda,r))}{\partial \phi^s(\Lambda,r)} \Omega(\mathrm{C}(\iota(\phi^s(\Lambda, r))))\right]}{\lim_{\Lambda \downarrow \Lambda'(r)} \left[\iota(\phi^s(\Lambda, r)) + \phi^s(\Lambda, r) \frac{\partial \iota(\phi^s(\Lambda,r))}{\partial \phi^s(\Lambda,r)}\right]} \\
&= \lim_{\Lambda \downarrow \Lambda'(r)} \Omega(\mathrm{C}(\iota(\phi^s(\Lambda, r)))) = 0,
\end{aligned}$$

where the last two equalities follow from (163) and

$$\lim_{\Lambda \downarrow \Lambda'(r)} \mathrm{C}'(\iota(\phi^s(\Lambda, r))) - 1 = 0.$$

This concludes the proof. ■

The following proposition considers an economy in which the equilibrium market valuation of marginal investment would be higher than the entrepreneur's valuation if there were no adverse selection, and shows that the presence of adverse selection causes the equilibrium market valuation of marginal investment to fall below the entrepreneur's valuation.

Proposition 4 *For any* $\Lambda \in [0,1]$*, let* $\phi^s(\Lambda, r)$ *be given by (161), and define* $\phi^s_Z \equiv \beta\varepsilon_e Z$ *for* $Z \in \{0, z\}$*. Assume* $1 < \min\{\phi^s_z, \phi^s(1, \bar{r})\}$ *and* $\phi^s_z < \phi^s(1, r)$*. For any* $r \in \mathbb{R}_+$*, let* $\Lambda'(r) \in (0,1)$ *be the number that satisfies* $\phi^s(\Lambda', r) = 1$*, and let* $\Lambda''(r) \in (\Lambda'(r), 1)$ *be the number that satisfies* $\phi^s(\Lambda'', r) = \phi^s_z$*. If* $\Lambda''(r) < \lambda < 1$*, there exists an equilibrium with equity issuance,* $(\phi^s(\Lambda^*, r), \Lambda^*)$*, with* $\Lambda^* \in (\Lambda'(r), \Lambda''(r)]$*, that is characerized by (161) and* $\Lambda^* = \Upsilon(\Lambda^*)$ *(with* Υ *as specified in Lemma 9), provided* $\Omega[\mathrm{C}(\iota(\phi^s(\Lambda^*, r)))] > 0$*. Moreover,* $\phi^s(\Lambda^*, r) \leq \phi^s_z$*, with "<" if*

$$\Lambda''(r) < \frac{\lambda}{\lambda + (1-\lambda)\frac{1}{\Theta(\Lambda''(r), r)}}. \tag{164}$$

Proof. In an equilibrium with equity issuance, the equity price is given by (161) (by Lemma 7), and the equilibrium belief, Λ^*, satisfies $\Lambda^* = \Upsilon(\Lambda^*)$ (with Υ as specified in Lemma 9). The assumption $1 < \min\{\phi^s_z, \phi^s(1, \bar{r})\}$ ensures that investment is always positive. The assumption $\phi^s_z < \phi^s(1, r)$ means that in the absence of adverse selection, the equilibrium market valuation of marginal investment would be higher than the entrepreneur's valuation of marginal investment, as in part (*ii*) of Lemma 9. In this case, it is immediate from part (*ii*) of Lemma 9, that if $\Lambda''(r) < \lambda < 1$, then there exists at least one value $\Lambda^* \in (\Lambda'(r), \Lambda''(r)]$ that satisfies $\Lambda^* = \Upsilon(\Lambda^*)$. Condition (164) implies $\Lambda^* < \Lambda''(r)$, which is equivalent to $\phi^s(\Lambda^*, r) < \phi^s_z$. ■

C Identification strategy

In this section we formalize the identification problem described in Section 3.3, and propose a strategy to address it. The *outcome variable* of interest for firm i in period t is denoted Y^i_t. In our application, Y^i_t represents either the log of the firm's investment rate at time t (i.e., x^i_t as defined in Section 4.2, which is the empirical counterpart of $\log \iota^*$ as defined in Proposition 1), or a measure of the firm's equity issuance in period t normalized by total assets (i.e., e^i_t as defined in Section 4.2, which is the empirical counterpart of $\phi^s \varsigma^*_{+1}$ as defined in Proposition 1). Let $\boldsymbol{v}^i_t \equiv (v^i_{1t}, ..., v^i_{Dt}) \in \mathbb{R}^D$ denote the D *transmission variables* through which a change in the nominal policy rate, r_t, may affect firm i's outcome variable Y^i_t in period t. To make this dependence explicit, we describe the outcome variable as a differentiable function, $Y : \mathbb{R}^D \to \mathbb{R}$, of the D transmission variables, i.e.,

$$Y^i_t = Y(\boldsymbol{v}^i_t). \tag{165}$$

We think of $\boldsymbol{v}_t^i$ as a vector of firm-specific and aggregate variables that influence the outcome variable Y_t^i. In our application, the first transmission variable, v_{1t}^i, is a measure of firm i's Tobin's q (e.g., the log of Tobin's q), which we denote q_t^i. Other elements of $\boldsymbol{v}_t^i$ could represent other firm-specific transmission variables, such as firm i's borrowing cost, the demand for its output, or the cost of inputs that firm i requires for production or investment, as well as marketwide transmission variables such as a baseline real interest rate, or other macro variables relevant for the firm's investment or capital-structure decisions. Firm i's transmission variables in period t may depend on the policy rate, r_t, as well as on a vector of *predetermined firm-level characteristics*, which we denote $\boldsymbol{\kappa}^i \equiv \left(\kappa_1^i, ..., \kappa_N^i\right) \in \mathbb{R}^N$. Formally, we describe each transmission variable $j \in \{1, ..., D\} \equiv \mathbb{D}$ as

$$v_{jt}^i = v_j\left(r_t, \boldsymbol{\kappa}^i\right) + \tilde{v}_{jt}^i, \tag{166}$$

where $v_j : \mathbb{R}^{N+1} \to \mathbb{R}$ is a differentiable function and $\tilde{v}_{jt}^i \in \mathbb{R}$, so we can write $\boldsymbol{v}_t^i = \boldsymbol{v}\left(r_t, \boldsymbol{\kappa}^i\right) + \tilde{\boldsymbol{v}}_t^i$, with $\boldsymbol{v}(\cdot) \equiv (v_1(\cdot), ..., v_D(\cdot))$ and $\tilde{\boldsymbol{v}}_t^i \equiv \left(\tilde{v}_{1t}^i, ..., \tilde{v}_{Dt}^i\right) \in \mathbb{R}^D$. (Our convention is to denote $v_1(\cdot)$ with $\mathrm{q}(\cdot)$.) The term $\tilde{v}_{jt}^i$ represents variation in transmission variable j (across firms and over time) that is independent of changes in the policy rate. The first characteristic, κ_1^i, is denoted $\mathcal{T}^i$, and represents the turnover rate of firm i's stock (i.e., the empirical counterpart of the variable $\mathcal{T}^i$ introduced in Section 3.2). Other elements of $\boldsymbol{\kappa}^i$ could represent other firm-level characteristics, such as the proportion of liquid assets relative to total assets in the firm's balance sheet (i.e., the empirical counterpart of the variable ω^i introduced in Section 2), other financial variables such as leverage, or non-financial variables such as firm i's sector, size, and age.

We allow for the possibility that only the first M firm-level characteristics are observed, while the last $N - M$ characteristics are unobserved. (We always treat stock turnover as an observed characteristic, so the integer M satisfies $1 \leq M \leq N$.) We also allow for the possibility that an unobserved characteristic may be related to the observed characteristics. Specifically, for each firm i we express an unobserved characteristic $s \in \{M+1, ..., N\}$ as

$$\kappa_s^i = \kappa_s(\kappa_1^i, ..., \kappa_M^i) + \varepsilon_{is}^\kappa, \tag{167}$$

where $\kappa_s : \mathbb{R}^M \to \mathbb{R}$ is a differentiable function, and $\varepsilon_{is}^\kappa \in \mathbb{R}$. The function κ_s describes the relation between the unobserved characteristic s and the observed characteristics. We interpret any unobserved characteristic s with $\partial\kappa_s(\cdot)/\partial\kappa_n = 0$ as uncorrelated with observed characteristic n. The term $\varepsilon_{is}^{\kappa}$ represents cross-sectional variation in the unobserved firm-level characteristic s that is independent of monetary policy shocks, uncorrelated with the observed firm-level characteristics, and satisfies $\mathbb{E}(\varepsilon_{is}^{\kappa}) = 0$. In what follows, it will be convenient to work with a first-order approximation around a point $(\bar{\kappa}_1, ..., \bar{\kappa}_M) \in \mathbb{R}^M$ to write each unobserved characteristic $s \in \{M+1, ..., N\}$ described in (167) as a linear function of observed characteristics, i.e.,

$$\kappa_s^i \approx \bar{\kappa}_s + \sum_{n=1}^{M} \varkappa_{sn}(\kappa_n^i - \bar{\kappa}_n) + \varepsilon_{is}^{\kappa}, \tag{168}$$

where $\bar{\kappa}_s \equiv \kappa_s(\bar{\kappa}_1, ..., \bar{\kappa}_M)$, and $\varkappa_{sn} \equiv \frac{\partial \kappa_s(\bar{\kappa}_1,...,\bar{\kappa}_M)}{\partial \kappa_n}$ represents the correlation between unobserved characteristic s and observed characteristic n. (Our convention is to denote $\bar{\kappa}_1$ with $\bar{\mathcal{T}}$, and $\varkappa_{s1}$ with $\varkappa_{s\mathcal{T}}$ for $s \in \{M+1, ..., N\}$.)

A first-order approximation to the function $Y(\boldsymbol{v}_t^i)$ (defined in (165)) around the point $\bar{\boldsymbol{v}} \equiv \boldsymbol{v}(\bar{r}, \bar{\boldsymbol{\kappa}}) \in \mathbb{R}^D$ for some $(\bar{r}, \bar{\boldsymbol{\kappa}}) \in \mathbb{R}^{N+1}$ gives

$$Y_t^i \approx \bar{Y} + \sum_{j=1}^{D} \gamma^j (v_{jt}^i - \bar{v}_j), \tag{169}$$

where $\bar{Y} \equiv Y(\bar{\boldsymbol{v}})$, and $\gamma^j \equiv \frac{\partial Y(\bar{\boldsymbol{v}})}{\partial v_j}$ for $j \in \mathbb{D}$. (Our convention is to denote $\gamma^1 \equiv \frac{\partial Y(\bar{\boldsymbol{v}})}{\partial v_1}$ by $\gamma^{\mathrm{q}} \equiv \frac{\partial Y(\bar{\boldsymbol{v}})}{\partial \mathrm{q}}$.) Intuitively, the coefficient γ^j measures the effect of a marginal increase in the transmission variable j on the outcome variable. We are interested in estimating γ^{q}, which quantifies the *q-channel* (i.e., gives the effect of an exogenous increase in q_t^i on the outcome variable Y_t^i).

