

CONSTRAINTS AND NON-EXISTENCE  
OF RATIONAL EXPECTATIONS EQUILIBRIA\*

by

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## ABSTRACT

In this article we show that in the presence of trading constraints, such as short sale constraints, the standard definition of a Rational Expectations Equilibrium allows for equilibrium prices that reveal information unknown to any *active* trader in the market. We propose a new definition of the Rational Expectations Equilibrium that incorporates a stronger measurability condition than measurability with respect to the join of the information sets of the agents and give an example of non-existence of equilibrium. The example is robust to perturbations on the data of the economy and the introduction of new assets.

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## 1. INTRODUCTION

The purpose of this article is to discuss the existence of rational expectations equilibrium (REE) in economies with asymmetric information and trading constraints such as leverage or short-sales constraints. We first argue that in the presence of trading constraints the definition of a rational expectations equilibrium which has so far been used by researchers can yield “unreasonable” equilibrium prices and allocations in the sense that prices reveal information not initially possessed by any *active* trader in the market. Furthermore, we exhibit a robust example where such an equilibrium is the *only* possible equilibrium. Hence if we strengthen the definition of a REE by incorporating a measurability condition that takes into account the fact that some traders are inactive<sup>1</sup> on a subset of the state space, the REE may fail to exist. This further suggests that the imposition of trading constraints on agents can be the cause rather than the solution for market breakdowns.

Researchers have been interested in models with trading constraints to solve asset pricing puzzles that arise in standard general equilibrium models such as the equity premium puzzle, the excess volatility puzzle or the asymmetric price-volume relation in stock markets. The typical framework used in this literature is a Lucas tree economy (Lucas, 1978) where agents face some type of liquidity or trading constraint<sup>2</sup>. More recently, researchers have become interested in the impact of trading constraints on asset prices in economies with asymmetric information. For instance, Diamond and Verrechia (1987) study the impact of short sales constraints on the speed of adjustment of asset prices in dealer markets<sup>3</sup>. Allen, Morris and Postlewaite (1993) argue that short sale constraints associated with asymmetric information can generate equilibrium price processes with finitely lived bubbles. Another strand of the literature has considered “ad hoc” specifications of trading constraints in asymmetric information set-ups. For instance, Marín (1993) analyzes the case of leverage constraints to explain some evidence on the relationship between trading volume and assets prices changes. Unfortunately, these examples of economies with asymmetric information, rational expectations and trading constraints where an equilibrium exists are all special in some sense. The purpose of this

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<sup>1</sup> Throughout the article we refer to an inactive trader as a trader who has a flat demand curve on some subset of the state space, although he may be buying or selling (a constant amount) in all those states.

<sup>2</sup> Given the amount of research done in this area, it is well beyond the scope of this paper to give a comprehensive or even a fair bibliography of work on these topics.

<sup>3</sup> Earlier work on the same topic, such as Jarrow (1980), does not assume rational expectations.

article is to point out why equilibrium exists in these papers and to show how more general specifications can lead to failures of existence in a robust manner.

Another motivation for this work comes from the literature on financial innovation. It is now well known that without some type of trading constraints, such as the prohibition of short sales, it is difficult to reconcile costly security design and price taking behaviour. The basic reason is that short sellers of a security can increase the supply of the security without bearing the cost the innovator faces<sup>4</sup>. On the other hand, there is now emerging a growing body of research dealing with the interaction between “spanning” and asymmetric information when new securities are created<sup>5</sup>. Yet, to the best of our knowledge, no researcher has yet attempted to incorporate both components, short sales and asymmetric information, in the same model. This article explains why this may be the case, as equilibrium typically fails to exist in such a framework when agents have rational expectations.