Suppose that for each transmission variable $j \in \mathbb{D}$ described in (166), we consider a first-order approximation to the function $v_j(\cdot)$ around $(\bar{r}, \bar{\boldsymbol{\kappa}})$,

$$v_{jt}^i \approx \bar{\mathrm{v}}_j^i + \alpha_r^j (r_t - \bar{r}) + \tilde{v}_{jt}^i, \tag{170}$$

where $\alpha_r^j \equiv \frac{\partial v_j(\bar{r}, \bar{\boldsymbol{\kappa}})}{\partial r}$, and $\bar{\mathrm{v}}_j^i$ is independent of r. (Our convention is to denote α_r^1 by α_r^{q}.) Intuitively, the coefficient α_r^j is an estimate of the (first-order) effect of an increase in the policy rate on a firm's transmission variable j. Next, suppose that from period $t-1$ to period t the policy rate changes from r_{t-1} to $r_t = r_{t-1} + \varepsilon_t^m$, where ε_t^m represents an unexpected policy shock, and at the same time, for firm i, $\tilde{v}_{jt-1}^i$ changes to $\tilde{v}_{jt}^i = \tilde{v}_{jt-1}^i + \varepsilon_{jit}^v$, where $\varepsilon_{jit}^v \in \mathbb{R}$. Intuitively, ε_{jit}^v represents time variation in the transmission variable v_{jt}^i that is independent of time variation in the policy rate. Conditions (169) and (170) imply

$$Y_t^i - Y_{t-1}^i \approx \gamma^{\mathrm{q}}(\mathrm{q}_t^i - \mathrm{q}_{t-1}^i) + \tilde{\mathrm{u}}_t^i \approx \delta_r^{\mathrm{q}} \varepsilon_t^m + \mathrm{u}_t^i, \tag{171}$$

where $\tilde{u}_t^i \equiv \sum_{j=2}^{D} \gamma^j (v_{jt}^i - v_{jt-1}^i) = \sum_{j=2}^{D} (\delta_r^j \varepsilon_t^m + \gamma^j \varepsilon_{jit}^v)$ and $u_t^i \equiv \tilde{u}_t^i + \gamma^{q} \varepsilon_{1it}^v$, with $\delta_r^j \equiv \gamma^j \alpha_r^j$ for $j \in \mathbb{D}$ (our convention is to use δ_r^{q} to denote δ_r^1). The first approximation in (171) can be thought of as the basis for a "structural form" that regresses the change in the outcome variable on the change in Tobin's q. The second approximation in (171) can be thought of as the basis for a "reduced form" that regresses the change in the outcome variable directly on a measure of the monetary shock, ε_t^m. Together, the two approximations in (171) suggest an identification strategy that uses ε_t^m as an instrument for $q_t^i - q_{t-1}^i$, which would solve the problem of isolating policy-driven variation in q_t^i. Our concern with this approach, however, is that it does not allay the problem of other omitted monetary transmission channels, since it would be difficult to argue that the instrument ε_t^m satisfies the *exclusion restriction*, i.e., that the money shock does not affect the outcome variable Y_t^i through transmission variables *other* than Tobin's q. In terms of (171), this exclusion restriction ensures there is no correlation between $\tilde{u}_t^i$ and (instrumented changes in) q_t^i, or equivalently, no correlation between the reduced-form residual, u_t^i, and the money shock, ε_t^m.[69]

The existing literature on monetary transmission offers many examples of channels that would lead to correlation between $\tilde{u}_t^i$ and changes in q_t^i instrumented with ε_t^m (or equivalently, correlation between u_t^i and ε_t^m). To illustrate, suppose the outcome variable Y_t^i is a measure of firm i's investment. According to the *interest-rate channel*, for instance, an unexpected decrease in the nominal policy rate that passes through to the real interest rate would have two effects: (a) decrease the user cost of capital, which increases investment, and *(b)* decrease the discount rate for future dividends, which increases the stock price, therefore leading to positive correlation between q_t^i and $\tilde{u}_t^i$. According to the *(heterogeneous) borrowing-cost channel*, an

[69]In Lemma 10 (Section C.2, in Appendix C) we show that

$$cov\left(q_t^i - q_{t-1}^i, \tilde{u}_t^i\right) = \alpha_r^{q} cov\left(\varepsilon_t^m, u_t^i\right) + \sum_{j=2}^{D} \gamma^j cov(\varepsilon_{1it}^v, \varepsilon_{jit}^v), \tag{172}$$

with $cov\left(\varepsilon_t^m, u_t^i\right) = var\left(\varepsilon_t^m\right) \sum_{j=2}^{D} \delta_r^j$. To calculate (172) we are using (170), which allows for variation in q_t^i that is caused not only by the monetary policy shock, ε_t^m, but also by the independent shock, ε_{1it}^v. Each of the last $D-1$ covariances in (172) will be nonzero if the exogenous shock to firm i's Tobin's q, i.e., ε_{1it}^v, is correlated with the exogenous shock to firm i's transmission variable $j \in \{2, ..., D\}$, i.e., ε_{jit}^v. In turn, each of these covariances will contribute to the covariance between q_t^i and $\tilde{u}_t^i$ if the transmission variable j affects the outcome variable (i.e., if $\gamma^j \neq 0$). The last $D-1$ terms in (172) would be absent if we focused on the covariance between $\tilde{u}_t^i$ and variation in q_t^i *that is caused exclusively by the monetary policy shock*, ε_t^m. But even in this case, given that $\alpha_r^{q} < 0$, the first term in (172) would vanish only if $cov\left(\varepsilon_t^m, u_t^i\right) = 0$, which is equivalent to the restriction $\delta_r^j = 0$ for all $j \in \{2, ..., D\}$. Thus, the exclusion restriction that ε_t^m would have to satisfy to be a valid instrument for $q_t^i - q_{t-1}^i$ is that the monetary policy shock does not affect the outcome variable through any transmission variable other than Tobin's q.

increase in the policy rate that passes through to the real interest rate and affects firm i's borrowing cost would have two effects: (a) change firm i's investment relative to other firms (due to the change in firm i's relative cost of borrowing to finance investment), and (b) decrease firm i's stock price (due to higher discounting of future dividends).[70] These examples illustrate that one cannot, in general, hope to estimate the causal effects of changes in a firm's equity prices on investment (or equity issuance)—the hallmark of the q-channel—simply from the comovement between equity prices and the outcome variable of interest that is induced by monetary policy shocks.

We meet this identification challenge by exploiting the cross-sectional variation in the responses of stock prices to monetary shocks, which we refer to as the *turnover channel.* Specifically, we will regress changes in the outcome variable on changes in stock prices induced by monetary-policy shocks, but our identification strategy will consist of using $\varepsilon_{it}^{\mathcal{T}m} \equiv (\mathcal{T}^i - \bar{\mathcal{T}})\varepsilon_t^m$ (i.e., the *product between a firm-specific predetermined measure of stock turnover and the money shock*) as an instrument for the change in the firm's stock price. Stock turnover has a strong effect on the passthrough of the policy shock to the stock price, which implies a strong correlation between the proposed instrument and the change in the stock price.[71] We will show that the relevant exclusion restriction will be satisfied as long as stock turnover (and any *unobserved* firm-level characteristic that is correlated with stock turnover) has no effect on the passthrough of the monetary-policy shock to transmission variables other than Tobin's q that influence the outcome variable.

Our identification strategy exploits the cross-sectional variation in the effects of the money shock on transmission variables that is associated with variation in firm-level characteristics. Thus, for each transmission variable $j \in \mathbb{D}$, we replace the first-order approximation to the function $v_j(\cdot)$ on the right side of (170) with a *second-order* approximation to the function $v_j(\cdot)$ around the point $(\bar{r}, \bar{\boldsymbol{\kappa}}) \in \mathbb{R}^{N+1}$, i.e.,

$$v_{jt}^i \approx \hat{\text{v}}_j^i + \left[\alpha_r^j + \alpha_{rr}^j (r_t - \bar{r})\right](r_t - \bar{r}) + \sum_{n=1}^{N} \alpha_{rn}^j \left(\kappa_n^i - \bar{\kappa}_n\right)(r_t - \bar{r}) + \tilde{v}_{jt}^i, \tag{173}$$

where $\alpha_r^j \equiv \frac{\partial v_j(\bar{r},\bar{\boldsymbol{\kappa}})}{\partial r}$, $\alpha_{rr}^j \equiv \frac{1}{2}\frac{\partial v_j(\bar{r},\bar{\boldsymbol{\kappa}})}{\partial r \partial r}$, $\alpha_{rn}^j \equiv \frac{\partial v_j(\bar{r},\bar{\boldsymbol{\kappa}})}{\partial \kappa_n \partial r}$ for $n \in \{1, ..., N\}$, and $\hat{\text{v}}_j^i$ is independent of

[70] Notice that in this third transmission mechanism, the change in firm i's investment is typically a function of firm i's idiosyncratic characteristics (such as its leverage, share of liquid assets, or other firm-level variables). For recent studies of the *(heterogeneous) borrowing-cost channel*, see Jeenas (2019) and Ottonello and Winberry (2020).

[71] This is essentially the *turnover-liquidity channel* documented in Lagos and Zhang (2020b).

r_t. Intuitively, the coefficients α_r^j and α_{rr}^j quantify the strength of the first- and second-order effects, respectively, of a marginal increase in the policy rate on a firm's transmission variable j, while controling for firm-level characteristics. The coefficient α_{rn}^j quantifies the part of the effect of a marginal increase in the policy rate on a firm's transmission variable j that varies with the firm-level characteristic $n \in \{1, ..., N\} \equiv \mathbb{N}$.[72] Our convention is to denote α_{r1}^j by $\alpha_{r\mathcal{T}}^j$, and α_r^1, α_{rr}^1, and α_{rn}^1 by $\alpha_r^{\mathrm{q}} \equiv \frac{\partial \mathrm{q}(\bar{r},\bar{\boldsymbol{\kappa}})}{\partial r}$, $\alpha_{rr}^{\mathrm{q}} \equiv \frac{1}{2}\frac{\partial \mathrm{q}(\bar{r},\bar{\boldsymbol{\kappa}})}{\partial r \partial r}$, and $\alpha_{rn}^{\mathrm{q}} \equiv \frac{\partial \mathrm{q}(\bar{r},\bar{\boldsymbol{\kappa}})}{\partial \kappa_n \partial r}$, respectively.

Suppose, again, that from period $t-1$ to period t the policy rate changes from r_{t-1} to $r_t = r_{t-1} + \varepsilon_t^m$, where ε_t^m represents an unexpected policy shock, and at the same time, for firm i, $\tilde{v}_{jt-1}^i$ changes to $\tilde{v}_{jt}^i = \tilde{v}_{jt-1}^i + \varepsilon_{jit}^v$, where $\varepsilon_{jit}^v \in \mathbb{R}$ (as before, ε_{jit}^v represents time variation in the transmission variable v_{jt}^i that is independent of time variation in the policy rate). If we let $\mathrm{q}_t^i \equiv v_1(r_t, \boldsymbol{\kappa}^i) + \tilde{v}_{1t}^i$, then (173) implies

$$\mathrm{q}_t^i - \mathrm{q}_{t-1}^i \approx \mathrm{a}_t^{\mathrm{q}} + \sum_{n=1}^{N} \alpha_{rn}^{\mathrm{q}} \left(\kappa_n^i - \bar{\kappa}_n\right) \varepsilon_t^m + \varepsilon_{it}^{\mathrm{q}}, \tag{174}$$

where $\varepsilon_{it}^{\mathrm{q}} \equiv \varepsilon_{1it}^v$ and $\mathrm{a}_t^j \equiv \{\alpha_r^j + \alpha_{rr}^j [\varepsilon_t^m + 2(r_{t-1} - \bar{r})]\}\varepsilon_t^m$ for any $j \in \mathbb{D}$ (our convention is to use $\mathrm{a}_t^{\mathrm{q}}$ to denote a_t^1). To account for the fact that only the first M predetermined firm-level characteristics are observable, we can use (168) to write (174) as

$$\mathrm{q}_t^i - \mathrm{q}_{t-1}^i \approx \mathrm{a}_t^{\mathrm{q}} + \hat{\alpha}_{r\mathcal{T}}^{\mathrm{q}} \varepsilon_{it}^{\mathcal{T}m} + \sum_{n=2}^{M} \hat{\alpha}_{rn}^{\mathrm{q}} \left(\kappa_n^i - \bar{\kappa}_n\right) \varepsilon_t^m + \epsilon_{it}^{\mathrm{q}}, \tag{175}$$

where $\epsilon_{it}^{\mathrm{q}} \equiv \varepsilon_{it}^{\mathrm{q}} + \sum_{s=M+1}^{N} \alpha_{rs}^{\mathrm{q}} \varepsilon_{is}^{\kappa} \varepsilon_t^m$, and

$$\hat{\alpha}_{rn}^j \equiv \alpha_{rn}^j + \sum_{s=M+1}^{N} \alpha_{rs}^j \varkappa_{sn}, \text{ for } j \in \mathbb{D} \text{ and } n \in \{1, ..., M\}. \tag{176}$$

(Our convention is to denote $\hat{\alpha}_{r1}^j$ by $\hat{\alpha}_{r\mathcal{T}}^j$, $\varkappa_{s1}$ by $\varkappa_{s\mathcal{T}}$, and $\hat{\alpha}_{rn}^1$ by $\hat{\alpha}_{rn}^{\mathrm{q}}$.) The representation (175) is reminiscent of the main regression in Bernanke and Kuttner (2005), who focus on estimating α_r^{q} (the coefficient in the first term of $\mathrm{a}_t^{\mathrm{q}}$), and one of the regression specifications in Lagos and Zhang (2020b), who focus on estimating $\hat{\alpha}_{r\mathcal{T}}^{\mathrm{q}}$. Intuitively, $\hat{\alpha}_{r\mathcal{T}}^{\mathrm{q}}$ measures the component of the effect of ε_t^m on q_t^i that varies with the turnover rate of the firm's stock, $\mathcal{T}^i$ (when controlling for

[72] Strictly speaking, α_r^j is the first-order effect of a marginal increase in the policy rate, r_t, on a firm's decision variable $j \in \mathbb{D}$ while controling for firm-level characteristics; $2r_t\alpha_{rr}^j$ is the second-order effect of a marginal increase in the policy rate, r_t, on a firm's decision variable $j \in \mathbb{D}$ while controling for firm-level characteristics; and $\alpha_{rn}^j \kappa_{nt-1}^i$ is the component of the effect of a marginal increase in the policy rate on firm i's decision variable $j \in \mathbb{D}$ that varies with the firm-level characteristic κ_{nt-1}^i for $n \in \mathbb{N}$.

the observed characteristics $n \in \{2, ..., M\}$). Notice that $\hat{\alpha}^{\mathrm{q}}_{r\mathcal{T}}$ captures not only the influence of stock turnover on the marginal effect of ε^m_t on q^i_t (i.e., through the term $\alpha^{\mathrm{q}}_{r\mathcal{T}}$), but also the influence of all other unobserved characteristics $s \in \{M+1, ..., N\}$ that are correlated with stock turnover (i.e., through the $N-M$ terms $\alpha^{\mathrm{q}}_{rs}\varkappa_{s\mathcal{T}}$ for $s \in \{M+1, ..., N\}$). Thus, we can think of $\hat{\alpha}^{\mathrm{q}}_{r\mathcal{T}}$ as an estimate of the *turnover-liquidity mechanism* through which monetary policy affects stock prices discussed in Section 3.2 and documented in Lagos and Zhang (2020b).

Similarly, if between period $t-1$ and period t the policy rate changes from r_{t-1} to $r_t = r_{t-1} + \varepsilon^m_t$ and $\tilde{v}^i_{jt-1}$ changes to $\tilde{v}^i_{jt} = \tilde{v}^i_{jt-1} + \varepsilon^v_{jit}$, then (169) and (173) imply

$$Y^i_t - Y^i_{t-1} \approx \mathrm{d}_t + \sum_{j=1}^{D}\sum_{n=1}^{N} \delta^j_{rn}\left(\kappa^i_n - \bar{\kappa}_n\right)\varepsilon^m_t + \bar{\varepsilon}_{it}, \tag{177}$$

where $\bar{\varepsilon}_{it} \equiv \sum_{j=1}^{D}\gamma^j\varepsilon^v_{jit}$, $\mathrm{d}_t \equiv \sum_{j=1}^{D}\gamma^j \mathrm{a}^j_t = \sum_{j=1}^{D}\{\delta^j_r + \delta^j_{rr}\left[\varepsilon^m_t + 2\left(r_{t-1} - \bar{r}\right)\right]\}\varepsilon^m_t$, and for all transmission variables $j \in \mathbb{D}$, $\delta^j_r \equiv \gamma^j\alpha^j_r$, $\delta^j_{rr} \equiv \gamma^j\alpha^j_{rr}$, and $\delta^j_{rn} \equiv \gamma^j\alpha^j_{rn}$ for all firm-level characteristics $n \in \mathbb{N}$. (Our convention is to denote δ^j_{r1} by $\delta^j_{r\mathcal{T}}$, and δ^1_r, δ^1_{rr}, and δ^1_{rn}, by δ^{q}_r, δ^{q}_{rr}, and δ^{q}_{rn}, respectively.) To account for the fact that only the first M predetermined firm-level characteristics are observable, we can use (168) to write (177) as

$$Y^i_t - Y^i_{t-1} \approx \mathrm{d}_t + \hat{\delta}^{\mathrm{q}}_{r\mathcal{T}}\varepsilon^{\mathcal{T}m}_{it} + \sum_{n=2}^{M}\tilde{\delta}_{rn}\left(\kappa^i_n - \bar{\kappa}_n\right)\varepsilon^m_t + \bar{\epsilon}_{it}, \tag{178}$$

where $\hat{\delta}^{\mathrm{q}}_{rn} \equiv \gamma^{\mathrm{q}}\hat{\alpha}^{\mathrm{q}}_{rn}$, $\tilde{\delta}_{rn} \equiv \gamma^{\mathrm{q}}\hat{\alpha}^{\mathrm{q}}_{rn} + \tilde{\delta}^{\sim\mathrm{q}}_{rn}$, $\bar{\epsilon}_{it} \equiv \tilde{\delta}^{\sim\mathrm{q}}_{r\mathcal{T}}\varepsilon^{\mathcal{T}m}_{it} + \bar{\varepsilon}_{it} + \sum_{j=1}^{D}\sum_{s=M+1}^{N}\delta^j_{rs}\varepsilon^{\kappa}_{is}\varepsilon^m_t$, and

$$\tilde{\delta}^{\sim\mathrm{q}}_{rn} \equiv \sum_{j=2}^{D}\gamma^j\hat{\alpha}^j_{rn} \tag{179}$$

for $n \in \{1, ..., M\}$ (with $\tilde{\delta}_{r1} \equiv \tilde{\delta}_{r\mathcal{T}}$, $\tilde{\delta}^{\sim\mathrm{q}}_{r1} \equiv \tilde{\delta}^{\sim\mathrm{q}}_{r\mathcal{T}}$, and $\hat{\delta}^{\mathrm{q}}_{r1} \equiv \hat{\delta}^{\mathrm{q}}_{r\mathcal{T}}$). The representation (178) decomposes the effect of the monetary shock on the outcome variable into two sets of mechanisms. The first, represented by the term d_t, consists of the first- and second-order effects of the policy shock (ε^m_t) that influence Y^i_t through all the transmission variables, $\left(v^i_1, ..., v^i_D\right)$, but that do not vary with firm-level characteristics. These are transmission channels that affect all firms in the same way, i.e., channels that induce no cross-sectional variation in the responses of the firms' outcome variable to the money shock. The second set of mechanisms, represented by the collection of terms in $\tilde{\delta}_{rn}$ for $n \in \{1, ..., M\}$, consists of all the transmission channels for the policy shock (that operate on Y^i_t through any transmission variable $j \in \mathbb{D}$) that vary with each observable firm-level characteristic n. In particular, $\tilde{\delta}_{r\mathcal{T}} \equiv \gamma^{\mathrm{q}}\hat{\alpha}^{\mathrm{q}}_{r\mathcal{T}} + \tilde{\delta}^{\sim\mathrm{q}}_{r\mathcal{T}}$ includes all the transmission channels (operating through any transmission variable $j \in \mathbb{D}$) that induce cross-sectional variation in the responses of the outcome variable to the money shock due to cross-sectional variation in stock turnover.[73]

Together, (175) and (178) imply

$$Y^i_t - Y^i_{t-1} \approx \mathrm{b}_t + \gamma^{\mathrm{q}}\left(\mathrm{q}^i_t - \mathrm{q}^i_{t-1}\right) + \sum_{n=2}^{M} \tilde{\delta}^{\sim\mathrm{q}}_{rn}\left(\kappa^i_n - \bar{\kappa}_n\right)\varepsilon^m_t + \tilde{\epsilon}_{it}, \tag{180}$$

where $\mathrm{b}_t \equiv \mathrm{d}_t - \gamma^{\mathrm{q}}\mathrm{a}^{\mathrm{q}}_t$ and $\tilde{\epsilon}_{it} \equiv \tilde{\delta}^{\sim\mathrm{q}}_{r\mathcal{T}}\varepsilon^{\mathcal{T}m}_{it} + \hat{\epsilon}_{it}$, with $\hat{\epsilon}_{it} \equiv \bar{\epsilon}_{it} - \gamma^{\mathrm{q}}\epsilon^{\mathrm{q}}_{it}$. The representation (180) can be thought of as the basis for a "structural form" that regresses the (change in the) outcome variable on the change in Tobin's q and some controls (i.e., a time dummy, b_t, and interactions of the shock with observed firm-level characteristics, $\left\{\left(\kappa^i_n - \bar{\kappa}_n\right)\varepsilon^m_t\right\}^M_{n=2}$). Together, the representations (175), (178), and (180) suggest an identification strategy that uses $\varepsilon^{\mathcal{T}m}_{it}$ as an instrument for $\mathrm{q}^i_t - \mathrm{q}^i_{t-1}$: Think of (178) as a "reduced form" that regresses the change in the outcome variable directly on the instrument and other controls (a time dummy, d_t, and interactions of the money shock with a vector of the other $M-1$ observed firm-level characteristics, $\left\{\left(\kappa^i_n - \bar{\kappa}_n\right)\varepsilon^m_t\right\}^M_{n=2}$); think of (175) as the "first stage" that projects $\mathrm{q}^i_t - \mathrm{q}^i_{t-1}$ onto the instrument $\varepsilon^{\mathcal{T}m}_{it}$ and other controls (a time dummy, $\mathrm{a}^{\mathrm{q}}_t$, and interactions of the money shock with a vector of the other $M-1$ observed firm-level characteristics); and think of (180) as the "structural form" estimated with the first-stage projection replacing $\mathrm{q}^i_t - \mathrm{q}^i_{t-1}$.[74] Two conditions need to be satisfied for $\varepsilon^{\mathcal{T}m}_{it}$ to be a valid instrument for $\mathrm{q}^i_t - \mathrm{q}^i_{t-1}$ in order to estimate γ^{q} by using (180) as the basis for an IV regression. First, $\varepsilon^{\mathcal{T}m}_{it}$ must be correlated with the change in firm i's stock price, $\mathrm{q}^i_t - \mathrm{q}^i_{t-1}$. This correlation is negative and strong (it is the

[73]For each firm-level observable characteristic $n \in \{1, ..., M\}$, $\tilde{\delta}_{rn} \equiv \gamma^{\mathrm{q}}\hat{\alpha}^{\mathrm{q}}_{rn} + \tilde{\delta}^{\sim\mathrm{q}}_{rn}$ gives a decomposition of all the transmission channels that affect the outcome variable differentially (depending on characteristic n), into two components: a component that consists exclusively of the q-channel (i.e., the channel for which the transmission variable is Tobin's q, represented by $\gamma^{\mathrm{q}}\hat{\alpha}^{\mathrm{q}}_{rn}$), and a component that contains all other channels that work through all transmission variables *other than* Tobin's q (represented by $\tilde{\delta}^{\sim\mathrm{q}}_{rn}$). Intuitively, the first component of $\tilde{\delta}_{r\mathcal{T}}$, i.e., $\gamma^{\mathrm{q}}\hat{\alpha}^{\mathrm{q}}_{r\mathcal{T}}$, is the sum of the effects of ε^m_t on Y^i_t that vary with firm i's stock turnover, $\mathcal{T}^i$, and are transmitted to Y^i_t *exclusively through Tobin's q*. In other words, we can think of $\gamma^{\mathrm{q}}\hat{\alpha}^{\mathrm{q}}_{r\mathcal{T}}$ as the portion of the q-channel of monetary transmission to Y^i_t that depends on the *turnover liquidity* of the firm's stock as documented in Lagos and Zhang (2020b). Notice that $\gamma^{\mathrm{q}}\hat{\alpha}^{\mathrm{q}}_{r\mathcal{T}}$ captures not only the influence of stock turnover on the marginal effect of ε^m_t on Y^i_t (i.e., through the term $\gamma^{\mathrm{q}}\alpha^{\mathrm{q}}_{r\mathcal{T}}$), but also the influence on the marginal effect of ε^m_t on Y^i_t of all other unobserved characteristics s that are correlated with stock turnover (i.e., through the $N-M$ terms $\gamma^{\mathrm{q}}\alpha^{\mathrm{q}}_{rs}\varkappa_{s\mathcal{T}}$ for $s \in \{M+1, ..., N\}$). The second component of $\tilde{\delta}_{r\mathcal{T}}$, i.e., $\tilde{\delta}^{\sim\mathrm{q}}_{r\mathcal{T}}$, is the sum of the effects of ε^m_t on Y^i_t that vary with firm i's stock turnover, and are transmitted to Y^i_t through all channels *other than* Tobin's q.