The basic intuition for the failure of the rational expectations equilibrium can be easily understood through the following example. Consider a world with two agents who face short sales constraints. The first agent, Mr. A, has some private information about an asset’s future payoffs; the second agent, Mr. B, can receive endowment shocks that make the asset more or less desirable to him for hedging purposes. The economy can be specified such that Mr. A initially holds no share of that asset; that if there were no trading constraints equilibrium prices would be fully revealing; and that Mr. A would like to short sell the asset if, and only if, the signal he receives is below some value  $\bar{s}$ . With short sales constraints, Mr. A will neither want to buy nor to sell the asset as long as  $s \leq \bar{s}$ . In that case, the equilibrium price should not reveal the *exact* signal Mr. A received but simply the fact that the signal was below  $\bar{s}$ . However, if the equilibrium price does not reveal his signal, there may exist realizations of the endowment shock of Mr. B for which Mr. A may want to purchase some shares<sup>6</sup> for  $s$  sufficiently close to  $\bar{s}$ . But if Mr. A is once again active on the market, then the price must reveal his information. Hence a contradiction and no equilibrium exists.

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<sup>4</sup>For a comprehensive discussion on this issue, see for instance Chapter 4 in Allen and Gale (1994).

<sup>5</sup>See Duffie and Rahi (1995) for a recent survey on the topic and Marín and Rahi (1996, 1997) and Ohashi (1997) for some posterior work on the interaction between information revelation and security design.

<sup>6</sup>For instance, if Mr. B has received a bad shock that decreases his desirability of the asset and, consequently, the associated price of the asset is low.

As in many examples of non-existence of REE (for instance, see Kreps (1977)), this story is somewhat reminiscent of the Hart (1975) example of non existence of equilibria in general equilibrium models with real assets. In the Hart example, equilibria fails to exist because of a drop in the dimensionality of the space generated by the assets' payoffs. Here a drop in the dimensionality of the information revealed by prices at the equilibrium generates the same phenomenon. However, in sharp contrast to the Hart example, it is not the case that generically equilibria will exist in the type of economies we consider. On the contrary, non-existence will creep in as soon as the trading constraint is binding for some trader with private information on a non-trivial subset of the state space<sup>7</sup>. Furthermore, contrary to already known examples of non-existence of REE, the result is robust to both perturbations of the data of the economy or the addition of new assets. In particular, if an asset whose payoffs are sufficiently correlated with those of the first asset is added to the economy then there still exists a non trivial subset of the state space where traders are constrained in their holdings of *both* assets and equilibrium still fails to exist.

The remainder of this article is organized as follows. In Section 2 we set up the primitives of a simple model of asset trading. In that framework we retake the standard definition of a REE and give an example illustrating how under this definition the REE can be such that the equilibrium price contains information unknown to the active traders in the market. With this example at hand we redefine the REE to eliminate this perverse feature of the old equilibrium definition. In Section 3 we present and discuss the robustness of an example of non-existence of equilibrium under the new definition of the REE. Finally, we reserve Section 4 for conclusions and for a discussion of some extensions of our simple analysis.

## 2. THE ECONOMY

In this section we define a very simple environment of asset trading. The framework is somewhat restrictive but allows us to develop simple examples with the advantage of admitting closed-form solutions that will, hopefully, make our points more transparent.

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<sup>7</sup>In particular, the reason why equilibrium exists in the example presented in Allen, Morris and Postlewaite (1993) is that their constraint is binding on a single point. In Marín (1993) there is no drop on the dimensionality of the information revealed by prices because only uninformed agents face the constraint.

We first define the basic framework and state the standard definition of a rational expectations equilibrium. We then give an example that highlights the need to modify that definition when agents face trading constraints.

## 2.1. The Basic Framework.

There is an underlying probability space  $(W, \mathcal{F}, P)$ , on which all random variables are defined. The economy lasts for only one period. At the beginning of the period agents trade in the asset market and at the end of the period all uncertainty is resolved and agents consume the only good in the economy whose price is normalized to one. We assume the existence of a safe storage technology (without loss of generality we assume that the rate of return is constant and equal to zero). There also exists a risky asset which is in constant net supply  $N^{\mathcal{S}0}$ . The payoff of the asset is given by the random variable  $f$ . We denote by  $p\hat{I}P$  the (equilibrium) price of the risky asset at time 0<sup>(8)</sup>. There is a finite set of agents who may participate in the asset market, indexed by  $i=1, \dots, I$ . Each agent is initially endowed with  $N/I$  shares of the risky asset and receives a (random) income  $e_i$  at the end of the period. This income can either be interpreted as labor income or just income from holding assets which do not have a spot market for trading. Denoting by  $w_i$  the end of period wealth of agent  $i$ , by  $w_{0,i}$  his initial wealth, and by  $n_i$  the number of shares of the risky asset he holds after trading, we have:

$$w_i = w_{0,i} + e_i + n_i(f - p) \quad (1)$$

Agents maximize the expected utility of their end of period consumption (wealth). In particular, each agent has a utility function  $U_i$ , where:

$$U_i : R \otimes R$$

We assume the utility function is twice differentiable, strictly increasing and strictly concave.