[74]Our baseline reduced-form formulation in Section 4.2 (i.e., equation (22)) does not control for firm-level characteristics other than stock turnover, so it can be thought of in terms of a version of (178) with $M = 1$.

turnover-liquidity mechanism documented by Lagos and Zhang (2020b)). Second, $\varepsilon_{it}^{\mathcal{T}m}$ must affect the outcome variable, Y_t^i, in the structural form (180) only through the transmission variable $\mathrm{q}_t^i - \mathrm{q}_{t-1}^i$. In other words, the instrument $\varepsilon_{it}^{\mathcal{T}m}$ must be uncorrelated with $\tilde{\epsilon}_{it}$. In Lemma 11 (Section C.2, in Appendix C) we show that, under our maintained assumptions (namely ε_t^m independent of ε_{jit}^v for $j \in \{1, ..., D\}$, independent of ε_{is}^κ for $s \in \{M+1, ..., N\}$, and $\mathbb{E}\left(\varepsilon_{is}^\kappa\right) = 0$ for $s \in \{M+1, ..., N\}$) imply $cov\left(\varepsilon_{it}^{\mathcal{T}m}, \tilde{\epsilon}_{it}\right) = \tilde{\delta}_{r\mathcal{T}}^{\sim \mathrm{q}} var\left(\varepsilon_{it}^{\mathcal{T}m}\right)$, so the *exclusion restriction* for $\varepsilon_{it}^{\mathcal{T}m}$ to be a valid instrument for $\mathrm{q}_t^i - \mathrm{q}_{t-1}^i$ is satisfied if and only if $\tilde{\delta}_{r\mathcal{T}}^{\sim \mathrm{q}} = 0$.[75] With (176) and (179), $\tilde{\delta}_{r\mathcal{T}}^{\sim \mathrm{q}} = 0$ is equivalent to

$$\sum_{j=2}^{D} \gamma^j \left(\alpha_{r\mathcal{T}}^j + \sum_{s=M+1}^{N} \alpha_{rs}^j \varkappa_{s\mathcal{T}} \right) = 0. \tag{181}$$

Condition (181) says that $(T^i - \bar{T})\varepsilon_t^m$ can serve as an instrument for Tobin's q if for every $j \in \{2, ..., D\}$ (i.e., for every transmission variable other than Tobin's q), either $\gamma^j = 0$, or $\alpha_{r\mathcal{T}}^j = \alpha_{rs}^j \varkappa_{s\mathcal{T}} = 0$ for all $s \in \{M+1, ..., N\}$.[76] In words: the exclusion restriction (181) is satisfied as long as stock turnover (and any unobserved firm-level characteristic that is correlated with turnover) has no effect on the passthrough of the monetary-policy shock to transmission variables other than Tobin's q that influence the outcome variable.[77]

C.1 Feedback from the outcome variable to the transmission variables

In this section we show that our identification strategy remains valid if we interpret specification (165) and (166) as the reduced form of a simultaneous system where, not only does q_t^i affect the outcome variable Y_t^i (as in (165)), but the outcome variable Y_t^i also affects q_t^i. To formalize

[75] In Corollary 6 we show that under our maintained assumptions, we have: (*i*) $cov\left(\varepsilon_{it}^{\mathcal{T}m}, \bar{\epsilon}_{it}\right) = cov\left(\varepsilon_{it}^{\mathcal{T}m}, \tilde{\epsilon}_{it}\right)$, so if the exclusion restriction $\tilde{\delta}_{r\mathcal{T}}^{\sim \mathrm{q}} = 0$ holds, (178) will deliver an unbiased OLS estimate of $\hat{\delta}_{r\mathcal{T}}^{\mathrm{q}}$; and (*ii*) $cov\left(\varepsilon_{it}^{\mathcal{T}m}, \epsilon_{it}^{\mathrm{q}}\right) = 0$, so specification (175) will deliver an unbiased OLS estimate of $\hat{\alpha}_{r\mathcal{T}}^{\mathrm{q}}$. Notice that $\frac{\hat{\delta}_{r\mathcal{T}}^{\mathrm{q}}}{\hat{\alpha}_{r\mathcal{T}}^{\mathrm{q}}} = \gamma^{\mathrm{q}}$, so if the exclusion restriction holds, the coefficient of interest, γ^{q}, can be obtained as the ratio of the OLS estimate of the effect of the instrument on the outcome variable in the reduced form, (178), and the OLS estimate of the effect of the instrument on Tobin's q in the first stage, (175).

[76] The condition $\gamma^j = 0$ means that j does not operate as a transmission variable for the outcome of interest. The condition $\alpha_{r\mathcal{T}}^j = 0$ means that firm i's stock turnover does not influence the marginal effect of the policy rate on transmission variable j. The condition $\alpha_{rs}^j \varkappa_{s\mathcal{T}} = 0$ for all $s \in \{M+1, ..., N\}$ means that every unobserved characteristic that is correlated with stock turnover has no influence on the marginal effect of the policy rate on transmission variable j.

[77] Notice that if transmission variable $j \in \{2, ..., D\}$ is an *aggregate* variable (rather than a *firm-specific* transmission variable), then $\alpha_{r\mathcal{T}}^j = \alpha_{rs}^j = 0$ for all $s \in \{M+1, ..., N\}$, so $\gamma^j \hat{\alpha}_{r\mathcal{T}}^j = 0$ is automatically satisfied. Thus, our identification strategy is very powerful to exclude transmission channels that operate through *aggregate* transmission variables (rather than firm-specific transmission variables), such as the *interest-rate channel* discussed above.

this, we keep the specifications of the outcome variable, (165), and firm-level characteristics, (167), unchanged, but generalize (166) to

$$v_{jt}^{i}=v_{j}\left(r_{t},\boldsymbol{\kappa}^{i}\right)+\tau_{j}Y_{t}^{i}+\tilde{v}_{jt}^{i},\tag{182}$$

where $\tau_j \in \mathbb{R}$, for each $j \in \{1,...,D\}$ (with $\tau_{\mathrm{q}} \equiv \tau_1$). The approximations (168) and (169) remain unchanged, and the approximation (173) generalizes to

$$v_{jt}^{i}\approx\hat{\mathrm{v}}_{j}^{i}+\left[\alpha_{r}^{j}+\alpha_{rr}^{j}\left(r_{t}-\bar{r}\right)\right]\left(r_{t}-\bar{r}\right)+\sum_{n=1}^{N}\alpha_{rn}^{j}\left(\kappa_{n}^{i}-\bar{\kappa}_{n}\right)\left(r_{t}-\bar{r}\right)+\tau_{j}Y_{t}^{i}+\tilde{v}_{jt}^{i}.\tag{183}$$

As before, suppose that from period $t-1$ to period t the policy rate changes from r_{t-1} to $r_t = r_{t-1}+\varepsilon_t^m$, where ε_t^m represents an unexpected policy shock, and at the same time, for firm i, $\tilde{v}_{jt-1}^{i}$ changes to $\tilde{v}_{jt}^{i}=\tilde{v}_{jt-1}^{i}+\varepsilon_{jit}^{v}$, where ε_{jit}^{v} represents time variation in the transmission variable v_{jt}^{i} that is independent of time variation in the policy rate. Then, (183) implies

$$v_{jt}^{i}-v_{jt-1}^{i}\approx\mathrm{a}_{t}^{j}+\sum_{n=1}^{N}\alpha_{rn}^{j}\left(\kappa_{n}^{i}-\bar{\kappa}_{n}\right)\varepsilon_{t}^{m}+\tau_{j}\left(Y_{t}^{i}-Y_{t-1}^{i}\right)+\varepsilon_{jit}^{v},\tag{184}$$

where $\mathrm{a}_t^j \equiv \{\alpha_r^j+\alpha_{rr}^j\left[\varepsilon_t^m+2\left(r_{t-1}-\bar{r}\right)\right]\}\varepsilon_t^m$ (with $\mathrm{a}_t^1\equiv\mathrm{a}_t^{\mathrm{q}}$), $\alpha_{rr}^j\equiv\frac{1}{2}\frac{\partial v_j(\bar{r},\bar{\boldsymbol{\kappa}})}{\partial r\partial r}$, and $\alpha_{rn}^j\equiv\frac{\partial v_j(\bar{r},\bar{\boldsymbol{\kappa}})}{\partial \kappa_n\partial r}$ for $n\in\{1,...,N\}$. With (167), we can write the change in transmission variable $j\in\{1,...,D\}$, i.e., (184), in terms of the interaction between the money shock and *observed* firm-level characteristics:

$$v_{jt}^{i}-v_{jt-1}^{i}\approx\mathrm{a}_{t}^{j}+\sum_{n=1}^{M}\hat{\alpha}_{rn}^{j}\left(\kappa_{n}^{i}-\bar{\kappa}_{n}\right)\varepsilon_{t}^{m}+\tau_{j}\left(Y_{t}^{i}-Y_{t-1}^{i}\right)+\tilde{\varepsilon}_{jit}^{v},\tag{185}$$

where $\hat{\alpha}_{rn}^j\equiv\alpha_{rn}^j+\sum_{s=M+1}^{N}\alpha_{rs}^j\varkappa_{sn}$ (with $\hat{\alpha}_{rn}^{\mathrm{q}}\equiv\hat{\alpha}_{rn}^1$ and $\hat{\alpha}_{r\mathcal{T}}^j\equiv\hat{\alpha}_{r1}^j$), and $\tilde{\varepsilon}_{jit}^v\equiv\varepsilon_{jit}^v+\sum_{s=M+1}^{N}\alpha_{rs}^j\varepsilon_{is}^{\kappa}\varepsilon_t^m$ (with $\tilde{\varepsilon}_{\mathrm{q}it}^v\equiv\tilde{\varepsilon}_{1it}^v$). Together, (185) and (169) imply

$$Y_{t}^{i}-Y_{t-1}^{i}\approx\mathrm{d}_{t}^{\prime}+\hat{\delta}_{r\mathcal{T}}^{\prime\mathrm{q}}\varepsilon_{it}^{\mathcal{T}m}+\sum_{n=2}^{M}\tilde{\delta}_{rn}^{\prime}\left(\kappa_{n}^{i}-\bar{\kappa}_{n}\right)\varepsilon_{t}^{m}+\bar{\epsilon}_{it}^{\prime},\tag{186}$$

where $\mathrm{d}_t^{\prime}\equiv\sum_{j=1}^{D}\bar{\gamma}^j\mathrm{a}_t^j$, $\varepsilon_{it}^{\mathcal{T}m}\equiv\left(\mathcal{T}^i-\bar{\mathcal{T}}\right)\varepsilon_t^m$, $\bar{\epsilon}_{it}^{\prime}\equiv\tilde{\delta}_{r\mathcal{T}}^{\prime\sim\mathrm{q}}\varepsilon_{it}^{\mathcal{T}m}+\sum_{j=1}^{D}\bar{\gamma}^j\tilde{\varepsilon}_{jit}^v$, $\bar{\gamma}^j\equiv\frac{\gamma^j}{1-\sum_{k=1}^{D}\tau_k\gamma^k}$ for $j\in\{1,...,D\}$ (with $\gamma^j\equiv\frac{\partial Y(\bar{\boldsymbol{v}})}{\partial v_j}$, and $\bar{\gamma}^{\mathrm{q}}\equiv\bar{\gamma}^1$), $\hat{\delta}_{rn}^{\prime\mathrm{q}}\equiv\bar{\gamma}^{\mathrm{q}}\hat{\alpha}_{rn}^{\mathrm{q}}$, $\tilde{\delta}_{rn}^{\prime}\equiv\hat{\delta}_{rn}^{\prime\mathrm{q}}+\tilde{\delta}_{rn}^{\prime\sim\mathrm{q}}$, and $\tilde{\delta}_{rn}^{\prime\sim\mathrm{q}}\equiv\sum_{j=2}^{D}\bar{\gamma}^j\hat{\alpha}_{rn}^j$ for $n\in\{1,...,M\}$ (with $\hat{\delta}_{r\mathcal{T}}^{\prime\mathrm{q}}\equiv\hat{\delta}_{r1}^{\prime\mathrm{q}}$ and $\tilde{\delta}_{r\mathcal{T}}^{\prime\sim\mathrm{q}}\equiv\tilde{\delta}_{r1}^{\prime\sim\mathrm{q}}$). By substituting (186) into (185) (for $j=1$) we obtain

$$\mathrm{q}_{t}^{i}-\mathrm{q}_{t-1}^{i}\approx\mathrm{a}_{t}^{\prime\mathrm{q}}+\hat{\alpha}_{r\mathcal{T}}^{\prime\mathrm{q}}\varepsilon_{it}^{\mathcal{T}m}+\sum_{n=2}^{M}\hat{\alpha}_{rn}^{\prime\mathrm{q}}\left(\kappa_{n}^{i}-\bar{\kappa}_{n}\right)\varepsilon_{t}^{m}+\epsilon_{it}^{\prime\mathrm{q}}\tag{187}$$

where $\mathrm{a}_t'^{\mathrm{q}} \equiv \mathrm{a}_t^{\mathrm{q}} + \tau_{\mathrm{q}} \mathrm{d}_t'$, $\hat{\alpha}_{rn}'^{\mathrm{q}} \equiv \hat{\alpha}_{rn}^{\mathrm{q}} + \tau_{\mathrm{q}} \tilde{\delta}_{rn}'$ for $n \in \{1, ..., M\}$ (with $\hat{\alpha}_{r1}'^{\mathrm{q}} \equiv \hat{\alpha}_{r\mathcal{T}}'^{\mathrm{q}} \equiv \hat{\alpha}_{r\mathcal{T}}^{\mathrm{q}} + \tau_{\mathrm{q}} \hat{\delta}_{r\mathcal{T}}'^{\mathrm{q}}$), and $\epsilon_{it}'^{\mathrm{q}} \equiv \tilde{\varepsilon}_{\mathrm{q}it}^{v} + \tau_{\mathrm{q}} \bar{\epsilon}_{it}'$. Finally, by substituting (187) into (186) we obtain

$$Y_t^i - Y_{t-1}^i \approx \mathrm{b}_t' + \hat{\gamma}_{\mathcal{T}}^{\mathrm{q}} \left(\mathrm{q}_t^i - \mathrm{q}_{t-1}^i\right) + \sum_{n=2}^{M} \Delta_{rn} \left(\kappa_n^i - \bar{\kappa}_n\right) \varepsilon_t^m + \tilde{\epsilon}_{it}', \tag{188}$$

where $\mathrm{b}_t' \equiv \mathrm{d}_t' - \hat{\gamma}_{\mathcal{T}}^{\mathrm{q}} \mathrm{a}_t'^{\mathrm{q}}$, $\Delta_{rn} \equiv \tilde{\delta}_{rn}' - \hat{\gamma}_{\mathcal{T}}^{\mathrm{q}} \hat{\alpha}_{rn}'^{\mathrm{q}}$, and $\tilde{\epsilon}_{it}' \equiv \bar{\epsilon}_{it}' - \hat{\gamma}_{\mathcal{T}}^{\mathrm{q}} \epsilon_{it}'^{\mathrm{q}}$, with

$$\hat{\gamma}_{\mathcal{T}}^{\mathrm{q}} \equiv \frac{\hat{\delta}_{r\mathcal{T}}'^{\mathrm{q}}}{\hat{\alpha}_{r\mathcal{T}}'^{\mathrm{q}}} = \Gamma^{\mathrm{q}} - \frac{\tau_{\mathrm{q}} \bar{\gamma}^{\mathrm{q}}}{1 + \tau_{\mathrm{q}} \bar{\gamma}^{\mathrm{q}}} \frac{\tilde{\delta}_{r\mathcal{T}}'^{\sim \mathrm{q}}}{\hat{\alpha}_{r\mathcal{T}}'^{\mathrm{q}}} \tag{189}$$

and

$$\Gamma^{\mathrm{q}} \equiv \frac{\bar{\gamma}^{\mathrm{q}}}{1 + \tau_{\mathrm{q}} \bar{\gamma}^{\mathrm{q}}} = \frac{\gamma^{\mathrm{q}}}{1 - \sum_{k=2}^{D} \tau_j \gamma^j}. \tag{190}$$

The representation (188) can be thought of as the basis for a "structural form" that regresses the (change in the) outcome variable on the change in Tobin's q and some controls (i.e., a time dummy, b_t', and interactions of the shock with observed firm-level characteristics, $\left\{\left(\kappa_n^i - \bar{\kappa}_n\right) \varepsilon_t^m\right\}_{n=2}^M$). Together, the representations (186), (187), and (188) suggest an identification strategy that uses $\varepsilon_{it}^{\mathcal{T}m}$ as an instrument for $\mathrm{q}_t^i - \mathrm{q}_{t-1}^i$: Think of (186) as a "reduced form" that regresses the change in the outcome variable directly on the instrument and other controls (a time dummy, d_t', and interactions of the money shock with a vector of the other $M-1$ observed firm-level characteristics, $\left\{\left(\kappa_n^i - \bar{\kappa}_n\right) \varepsilon_t^m\right\}_{n=2}^M$); think of (187) as the "first stage" that projects $\mathrm{q}_t^i - \mathrm{q}_{t-1}^i$ onto the instrument $\varepsilon_{it}^{\mathcal{T}m}$ and other controls (a time dummy, $\mathrm{a}_t'^{\mathrm{q}}$, and interactions of the money shock with a vector of the other $M-1$ observed firm-level characteristics); and think of (188) as the "structural form" estimated with the first-stage projection replacing $\mathrm{q}_t^i - \mathrm{q}_{t-1}^i$. Two conditions need to be satisfied for $\varepsilon_{it}^{\mathcal{T}m}$ to be a valid instrument for $\mathrm{q}_t^i - \mathrm{q}_{t-1}^i$ in order to estimate the q-channel by using (188) as the basis for an IV regression. First, $\varepsilon_{it}^{\mathcal{T}m}$ must be correlated with the change in firm i's stock price, $\mathrm{q}_t^i - \mathrm{q}_{t-1}^i$. This correlation is negative and strong (it is the *turnover-liquidity mechanism* documented by Lagos and Zhang (2020b)). Second, $\varepsilon_{it}^{\mathcal{T}m}$ must affect the outcome variable, Y_t^i, in the structural form (188) only through the transmission variable $\mathrm{q}_t^i - \mathrm{q}_{t-1}^i$, i.e., the instrument $\varepsilon_{it}^{\mathcal{T}m}$ must be uncorrelated with $\tilde{\epsilon}_{it}'$.

In Corollary 7 (Section C.2, in Appendix C) we show that, under our maintained assumptions (namely ε_t^m independent of ε_{jit}^v for $j \in \{1, ..., D\}$, independent of ε_{is}^κ for $s \in \{M+1, ..., N\}$, and $\mathbb{E}\left(\varepsilon_{is}^\kappa\right) = 0$ for $s \in \{M+1, ..., N\}$) imply $cov\left(\varepsilon_{it}^{\mathcal{T}m}, \tilde{\epsilon}_{it}'\right) = \tilde{\delta}_{r\mathcal{T}}'^{\sim \mathrm{q}} \left(1 - \tau_{\mathrm{q}} \hat{\gamma}_{\mathcal{T}}^{\mathrm{q}}\right) var\left(\varepsilon_{it}^{\mathcal{T}m}\right)$, so the *exclusion restriction* for $\varepsilon_{it}^{\mathcal{T}m}$ to be a valid instrument for $\mathrm{q}_t^i - \mathrm{q}_{t-1}^i$ is satisfied if and only if

$\tilde{\delta}_{r\mathcal{T}}^{\prime\sim\mathrm{q}} = 0$.[78] Under this condition specification (188) delivers an unbiased estimate of Γ^{q}, which is the generalization of the coefficient γ^{q} estimated from specification (20). The difference is that the coefficient Γ^{q} now also encodes the *feedback effects* from *other* transmission variables that are triggered by the effect of the *turnover-q* channel on $Y_t^i \in \{\mathrm{x}_t^i, \mathrm{e}_t^i\}$. Specifically, in this case our estimate of the q-channel, i.e., the coefficent Γ^{q}, includes not only the direct ("first-round") effect of the *turnover-q* channel on Y_t^i, i.e., γ^{q}, but also the indirect ("second-round") effects on Y_t^i associated with the variation in other transmission variables caused by the feedback to those variables of the direct change in the outcome variable Y_t^i originally triggered by the *turnover-q* effect. The indirect "second-round" effects due to the feedback of the outcome variable to other transmission variables are captured by the factor $1 - \sum_{k=2}^{D} \tau_j \gamma^j$ in the denominator of (190).

In Corollary 7 (Section C.2, in Appendix C) we show that under our maintained assumptions, we have: (i) $cov\left(\varepsilon_{it}^{\mathcal{T}m}, \bar{\epsilon}_{it}'\right) = \left(1 - \tau_{\mathrm{q}}\hat{\gamma}_{\mathcal{T}}^{\mathrm{q}}\right) cov\left(\varepsilon_{it}^{\mathcal{T}m}, \bar{\epsilon}_{it}'\right)$, so if the exclusion restriction $\tilde{\delta}_{r\mathcal{T}}^{\prime\sim\mathrm{q}} = 0$ holds, (186) will deliver an unbiased OLS estimate of $\hat{\delta}_{r\mathcal{T}}^{\prime\mathrm{q}}$; and (ii) $cov(\varepsilon_{it}^{\mathcal{T}m}, \epsilon_{it}^{\prime\mathrm{q}}) = \tau_{\mathrm{q}}\tilde{\delta}_{r\mathcal{T}}^{\prime\sim\mathrm{q}} var\left(\varepsilon_{it}^{\mathcal{T}m}\right)$, so if the exclusion restriction $\tilde{\delta}_{r\mathcal{T}}^{\prime\sim\mathrm{q}} = 0$ holds, (187) will deliver an unbiased OLS estimate of $\hat{\alpha}_{r\mathcal{T}}^{\prime\mathrm{q}}$. Hence, if the exclusion restriction $\tilde{\delta}_{r\mathcal{T}}^{\prime\sim\mathrm{q}} = 0$ holds, then we see from (189) that $\Gamma^{\mathrm{q}} = \hat{\delta}_{r\mathcal{T}}^{\prime\mathrm{q}} / \hat{\alpha}_{r\mathcal{T}}^{\prime\mathrm{q}}$. That is, the coefficient of interest, Γ^{q}, can be obtained as the ratio of the OLS estimate of the effect of the instrument on the outcome variable in the reduced form, (186), and the OLS estimate of the effect of the instrument on Tobin's q in the first stage, (175).

C.2 Proofs of identification results

Lemma 10 *Consider a firm i, and suppose that $cov(\varepsilon_t^m, \varepsilon_{jit}^v) = 0$ for all $j \in \{1, ..., D\}$. Then, formulation (171) implies*

$$cov\left(\mathrm{q}_t^i - \mathrm{q}_{t-1}^i, \tilde{\mathrm{u}}_t^i\right) = \alpha_r^q \bar{\delta}_r^{\sim\mathrm{q}} var\left(\varepsilon_t^m\right) + \sum_{j=2}^{D} \gamma^j cov(\varepsilon_{1it}^v, \varepsilon_{jit}^v) \tag{191}$$

and

$$cov\left(\varepsilon_t^m, \mathrm{u}_t^i\right) = \bar{\delta}_r^{\sim\mathrm{q}} var\left(\varepsilon_t^m\right), \tag{192}$$

where $\bar{\delta}_r^{\sim\mathrm{q}} \equiv \sum_{j=2}^{D} \delta_r^j$.

[78]The identifying restriction, $\tilde{\delta}_{r\mathcal{T}}^{\prime\sim\mathrm{q}} = 0$, can be written explicitly as

$$\frac{1}{1 - \sum_{k=1}^{D} \tau_k \gamma^k} \sum_{j=2}^{D} \gamma^j \left(\alpha_{r\mathcal{T}}^j + \sum_{s=M+1}^{N} \alpha_{rs}^j \varkappa_s \mathcal{T} \right) = 0,$$

which is equivalent to (21) in terms of the restrictions it imposes on the "theory" $\left\{\gamma^j, \alpha_{r\mathcal{T}}^j, \left\{\alpha_{rs}^j \varkappa_s \mathcal{T}\right\}_{s=M+1}^{N}\right\}_{j=2}^{D}$.

Proof. From (170) we have

$$\mathrm{q}_t^i - \mathrm{q}_{t-1}^i \approx \alpha_r^{\mathrm{q}} \varepsilon_t^m + \varepsilon_{1it}^v,$$

and $\tilde{\mathrm{u}}_t^i \equiv \sum_{j=2}^D \gamma^j (v_{jt}^i - v_{jt-1}^i) = \sum_{j=2}^D (\delta_r^j \varepsilon_t^m + \gamma^j \varepsilon_{jit}^v)$, so

$$\begin{aligned} cov\left(\mathrm{q}_t^i - \mathrm{q}_{t-1}^i, \tilde{\mathrm{u}}_t^i\right) &= cov\left(\alpha_r^{\mathrm{q}} \varepsilon_t^m + \varepsilon_{1it}^v, \sum_{j=2}^D (\delta_r^j \varepsilon_t^m + \gamma^j \varepsilon_{jit}^v)\right) \\ &= \alpha_r^{\mathrm{q}} \sum_{j=2}^D \delta_r^j var\left(\varepsilon_t^m\right) + \alpha_r^{\mathrm{q}} \sum_{j=2}^D \gamma^j cov(\varepsilon_{jit}^v, \varepsilon_t^m) \\ &\quad + \sum_{j=2}^D \delta_r^j cov\left(\varepsilon_{1it}^v, \varepsilon_t^m\right) + \sum_{j=2}^D \gamma^j cov(\varepsilon_{1it}^v, \varepsilon_{jit}^v). \end{aligned}$$

The assumption that $cov(\varepsilon_t^m, \varepsilon_{jit}^v) = 0$ for all $j \in \{1, ..., D\}$ implies

$$cov\left(\varepsilon_{1it}^v, \varepsilon_t^m\right) = \sum_{j=2}^D \gamma^j cov(\varepsilon_{jit}^v, \varepsilon_t^m) = 0.$$

This establishes (191). To obtain (192), notice that

$$\begin{aligned} cov\left(\varepsilon_t^m, \mathrm{u}_t^i\right) &= cov\left(\varepsilon_t^m, \tilde{\mathrm{u}}_t^i + \gamma^{\mathrm{q}} \varepsilon_{1it}^v\right) \\ &= cov\left(\varepsilon_t^m, \sum_{j=2}^D (\delta_r^j \varepsilon_t^m + \gamma^j \varepsilon_{jit}^v) + \gamma^{\mathrm{q}} \varepsilon_{1it}^v\right) \\ &= var\left(\varepsilon_t^m\right) \sum_{j=2}^D \delta_r^j + \sum_{j=1}^D \gamma^j cov\left(\varepsilon_{jit}^v, \varepsilon_t^m\right) \\ &= var\left(\varepsilon_t^m\right) \sum_{j=2}^D \delta_r^j. \end{aligned}$$