Before the market for trading assets opens, each agent receives a signal  $s_i \hat{I} S_i$ ,  $i=1, \dots, I$ . These signals are payoff relevant for agents because they are statistically related to agents' endowments and the assets payoffs. Let  $S = S_1 \times \dots \times S_I$  and denote by  $s = (s_1, \dots, s_I)$  a generic element of  $S$ . We will often commit a slight abuse of language and refer to these elements in  $S$  as "states of the world". For clarity in our presentation we also define the set  $S^{-i} = S_1 \times \dots \times S_{i-1} \times S_{i+1} \times \dots \times S_I$ , which is the section of  $S$  that excludes  $S_i$ , and denote by  $s^{-i}$

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<sup>8</sup>Alternatively, one could consider an economy without a storage technology but with a futures contract in zero net supply. The results would be identical.

${}^i\hat{I}S^{-i}$ , a typical element of this set. This allows to describe a state of the world as  $s=(s_i, s^{-i})$ , the signal received by some trader  $i$ ,  $s_i$ , and the signal received by the rest of traders,  $s^{-i}$ . In the rest of the analysis we assume that  $S_i=R^{h_i}$  (where  $R^{h_i}$  is the  $h_i$ -dimensional Euclidean space), with the convention that  $R^0=\{0\}$ , and that  $P=R^{\circ}$ .

We now define a *rational expectations equilibrium (REE)* for this economy.

DEFINITION A. A REE is a price  $p \in R$  and a set of equilibrium holdings  $\{n_i\}_{i=1}^I$  such that for a.e.  $s \in S$ :

A.1) (measurability)

$p: S \rightarrow R$  is a (Borel) measurable function.

A.2) (optimality)

For each  $i$ ,  $n_i: S \rightarrow P \subseteq R$  is such that:

$$n_i(s_i, p; p(s_i, s^{-i})) = \text{Argmax}_{n_i} E[U_i(w_i) \text{ s.t. } (1) \mid s_i, p(s_i, s^{-i}) = p]$$

A.3) (clearing)

$$\sum_{i=1}^I n_i = N$$

*Condition A.1* is the usual measurability condition first introduced by Radner (1979) and then commonly used in the literature even in the presence of trading constraints (see for instance condition (d) in the definition of the REE in Allen, Morris and Postlewaite (1993)). This condition rules out equilibrium prices which are not measurable with respect to the join of the information sets of the agents. Without this condition, a price like  $p=f$  could be an equilibrium price with no trading, even in an economy in which no agent has private information ( $h_i=0$ ,  $i=1, \dots, I$ ). Hence, this measurability condition prevents equilibrium prices from containing more information than that already existing in the economy.

Suppose now that agents must satisfy the following constraint on their holdings of the risky asset:

$$n_i \in C_i \subseteq R \tag{2}$$

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<sup>9</sup> We are not assuming compactness of the sets  $S$  and  $P$ . However, in the examples we will present we will put enough structure on the distributions and utility functions guaranteeing the existence of expected utility. None of our non-existence results will rely on the lack of compactness of these sets.

Then it may be the case that some agent will trade exactly the same number of shares for many different realizations of his signal  $s_i$ . According to the measurability *condition A.1*, equilibrium prices could potentially reveal which of these signals the agent actually received, *even though the agent demands exactly the same number of shares in all these different states*<sup>10</sup>. Hence in the presence of trading constraints the measurability condition may not serve its role of eliminating “unreasonable” equilibria in the sense of equilibria where prices reveal more information than owned by *active* traders in the market. The next section clarifies this point with a simple example.