The last equality follows from the assumption that $cov(\varepsilon_t^m, \varepsilon_{jit}^v) = 0$ for all $j \in \{1, ..., D\}$. ■

Lemma 11 *Suppose that: (i) $cov(\varepsilon_t^m, \varepsilon_{jit}^v) = 0$ for each $j \in \{1, ..., D\}$, (ii) ε_t^m is independent of ε_{is}^κ for each $s \in \{M+1, ..., N\}$, and (iii) $\mathbb{E}\left(\varepsilon_{is}^\kappa\right) = 0$ for each $s \in \{M+1, ..., N\}$. Then,*

$$cov\left(\varepsilon_{it}^{\mathcal{T}m}, \tilde{\epsilon}_{it}\right) = \tilde{\delta}_{r\mathcal{T}}^{\sim\mathrm{q}} var\left(\varepsilon_{it}^{\mathcal{T}m}\right). \tag{193}$$

Proof. Since $\tilde{\epsilon}_{it} \equiv \tilde{\delta}_{r\mathcal{T}}^{\sim\mathrm{q}} \varepsilon_{it}^{\mathcal{T}m} + \hat{\epsilon}_{it}$,

$$cov\left(\varepsilon_{it}^{\mathcal{T}m}, \tilde{\epsilon}_{it}\right) = \tilde{\delta}_{r\mathcal{T}}^{\sim\mathrm{q}} var\left(\varepsilon_{it}^{\mathcal{T}m}\right) + cov\left(\varepsilon_{it}^{\mathcal{T}m}, \hat{\epsilon}_{it}\right). \tag{194}$$

Also, since $\hat{\epsilon}_{it} \equiv \sum_{j=2}^{D} \gamma^j \varepsilon_{jit}^v + \sum_{j=2}^{D} \sum_{s=M+1}^{N} \delta_{rs}^j \varepsilon_{is}^\kappa \varepsilon_t^m$,

$$\begin{aligned} cov\left(\varepsilon_{it}^{\mathcal{T}m}, \hat{\epsilon}_{it}\right) &= cov\left(\varepsilon_{it}^{\mathcal{T}m}, \sum_{j=2}^{D} \gamma^j \varepsilon_{jit}^v\right) + cov\left(\varepsilon_{it}^{\mathcal{T}m}, \sum_{j=2}^{D} \sum_{s=M+1}^{N} \delta_{rs}^j \varepsilon_{is}^\kappa \varepsilon_t^m\right) \\ &= \sum_{j=2}^{D} \gamma^j cov\left(\varepsilon_{it}^{\mathcal{T}m}, \varepsilon_{jit}^v\right) + \sum_{s=M+1}^{N} \bar{\delta}_{rs}^{\sim q} cov\left(\varepsilon_{it}^{\mathcal{T}m}, \varepsilon_{is}^\kappa \varepsilon_t^m\right), \end{aligned} \tag{195}$$

with $\bar{\delta}_{rs}^{\sim q} \equiv \sum_{j=2}^{D} \delta_{rs}^j$. Since $\varepsilon_{it}^{\mathcal{T}m} \equiv (\mathcal{T}^i - \bar{\mathcal{T}})\varepsilon_t^m$,

$$\begin{aligned} cov\left(\varepsilon_{it}^{\mathcal{T}m}, \varepsilon_{jit}^v\right) &= (\mathcal{T}^i - \bar{\mathcal{T}}) cov\left(\varepsilon_t^m, \varepsilon_{jit}^v\right) \\ &= 0 \text{ for all } j \in \{1, ..., D\}. \end{aligned} \tag{196}$$

(The second equality in (196) follows from assumption (i) in the statement of the proposition.) Also,

$$\begin{aligned} cov\left(\varepsilon_{it}^{\mathcal{T}m}, \varepsilon_{is}^\kappa \varepsilon_t^m\right) &= (\mathcal{T}^i - \bar{\mathcal{T}}) \left\{\mathbb{E}\left[\varepsilon_{is}^\kappa \left(\varepsilon_t^m\right)^2\right] - \mathbb{E}\left(\varepsilon_t^m\right) \mathbb{E}\left(\varepsilon_{is}^\kappa \varepsilon_t^m\right)\right\} \\ &= (\mathcal{T}^i - \bar{\mathcal{T}}) \mathbb{E}\left(\varepsilon_{is}^\kappa\right) var\left(\varepsilon_t^m\right) \\ &= 0 \text{ for all } s \in \{M+1, ..., N\}. \end{aligned} \tag{197}$$

(The second equality in (197) follows from assumption (ii), and the third equality from assumption (iii) in the statement of the proposition.) Conditions (195), (196), and (197) imply

$$cov\left(\varepsilon_{it}^{\mathcal{T}m}, \hat{\epsilon}_{it}\right) = 0,$$

and therefore (193) follows from (194). ■

Corollary 6 *Under the assumptions of Lemma 11: (i)* $cov\left(\varepsilon_{it}^{\mathcal{T}m}, \bar{\epsilon}_{it}\right) = \tilde{\delta}_{r\mathcal{T}}^{\sim q} var\left(\varepsilon_{it}^{\mathcal{T}m}\right)$*, and (ii)* $cov\left(\varepsilon_{it}^{\mathcal{T}m}, \epsilon_{it}^{q}\right) = 0$.

Proof. (i) Recall that $\bar{\epsilon}_{it} \equiv \tilde{\delta}_{r\mathcal{T}}^{\sim q} \varepsilon_{it}^{\mathcal{T}m} + \bar{\varepsilon}_{it} + \sum_{j=1}^{D} \sum_{s=M+1}^{N} \delta_{rs}^j \varepsilon_{is}^\kappa \varepsilon_t^m$ and $\bar{\varepsilon}_{it} \equiv \sum_{j=1}^{D} \gamma^j \varepsilon_{jit}^v$, so

$$\begin{aligned} cov\left(\varepsilon_{it}^{\mathcal{T}m}, \bar{\epsilon}_{it}\right) &= cov\left(\varepsilon_{it}^{\mathcal{T}m}, \tilde{\delta}_{r\mathcal{T}}^{\sim q} \varepsilon_{it}^{\mathcal{T}m} + \bar{\varepsilon}_{it} + \sum_{j=1}^{D} \sum_{s=M+1}^{N} \delta_{rs}^j \varepsilon_{is}^\kappa \varepsilon_t^m\right) \\ &= \tilde{\delta}_{r\mathcal{T}}^{\sim q} var\left(\varepsilon_{it}^{\mathcal{T}m}\right) + \sum_{j=1}^{D} \gamma^j cov\left(\varepsilon_{it}^{\mathcal{T}m}, \varepsilon_{jit}^v\right) + \sum_{s=M+1}^{N} \left(\sum_{j=1}^{D} \delta_{rs}^j\right) cov\left(\varepsilon_{it}^{\mathcal{T}m}, \varepsilon_{is}^\kappa \varepsilon_t^m\right). \end{aligned}$$

The result follows from (196) and (197).

(*ii*) Since $\epsilon_{it}^{\mathrm{q}} \equiv \varepsilon_{it}^{\mathrm{q}} + \sum_{s=M+1}^{N} \alpha_{rs}^{\mathrm{q}} \varepsilon_{is}^{\kappa} \varepsilon_{t}^{m}$ and $\varepsilon_{it}^{\mathrm{q}} \equiv \varepsilon_{1it}^{v}$,

$$\begin{aligned} cov\left(\varepsilon_{it}^{\mathcal{T}m}, \epsilon_{it}^{\mathrm{q}}\right) &= cov\left(\varepsilon_{it}^{\mathcal{T}m}, \varepsilon_{1it}^{v} + \sum_{s=M+1}^{N} \alpha_{rs}^{\mathrm{q}} \varepsilon_{is}^{\kappa} \varepsilon_{t}^{m}\right) \\ &= cov\left(\varepsilon_{it}^{\mathcal{T}m}, \varepsilon_{1it}^{v}\right) + \sum_{s=M+1}^{N} \alpha_{rs}^{\mathrm{q}} cov\left(\varepsilon_{it}^{\mathcal{T}m}, \varepsilon_{is}^{\kappa} \varepsilon_{t}^{m}\right). \end{aligned}$$

The result follows from (196) and (197). ■

Corollary 7 *Under the assumptions of Lemma 11: (i)* $cov\left(\varepsilon_{it}^{\mathcal{T}m}, \tilde{\epsilon}_{it}'\right) = \left(1 - \tau_{\mathrm{q}} \hat{\gamma}_{\mathcal{T}}^{\mathrm{q}}\right) cov\left(\varepsilon_{it}^{\mathcal{T}m}, \vec{\epsilon}_{it}'\right) = \left(1 - \tau_{\mathrm{q}} \hat{\gamma}_{\mathcal{T}}^{\mathrm{q}}\right) \tilde{\delta}_{r\mathcal{T}}^{\prime\sim\mathrm{q}} var\left(\varepsilon_{it}^{\mathcal{T}m}\right)$, *and (ii)* $cov(\varepsilon_{it}^{\mathcal{T}m}, \epsilon_{it}^{\prime\mathrm{q}}) = \tau_{\mathrm{q}} \tilde{\delta}_{r\mathcal{T}}^{\prime\sim\mathrm{q}} var\left(\varepsilon_{it}^{\mathcal{T}m}\right)$.

Proof. (*i*) Recall that $\tilde{\epsilon}_{it}' \equiv \vec{\epsilon}_{it}' - \hat{\gamma}_{\mathcal{T}}^{\mathrm{q}} \epsilon_{it}^{\prime\mathrm{q}}$, $\epsilon_{it}^{\prime\mathrm{q}} \equiv \tilde{\varepsilon}_{\mathrm{q}it}^{v} + \tau_{\mathrm{q}} \vec{\epsilon}_{it}'$, $\vec{\epsilon}_{it}' \equiv \tilde{\delta}_{r\mathcal{T}}^{\prime\sim\mathrm{q}} \varepsilon_{it}^{\mathcal{T}m} + \sum_{j=1}^{D} \bar{\gamma}^{j} \tilde{\varepsilon}_{jit}^{v}$, and $\tilde{\varepsilon}_{jit}^{v} \equiv \varepsilon_{jit}^{v} + \sum_{s=M+1}^{N} \alpha_{rs}^{j} \varepsilon_{is}^{\kappa} \varepsilon_{t}^{m}$, so

$$\begin{aligned} cov\left(\varepsilon_{it}^{\mathcal{T}m}, \tilde{\epsilon}_{it}'\right) &= cov\left(\varepsilon_{it}^{\mathcal{T}m}, \left(1 - \tau_{\mathrm{q}} \hat{\gamma}_{\mathcal{T}}^{\mathrm{q}}\right) \vec{\epsilon}_{it}' - \hat{\gamma}_{\mathcal{T}}^{\mathrm{q}} \tilde{\varepsilon}_{\mathrm{q}it}^{v}\right) \\ &= \left(1 - \tau_{\mathrm{q}} \hat{\gamma}_{\mathcal{T}}^{\mathrm{q}}\right) cov\left(\varepsilon_{it}^{\mathcal{T}m}, \vec{\epsilon}_{it}'\right) - \hat{\gamma}_{\mathcal{T}}^{\mathrm{q}} cov\left(\varepsilon_{it}^{\mathcal{T}m}, \tilde{\varepsilon}_{\mathrm{q}it}^{v}\right) \\ &= \left(1 - \tau_{\mathrm{q}} \hat{\gamma}_{\mathcal{T}}^{\mathrm{q}}\right) cov\left(\varepsilon_{it}^{\mathcal{T}m}, \tilde{\delta}_{r\mathcal{T}}^{\prime\sim\mathrm{q}} \varepsilon_{it}^{\mathcal{T}m} + \sum_{j=1}^{D} \bar{\gamma}^{j} \tilde{\varepsilon}_{jit}^{v}\right) - \hat{\gamma}_{\mathcal{T}}^{\mathrm{q}} cov\left(\varepsilon_{it}^{\mathcal{T}m}, \tilde{\varepsilon}_{\mathrm{q}it}^{v}\right) \\ &= \left(1 - \tau_{\mathrm{q}} \hat{\gamma}_{\mathcal{T}}^{\mathrm{q}}\right) \tilde{\delta}_{r\mathcal{T}}^{\prime\sim\mathrm{q}} var\left(\varepsilon_{it}^{\mathcal{T}m}\right) + \left(1 - \tau_{\mathrm{q}} \hat{\gamma}_{\mathcal{T}}^{\mathrm{q}}\right) \sum_{j=1}^{D} \bar{\gamma}^{j} cov\left(\varepsilon_{it}^{\mathcal{T}m}, \tilde{\varepsilon}_{jit}^{v}\right) - \hat{\gamma}_{\mathcal{T}}^{\mathrm{q}} cov\left(\varepsilon_{it}^{\mathcal{T}m}, \tilde{\varepsilon}_{\mathrm{q}it}^{v}\right) \\ &= \tilde{\delta}_{r\mathcal{T}}^{\prime\sim\mathrm{q}} \left(1 - \tau_{\mathrm{q}} \hat{\gamma}_{\mathcal{T}}^{\mathrm{q}}\right) var\left(\varepsilon_{it}^{\mathcal{T}m}\right) \\ &\quad + \left(1 - \tau_{\mathrm{q}} \hat{\gamma}_{\mathcal{T}}^{\mathrm{q}}\right) \left[\sum_{j=1}^{D} \bar{\gamma}^{j} cov\left(\varepsilon_{it}^{\mathcal{T}m}, \varepsilon_{jit}^{v}\right) + \sum_{s=M+1}^{N} \left(\sum_{j=1}^{D} \bar{\gamma}^{j} \alpha_{rs}^{j}\right) cov\left(\varepsilon_{it}^{\mathcal{T}m}, \varepsilon_{is}^{\kappa} \varepsilon_{t}^{m}\right)\right] \\ &\quad - \hat{\gamma}_{\mathcal{T}}^{\mathrm{q}} \left[cov\left(\varepsilon_{it}^{\mathcal{T}m}, \varepsilon_{1it}^{v}\right) + \sum_{s=M+1}^{N} \alpha_{rs}^{\mathrm{q}} cov\left(\varepsilon_{it}^{\mathcal{T}m}, \varepsilon_{is}^{\kappa} \varepsilon_{t}^{m}\right)\right]. \end{aligned}$$