## 2.2. A First Example.

We now present a very simple example to illustrate how the introduction of constraints requires reformulating the measurability condition. For this example, we assume that all random variables defined in the (fixed) probability space  $(\mathbf{W}, \mathcal{F}, \mathbf{P})$  are normally distributed and belong to a linear space  $\mathcal{N}$  of jointly normally distributed random variables on  $\mathbf{W}$ . Furthermore, we assume that the asset payoff can be split into two components and that there are just two agents in the economy ( $i=1,2$ ) whose utility functions display constant absolute risk aversion. In particular:

$$f = y + \mathbf{e} \quad (3)$$

$$U_i(w_i) = - \exp\{-w_i\} \quad i=1, \dots, I \quad (4)$$

We also assume that the risky asset is in zero net supply ( $N=0$ ); that agent 1 has no (risky) endowments ( $e_1=0$ ) and agent 2 has endowments  $e_2=z$ ; and that agents do not receive any signal (no private information or  $h_1=h_2=0$ ). To simplify the analysis, we make the following distributional assumptions:

$$\begin{pmatrix} y \\ z \\ \mathbf{e} \end{pmatrix} \rightarrow N \left( \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 & V_{yz} & 0 \\ V_{yz} & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \right)$$

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<sup>10</sup> Of course, implicit in this statement is the assumption that no other agent receives the same signal  $s_i$ .

For simplicity we assume that  $V_{yz} > 0$ . Under these assumptions, it is easy to see that:

$$n_1 = \frac{E(f) - p}{\text{Var}(f)} = \frac{-p}{2}$$

$$n_2 = \frac{E(f) - p - V_{yz}}{\text{Var}(f)} = \frac{-p - V_{yz}}{2}$$

and that the equilibrium price and equilibrium holdings for each agent are:

$$p = -\frac{V_{yz}}{2}; \quad n_1^* = \frac{V_{yz}}{4}; \quad n_2^* = -\frac{V_{yz}}{4}$$

As expected, in this equilibrium agent 1 buys half of the endowment risk of agent 2 that can be shared with the risky asset ( $V_{yz}/V_f$ ) and the price is smaller than the conditional expectation of the asset payoff (in this case zero), which implies the existence of a strictly positive risk premium (even though the asset is in zero net supply).

Now suppose we introduce a third agent in this economy ( $i=3$ ). This agent has no endowment risk,  $e_3=0$ , and receives a signal  $s_3=y$  before the asset market opens for trading. What is peculiar about this agent is that he faces a strong trading constraint, namely, he is **constrained never to trade**. Once we see the nature of the constraint this agent faces we should expect nothing to change in the economy. In particular, we should expect that the nature of the equilibrium price and original agents' allocations does not change because of the introduction of a third agent who cannot trade. However, under *Definition 1*, the following is also an equilibrium in the economy with three agents:

$$p(y) = y; \quad n_1^* = 0; \quad n_2^* = 0$$

For this, note that the equilibrium price fully reveals agent 3 private information,  $y$ , and that  $\text{Cov}(y, z|y)=0$ . In this equilibrium agents do not trade. Obviously, the former equilibrium *ex ante* Pareto dominates this last one. One might think of the equilibrium price of this new equilibrium as a self-fulfilling price set by a "malevolent", omniscient

and yet invisible auctioneer.<sup>11</sup> We take the view in this paper that these extraordinary abilities of the hypothetical auctioneer should be restricted. Once we introduce trading constraints that can exclude some agents from trading in some states of the world, it seems economically reasonable to restrict attention to price schemes  $p$  that contains no more information than is possessed by *active* traders in the market, as opposed to *existing* traders in the economy.

### 2.3. A new measurability condition.

In order to avoid situations like in the previous example we propose a new measurability condition to replace *Condition A.1*. The idea is to require the equilibrium price not to contain information which is not incorporated in the optimal rational expectation demand of traders. In order to do this, for every signal  $s_i$  and price  $p$ , we define the set  $\bar{S}_i^p \subseteq S_i$  as:

$$\bar{S}_i^p(s_i) = \left\{ s'_i \in S_i \mid n_i(s_i, p; p(s_i, s^{-i})) = n_i(s'_i, p; p(s_i, s^{-i})) \right\}$$

where  $n_i(s_i, p; p(s_i, s^{-i}))$  is the optimal rational expectations demand of trader  $i$  when he faces the constraint (2) —see the formal definition below. The set  $\bar{S}_i^p$  includes all the signals received by trader  $i$  for which his optimal demand is identical to the one he chooses when he receives the signal  $s_i$  and learns from the price associated to the (fixed) signal  $s^{-i}$  received by the other traders. An interesting particular case is the typical framework in the finance literature where the set of traders is divided into two types, informed and uninformed traders, with a representative agent for each type. In this set-up, only the uninformed group learns from prices and the set  $\bar{S}_i^p$  corresponding to an informed agent depends only on that agent's walrasian demand<sup>12</sup>.

We now propose a new definition of a REE where we impose a new measurability condition that restricts the equilibrium price not to depend on different signals in the same set  $\bar{S}_i^p$ , that is, not to reveal information that is not differentiated in the optimal demands.

<sup>11</sup>In some sense, it can be also thought of as a very special type of sunspot equilibria.

<sup>12</sup>Formally, agent  $i$  walrasian demand is a function  $n_i^w: S \times P \rightarrow R$  such that:

$$n_i^w(s_i, p) = \text{Argmax}_{n_i} E[U_i(w_i) \text{ s.t. (1) \& (2)} \mid s_i]$$

In this case, we have:  $\bar{S}_i^p(s_i) = \left\{ s'_i \in S_i \mid n_i^w(s_i, p) = n_i^w(s'_i, p) \right\}$

DEFINITION B. A REE is a price  $p \in \mathbb{R}^I$  and a set of equilibrium holdings  $\{n_i\}_{i=1}^I$  such that

for a.e.  $s \in \mathcal{S}$ :

B.1)(Constrained measurability)

$p: \mathcal{S} \rightarrow \mathbb{R}^I$  is a (Borel) measurable function subject to the restriction:

$$\forall i \in I, p(s_i, s^{-i}) = p \text{ and } s'_i \in \bar{S}_i^p(s_i) \Rightarrow p(s'_i, s^{-i}) = p$$

B.2)(optimality)

For each  $i$ ,  $n_i: \mathcal{S} \rightarrow \mathbb{R}^P$  is such that:

$$n_i(s_i, p; p(s_i, s^{-i})) = \text{Argmax}_{n_i} E[U_i(w_i) \text{ s.t. } (1) \& (2) | s_i, p(s_i, s^{-i}) = p]$$

B.3)(clearing)

$$\sum_{i=1}^I n_i = N$$

### 3. NON-EXISTENCE OF REE.

In this section we argue that under *Definition B* (or any other that restrict the price in a similar fashion to condition B.1), rational expectations equilibrium may fail to exist (or, equivalently, that the only equilibrium that may exist under *Definition A* is the one that we have argued is unreasonable on pure economic grounds). Differently to well known non-existence cases, the failure in the present example is robust to perturbations of the data of the economy, the introduction of more assets or the introduction of noise. We now give one of these robust examples.

#### 3.1. An example of non-existence of equilibrium.

Consider again the economy of the previous example with two traders ( $i=1,2$ ), both with preferences exhibiting constant absolute risk aversion (4) and asset payoffs  $f$  given by (3). Furthermore, assume now that  $N \geq 0$  and that agent 2 (risky) endowment is given by

$$e_2 = x z$$

where  $x$  is a standard normal uncorrelated with all the other random variables in the economy. Also, without loss of generality, assume that  $Cov(\mathbf{e}, z) = V_{e_z} > 0$ . To summarize, the distributional assumptions on all the random variables are:

$$\begin{pmatrix} y \\ z \\ \mathbf{e} \\ x \end{pmatrix} \rightarrow N \left( \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 & V_{yz} & 0 & 0 \\ V_{yz} & 1 & V_{e_z} & 0 \\ 0 & V_{e_z} & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \right)$$

Finally, assume that  $s_1 = y$  and  $s_2 = x$ . That is agent 2 has some information on his endowment risk while agent 1 has private information on asset payoffs (and, consequently, some extra information on agent 2's endowment risk)<sup>13</sup>. We denote by  $s = (y, x)$  a state of the world. We now look at the equilibrium in this economy for two cases. In the first case no trader faces constraints. In the second, some traders face a lower bound constraint on asset holdings ( $n_i \geq v$ , for some  $-v < v < v$ , for some  $i$ ).