The result $cov\left(\varepsilon_{it}^{\mathcal{T}m}, \tilde{\epsilon}_{it}'\right) = \tilde{\delta}_{r\mathcal{T}}^{\prime\sim\mathrm{q}} \left(1 - \tau_{\mathrm{q}} \hat{\gamma}_{\mathcal{T}}^{\mathrm{q}}\right) var\left(\varepsilon_{it}^{\mathcal{T}m}\right)$ follows from (196) and (197). Also, notice that from the second equality in the above derivation, we have

$$\begin{aligned} cov\left(\varepsilon_{it}^{\mathcal{T}m}, \vec{\epsilon}_{it}'\right) &= \frac{1}{1 - \tau_{\mathrm{q}} \hat{\gamma}_{\mathcal{T}}^{\mathrm{q}}} \left[cov\left(\varepsilon_{it}^{\mathcal{T}m}, \tilde{\epsilon}_{it}'\right) + \hat{\gamma}_{\mathcal{T}}^{\mathrm{q}} cov\left(\varepsilon_{it}^{\mathcal{T}m}, \tilde{\varepsilon}_{\mathrm{q}it}^{v}\right)\right] \\ &= \tilde{\delta}_{r\mathcal{T}}^{\prime\sim\mathrm{q}} var\left(\varepsilon_{it}^{\mathcal{T}m}\right) + \frac{1}{1 - \tau_{\mathrm{q}} \hat{\gamma}_{\mathcal{T}}^{\mathrm{q}}} \hat{\gamma}_{\mathcal{T}}^{\mathrm{q}} cov\left(\varepsilon_{it}^{\mathcal{T}m}, \tilde{\varepsilon}_{\mathrm{q}it}^{v}\right) \\ &= \tilde{\delta}_{r\mathcal{T}}^{\prime\sim\mathrm{q}} var\left(\varepsilon_{it}^{\mathcal{T}m}\right) + \frac{1}{1 - \tau_{\mathrm{q}} \hat{\gamma}_{\mathcal{T}}^{\mathrm{q}}} \hat{\gamma}_{\mathcal{T}}^{\mathrm{q}} \left[cov\left(\varepsilon_{it}^{\mathcal{T}m}, \varepsilon_{1it}^{v}\right) + \sum_{s=M+1}^{N} \alpha_{rs}^{\mathrm{q}} cov\left(\varepsilon_{it}^{\mathcal{T}m}, \varepsilon_{is}^{\kappa} \varepsilon_{t}^{m}\right)\right]. \end{aligned}$$

The result $cov\left(\varepsilon_{it}^{\mathcal{T}m}, \vec{\epsilon}_{it}'\right) = \tilde{\delta}_{r\mathcal{T}}^{\prime\sim q} var\left(\varepsilon_{it}^{\mathcal{T}m}\right)$ follows from (196) and (197).

(ii) Since $\tilde{\epsilon}_{it}' \equiv \vec{\epsilon}_{it}' - \hat{\gamma}_{\mathcal{T}}^{q}\epsilon_{it}^{\prime q}$, we have $\epsilon_{it}^{\prime q} \equiv \frac{1}{\hat{\gamma}_{\mathcal{T}}^{q}}\left(\vec{\epsilon}_{it}' - \tilde{\epsilon}_{it}'\right)$, so

$$
\begin{aligned}
cov(\varepsilon_{it}^{\mathcal{T}m}, \epsilon_{it}^{\prime q}) &= cov\left[\varepsilon_{it}^{\mathcal{T}m}, \frac{1}{\hat{\gamma}_{\mathcal{T}}^{q}}\left(\vec{\epsilon}_{it}' - \tilde{\epsilon}_{it}'\right)\right] \\
&= \frac{1}{\hat{\gamma}_{\mathcal{T}}^{q}}\left[cov\left(\varepsilon_{it}^{\mathcal{T}m}, \vec{\epsilon}_{it}'\right) - cov\left(\varepsilon_{it}^{\mathcal{T}m}, \tilde{\epsilon}_{it}'\right)\right] \\
&= \tilde{\delta}_{r\mathcal{T}}^{\prime\sim q}\tau_{q} var\left(\varepsilon_{it}^{\mathcal{T}m}\right),
\end{aligned}
$$

where the last equality follows from part (i). ■

D Robustness of empirical findings

In this section we assess the robustness of the empirical findings reported in Section 4.

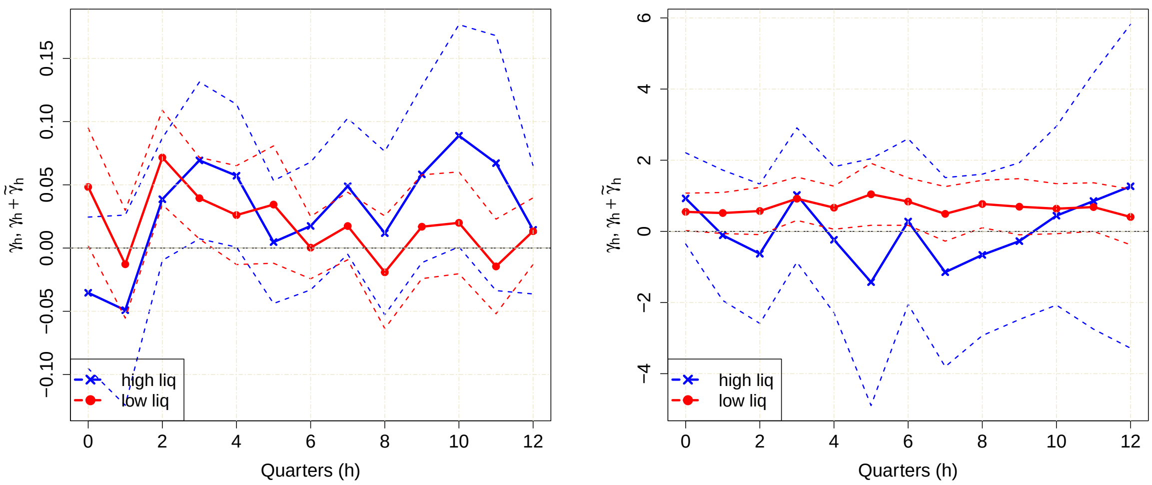
Figure 8 reports the results from including additional firm-level controls (measures of size, leverage, and liquidity ratios) interacted with the monetary shock in specification (25). The results from this specification indicate that the predicted heterogeneity in issuance and investment responsiveness across high- and low-liquidity firms is not explained by other firm-level covariates. By comparing the impulse responses in Figures 4 and 8, we verify that the point estimates are in large part unchanged, so our main results are robust to introducing these controls.

Figure 9 reports the results from including firm age as an additional control. Because of worse coverage of the age variable, we lose almost a fifth of the firm-quarter observations from the full sample behind the results in Figure 4. Nevertheless, the main finding is robust: An increase in firms' Tobin's q (instrumented with the interaction between stock turnover and monetary policy shocks) leads to significantly higher equity issuance and investment among low-liquidity firms, and this finding does not appear to be explained by heterogeneous responsiveness to monetary shocks accounted for by the other firm-level covariates.

Figure 10 reports the main OLS and IV coefficient estimates using an alternative series for the money shock, ε_t^m, identified based on the "poor man's sign restrictions"proposed by Jarociński and Karadi (2020).[79] Again, our main findings are robust—this time to purging potential informational policy-announcement effects from the monetary shock series.

[79]We focus on the "poor man's sign restrictions"series by Jarociński and Karadi (2020) since their benchmark identification approach relies on (set-)identification with a linear model which can lead to further imprecisions during the financial crisis and zero lower bound periods after 2008 during which nonlinear dynamics most likely played a central role in the economy.

Figure 8: Dynamic responses of equity issuance and investment rate to instrumented changes in Tobin's q (conditional on liquidity ratio, with additional firm controls)



(a) $y_t^i = \mathrm{e}_t^i$ (b) $y_t^i = \mathrm{x}_t^i$

Notes: Point estimates and 95% confidence intervals for γ_h and $\gamma_h + \tilde{\gamma}_h$ from specification

$$y_{t+h}^i = f_h^i + d_{s,h,t+h} + \alpha_h \mathbb{I}_{L,t}^i + \left(\rho_h + \tilde{\rho}_h \mathbb{I}_{L,t-1}^i\right) y_{t-1}^i + \left(\Lambda_h + \tilde{\Lambda}_h \mathbb{I}_{L,t-1}^i\right) Z_{t-1}^i + \left(\Psi_h + \tilde{\Psi}_h \mathbb{I}_{L,t-1}^i\right) Z_{t-1}^i \varepsilon_t^m + \left(\beta_h + \tilde{\beta}_h \mathbb{I}_{L,t-1}^i\right) \mathcal{T}_{t-1}^i + \left(\gamma_h + \tilde{\gamma}_h \mathbb{I}_{L,t-1}^i\right) \mathrm{q}_t^i + u_{h,t+h}^i,$$

where Z_t^i is a vector containing the firm's liquidity ratio, log total assets as a measure of firm size, and $\frac{\text{total debt}_t^i}{\text{total assets}_t^i}$ as a measure of leverage. The measure of Tobin's q, q_t^i, is instrumented with $\mathcal{T}_{t-1}^i \varepsilon_t^m$. Confidence intervals constructed based on two-way clustered standard errors at firm and SIC 3-digit industry-quarter levels.

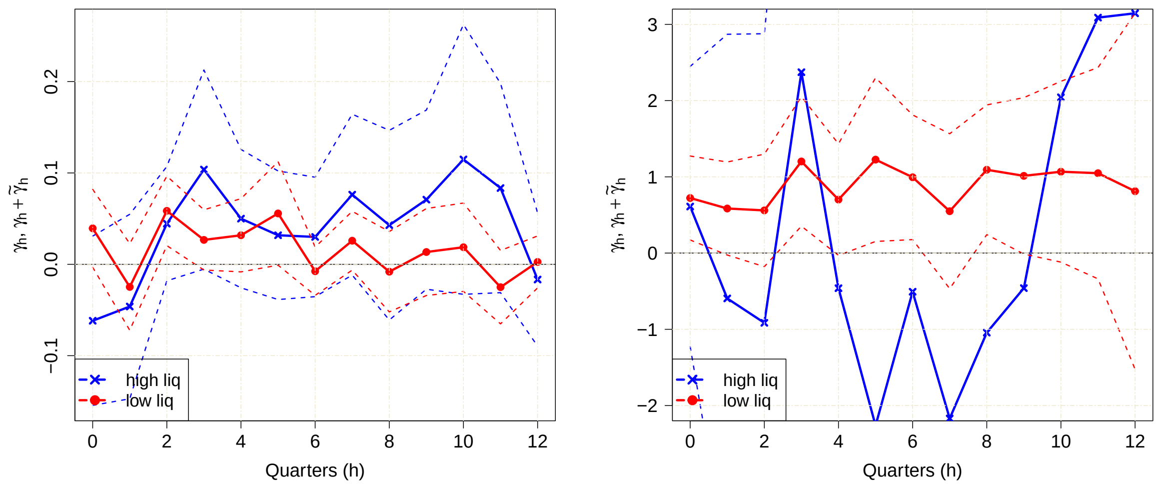
Figure 11 shows that our main findings are also robust to alternative variable transformations, such as using q_t^i instead of $\mathrm{q}_t^i \equiv \log\left(q_t^i\right)$ as the measure of Tobin's q, or the investment rate without taking logs and employing capital expenditures net of sales of property, plant, and equipment.