### 3.1.1. The unconstrained case.

In this case it is easy to see that a fully revealing REE exists in which agents trade the risky asset to share their endowments risk. First, note that conditional on their private information, each agent's end of period wealth is normally distributed. This means that agents choose again the typical mean-variance portfolio given by:

$$n_1 = \frac{E(f|s_1) - p}{Var(f|s_1)} = y - p \quad (5)$$

$$n_2 = \frac{E(f|s_2, p(s) = p) - p - Cov(f, e_2|s_2, p(s) = p)}{Var(f|s_2, p(s) = p)} \quad (6)$$

Now, it is easy to see that there exist a (unique) linear fully revealing REE with associated equilibrium price:

$$p(y, x) = y - \frac{N + xV_{e_z}}{2} \quad (7)$$

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<sup>13</sup>It can be easily verified that exactly the same non-existence result would prevail if both agents were endowed with (different) private information.

Given this equilibrium price functional, the rational expectations optimal demands of agents 1 and 2, as defined in *Condition B.2* and *Condition B.1*, are given by:

$$n_1 = y - p \quad (8)$$

$$n_2 = \frac{\left( p + \frac{N + xV_{e_z}}{2} \right) - p - xV_{e_z}}{1} \quad (9)$$

Obviously, in this economy measurability conditions A.1 and B.1. impose identical restrictions on the equilibrium price as  $\bar{\cdot}$  is a singleton for every  $s_i$  and  $p$ . The equilibrium holdings of agents 1 and 2 are obtained by substituting the equilibrium price into the their optimal demands and are given by:

$$n_1^* = \frac{N + xV_{e_z}}{2}; \quad n_2^* = \frac{N - xV_{e_z}}{2} \quad (10)$$

One can easily see why this equilibrium price is fully revealing. While only agent 1 knows  $y$ , agent 2 can invert the price function and *learn*  $y$ . Hence the private information of the trader with superior information is transmitted to the trader with inferior information via prices. As expected, in this equilibrium agent 1 buys half the endowment risk of agent 2 that can be shared with the existing asset ( $xV_{e_z}$ ).

### 3.1.2. The constrained case.

We now impose a trading constraint in the previous economy. For simplicity, we assume that only agent 1 faces the constraint. The constraint is assumed to be of the form.

$$n_1 \in \mathbf{n}, \quad -\Psi < v < \Psi \quad (11)$$

Equation (11) represents a fairly general type of constraint, an example of which is short sales constraint ( $\mathbf{n}=0$ ). It is important to note that exactly the same result arises if we constrain  $n_1$  to lie in any non-degenerate interval not equal to  $(-\infty, \infty)$ .

Before stating the main result of the paper, namely that under *Definition B* there is no REE in this economy, we analyze the nature of the “potential” equilibrium in this

economy. The first thing to notice is that the constraint may be binding in a non-trivial subset of the state space. In particular, from (5) we can see that agent 1 will be constrained in all states for which  $y - p - v \leq 0$ . Now, suppose  $y$  is such that the associated equilibrium price satisfies  $y - p - v > 0$ . In this case, agent 1 is unconstrained and, since both traders will be active in the market, the equilibrium price will have the same functional form as in (7). Given this form for the price, it is easy to see that if agent 1 is unconstrained, then necessarily  $x > \frac{2v - N}{V_{ez}}$ . This allows us to determine a subset of the state space where

agent 1 is unconstrained. Let's denote this subset as the  $U$  region:

$$U = \left\{ (y, x) \in \mathbb{R}^2 \mid x > \frac{2v - N}{V_{ez}} \right\}$$

Now, suppose  $y$  is such that the associated equilibrium price satisfies  $y - p - v \leq 0$ . Then, necessarily  $x \leq \frac{2v - N}{V_{ez}}$ . Furthermore, since agent 1 is constrained in this case, by

measurability *Condition B.1*,  $p$  cannot be a function of  $y$ . This means that agent 2 cannot learn  $y$  from the equilibrium price. Therefore, his demand will be given by:

$$n_2 = \frac{E(f|x) - p - x \text{Cov}(f, z)}{\text{Var}(f|x)} = \frac{0 - p - x(V_{yz} + V_{ez})}{2}$$