E Data

E.1 Stock turnover from CRSP

We use daily data from the CRSP US Stock Database, to construct the *Daily Turnover*, $DTOVER_{t_d}^s$, for security s on day t_d as the ratio of daily *Volume Traded* (CRSP data item

Figure 9: Dynamic responses of equity issuance and investment rate to instrumented changes in Tobin's q (conditional on liquidity ratio, with additional firm controls including age)



(a) $y_t^i = \mathrm{e}_t^i$ (b) $y_t^i = \mathrm{x}_t^i$

Notes: Point estimates and 95% confidence intervals for γ_h and $\gamma_h + \tilde{\gamma}_h$ from specification

$$y_{t+h}^i = f_h^i + d_{s,h,t+h} + \alpha_h \mathbb{I}_{L,t}^i + \left(\rho_h + \tilde{\rho}_h \mathbb{I}_{L,t-1}^i\right) y_{t-1}^i + \left(\Lambda_h + \tilde{\Lambda}_h \mathbb{I}_{L,t-1}^i\right) Z_{t-1}^i + \left(\Psi_h + \tilde{\Psi}_h \mathbb{I}_{L,t-1}^i\right) Z_{t-1}^i \varepsilon_t^m$$
$$+ \left(\beta_h + \tilde{\beta}_h \mathbb{I}_{L,t-1}^i\right) \mathcal{T}_{t-1}^i + \left(\gamma_h + \tilde{\gamma}_h \mathbb{I}_{L,t-1}^i\right) \mathrm{q}_t^i + u_{h,t+h}^i,$$

where Z_t^i is a vector containing the firm's liquidity ratio, log total assets as a measure of firm size, $\frac{\text{total debt}_t^i}{\text{total assets}_t^i}$ as a measure of leverage, and time since incorporation as a measure of age. The measure of Tobin's q, q_t^i, is instrumented with $\mathcal{T}_{t-1}^i \varepsilon_t^m$. Confidence intervals constructed based on two-way clustered standard errors at firm and SIC 3-digit industry-quarter levels.

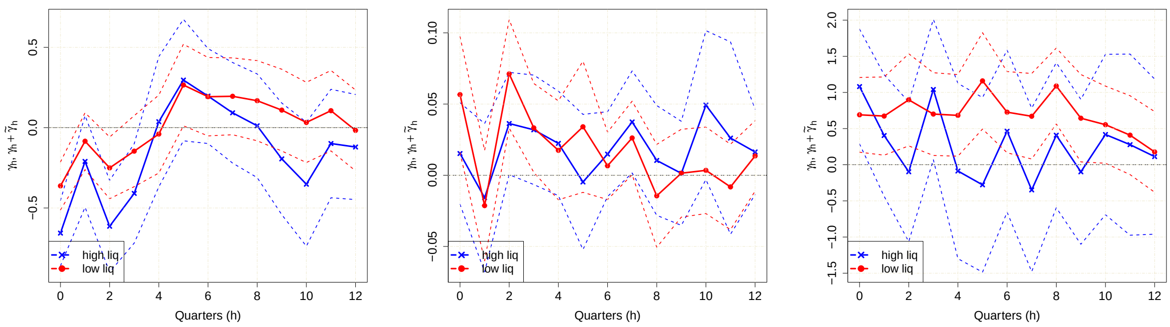
$VOL_{t_d}^s$) relative to Shares Outstanding, $SHROUT_{t_d}^s$ (in thousands), i.e.,

$$DTOVER_{t_d}^s = \frac{VOL_{t_d}^s}{1000 \times SHROUT_{t_d}^s}.$$

We aggregate the *Daily Turnover* series into *Quarterly Turnover*, $TOVER_t^s$, for security s, quarter t by taking the mean of *Daily Turnover* over the corresponding calendar quarter.

We then link the quarterly stock turnover data to the quarterly Compustat firm database using the CCM Link Table provided by WRDS, dropping all securities that are not marked as *Primary Security* in Compustat ($LINKPRIM$ not equal to P or C in CCM Link Table).

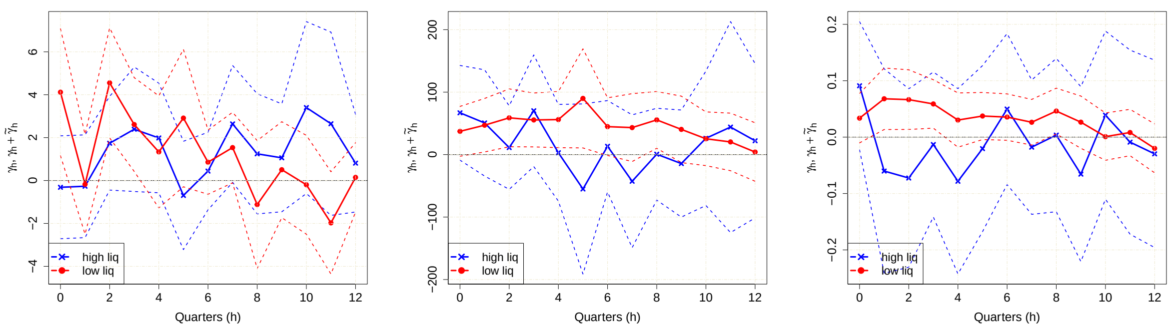
Figure 10: OLS and IV regression estimates (conditional on liquidity ratio), using Jarociński and Karadi (2020) “poor man’s sign restrictions”



(a) $y_t^i = \mathrm{q}_t^i$ (b) $y_t^i = \mathrm{e}_t^i$ (c) $y_t^i = \mathrm{x}_t^i$

Notes: Point estimates and 95% confidence intervals for γ_h and $\gamma_h + \tilde{\gamma}_h$ from specification (23) in panel (a), and specification (25) in panels (b) and (c) with $y_{i,t+h}$ as dependent variable. The shock series ε_t^m is inferred based on the “poor man’s sign restrictions” of Jarociński and Karadi (2020), for 1990Q1–2016Q4. Confidence intervals constructed based on two-way clustered standard errors at firm and SIC 3-digit industry-quarter levels.

Figure 11: Dynamic responses of equity issuance and investment to instrumented changes in Tobin’s q (for alternative variable transformations)



(a) $y_t^i = \mathrm{e}_t^i$, on q_t^i (b) $y_t^i = \mathrm{x}_t^i$, on q_t^i (c) $y_t^i = x_t^i/k_t^i$, on q_t^i

Notes: Point estimates and 95% confidence intervals for γ_h and $\gamma_h + \tilde{\gamma}_h$ from specification (25) with y_{t+h}^i as dependent variable. In panels (a) and (b), q_t^i is included in the regression in levels, in panel (c) as $\log(q_t^i)$. In panel (c), investment x_t^i in is constructed as capital expenditures net of sales of property, plant, and equipment. Confidence intervals constructed based on two-way clustered standard errors at firm and SIC 3-digit industry-quarter levels.

E.2 Compustat

In this section we explain the sample selection of Compustat firm-quarters, the construction of the variables used in the empirical analysis, and the calculation of the calibration targets.

E.2.1 Sample Selection

Our sample selection criteria follow standard practice in the literature. We exclude all firm-quarters for which:

1. The firm is not incorporated in the United States.
2. The firm is in the financial (SIC code between 6000 and 6999) or utilities sector (SIC between 4900 and 4999).
3. The measurements of *Total Assets* (Compustat data item 44, ATQ_t^i) and *Property, Plant and Equipment (Net)* (item 42, $PPENTQ_t^i$) are missing or not positive.
4. The measurements of *Debt in Current Liabilities* (item 45, $DLCQ_t^i$), *Total Long-Term Debt* (item 51, $DLTTQ_t^i$), and *Cash and short-term investments* ($CHEQ_{i,t}$, item 38) are missing or negative.

We also exclude:

5. All firm-quarters before a firm's first observation of *Property, Plant and Equipment (Gross)* (item 118, $PPEGTQ_t^i$) in the full quarterly Compustat dataset.
6. All firms which are observed for less than 40 quarters between 1990Q1–2016Q4.

E.2.2 Construction of variables

We construct the key variables employed in the empirical analysis as follows.

1. We measure *investment* for firm i in quarter t as the quarterly *Capital Expenditures* ($CAPXQ_t^i$), constructed based on the Compustat reported *Year-to-date Capital Expenditures* (item 90, $CAPXY_t^i$). We construct the *Investment Rate* of firm i in quarter t as the ratio of *Capital Expenditures* to *Property, Plant and Equipment – Total (Net)*, as measured at the end of the previous quarter (item 42, $PPENTQ_{t-1}^i$): $\frac{CAPXQ_t^i}{PPENTQ_{t-1}^i}$.

   In robustness analysis, we have also verified that all our results remain virtually unchanged, both qualitatively and quantitatively, when considering the following variations to the construction of the *Investment Rate*:

(a) Measure quarterly *Investment* as $CAPXQ_t^i - SPPEQ_t^i$ where $SPPEQ_t^i$ is the quarterly *Sale of Property, Plant and Equipment*, constructed based on the Compustat reported *Year-to-date Sale of Property, Plant and Equipment* (item 83, $SPPEY_t^i$).

(b) Instead of using Compustat's $PPENTQ_t^i$ as the measure of the firm's *Capital Stock*, construct a measure using the perpetual inventory method, as is commonly done for Compustat data, as for example by Ottonello and Winberry (2020). In doing so, the initial value of firm i's capital stock is measured as the earliest available entry of $PPEGTQ_{i,t}$ (item 118), and then iteratively construct K_t^i from $PPENTQ_t^i$ as:[80]

$$K_{t+1}^i = K_t^i + PPENTQ_t^i - PPENTQ_{t-1}^i$$

2. We measure *(Net) Equity Issuance* for firm i in quarter t as $SSTKQ_t^i - PRSTKCQ_t^i$, where $SSTKQ_t^i$ is the quarterly *Sale of Common and Preferred Stock*, constructed based on the Compustat reported *Year-to-date Sale of Common and Preferred Stock* (item 84, $SSTKY_t^i$); $PRSTKCQ_t^i$ is the quarterly *Purchase of Common and Preferred Stock*, constructed based on the Compustat reported *Year-to-date Purchase of Common and Preferred Stock* (item 93, $PRSTKCY_t^i$).

   In our empirical work, we normalize these quarterly net issuances by the *Total Assets* at the beginning of quarter t, i.e. ATQ_{t-1}^i.

3. We measure *Tobin's q* for firm i in quarter t as the market-to-book ratio:

$$q_t^i = \frac{ATQ_t^i + CSHOQ_t^i \times PRCCQ_t^i - CEQQ_t^i}{ATQ_t^i}$$

   where $CSHOQ_t^i$ is the number of *Common Shares Outstanding* (item 61), $PRCCQ_t^i$ is the *Share Price (Close)*, and $CEQQ_t^i$ is *Common/Ordinary Equity - Total* (item 59). We do not subtract deferred taxes from the numerator due to many missing values for the deferred taxes variable in the quarterly Compustat data.

4. As the measure of firm *Size*, we employ *Total Assets* ATQ_{it}^i.

5. We define *Leverage* as *Total Debt* divided by ATQ_t^i, with *Total Debt* computed as the sum of *Debt in Current Liabilities* and *Total Long-Term Debt* ($DLCQ_t^i + DLTTQ_t^i$).

6. We define the *Liquidity Ratio* for firm i in quarter t as $\frac{CHEQ_t^i}{ATQ_t^i}$.

[80] Note again that we use timing convention that K_t^i measures the capital stock in place at the beginning of t, corresponding to the Compustat reported $PPENTQ_{t-1}^i$ at the end of $t-1$.

7. We measure *Liabilities* as Compustat's variable *Liabilities – Total* (item 54, LTQ^i_{it}).
8. To construct a measure of firm *Age*, we follow Cloyne et al. (2018) and use data from Thomson Reuters' WorldScope database to infer *time since the firm's incorporation*.

Dropping outliers. For all the above variables defined as ratios in the empirical analysis, we drop outliers by trimming, assigning the outlier values to missing. For ratios where the numerator can take values on both sides of zero, such as the *Investment Rate*, or the *(Net) Equity Issuance* to *Total Assets* ratio, we trim the highest and lowest 0.5% of observations, by quarter. For ratios where the numerator can take only non-negative values, such as *Tobin's q*, *Leverage*, or *Liquidity Ratio*, we trim the highest 1% of observations, by quarter.

Deflating. Whenever the deflating of variables is necessary, such as for constructing ratios of variables in adjacent quarters (e.g. $CAPXQ^i_t/PPENTQ^i_{t-1}$), or employing the measures of gross and net fixed capital in the perpetual inventory method, we deflate them using the *Implied Price Index of Gross Value Added in the U.S. Nonfarm Business Sector* (BEA-NIPA Table 1.3.4 Line 3).