In equilibrium, the market clearing condition,  $v + n_2 = N$ , implies:

$$p(x) = -\left[ 2(N - v) + x(V_{yz} + V_{ez}) \right] \quad (12)$$

Note that when this is the equilibrium price, agent 1 optimally demands a number of shares no larger than  $v$ , and hence it must be the case that  $v \leq -\left[ \frac{2v - N}{V_{ez}} + \frac{2(N - v) + x(V_{yz} + V_{ez})}{2} \right]$ . This allows us to identify another subset of the state space in which agent 1 is constrained, which we denote as the  $C$  region:

$$C = \left\{ (y, x) \in \mathbb{R}^2 \mid x \leq \frac{2v - N}{V_{ez}}, \quad y \leq 3v - \left[ 2N + x(V_{yz} + V_{ez}) \right] \right\}$$

The  $C$  and  $U$  regions give the set of states for which agent 1 is constrained and unconstrained, respectively. However, these two sets do not exhaust the whole state space. The previous analysis does not characterize the equilibrium in a positive measure set of the state space. Indeed, as the following proposition states, in that set there is no equilibrium price, which implies the inexistence of an equilibrium in this economy.

**PROPOSITION 1:** There does not exist a REE, as defined in *Definition B*, in the economy with trading constraints.

*Proof:*

Consider the set:

$$NE = \left\{ (y, x) \in R^2 \left| x \leq \frac{2v - N}{V_{ez}}, \quad y > 3v - [2N + x(V_{yz} + V_{ez})] \right. \right\}$$

We show that there is no equilibrium price for any state in this set. By contradiction, assume there is. Then, for every  $(y, x) \in NE$  agent 1 is either constrained or unconstrained.

- i) Suppose agent 1 is unconstrained. Then,  $y - p - v > 0$  and the equilibrium price must be given by (7). This implies  $x > \frac{2v - N}{V_{ez}}$ , which contradicts the assumption  $(y, x) \in NE$ .
- ii) Suppose agent 1 is constrained. Then,  $y - p - v \leq 0$  and the equilibrium price is given by (12). This implies  $y \leq 3v - [2N + x(V_{yz} + V_{ez})]$ , which contradicts the assumption  $(y, x) \in NE$ .

*QED.*

One important point to notice is that under *measurability condition A.1*, there exists a fully revealing rational expectations equilibrium. In particular, the following is an equilibrium:

$$p(x, y) = \begin{cases} y - \frac{(N + xV_{ez})}{2} & \forall (y, x) \in U \\ y - (N - v + xV_{ez}) & \forall (y, x) \in C \cup NE \end{cases}$$

$$n_1^* = \begin{cases} \frac{(N + xV_{ez})}{2} & \forall (y, x) \in U \\ v & \forall (y, x) \in C \cup NE \end{cases}$$

$$n_2^* = \begin{cases} \frac{(N - xV_{ez})}{2} & \forall (y, x) \in U \\ N - v & \forall (y, x) \in C \cup NE \end{cases}$$

However, once again, such an equilibrium requires the price to reveal the signal  $y$  received by agent 1 even when this agent sells a *constant* amount of shares  $N/2 - n$  for *any* signal he receives.

### 3.2. Discussion.

The example we have presented here is fairly specific but the same reasoning applies to any economy with asymmetric information where, conditional on prices revealing some trader's information, the constraint is binding on a non-empty, non-singleton subset of the state space. By measurability *Condition B.1.*, equilibrium prices can not be fully revealing and must look like the conditional expectation of  $f$  given that  $s \in C$ . However, given that the equilibrium price takes this form, the trader may want to hold more of the asset than the minimum allowed by the constraint for the “best” signals in  $C$ . Hence no equilibrium exists. The existing literature on the impact of trading constraints and asymmetric information has gone around this problem in several ways. In the example given in Allen, Morris and Postlewaite (1993), the constraint is binding on a singleton state. Hence, the problem we point out in this note does not appear. However, the “reasonable” equilibrium we suggest generally fails to exist in the general set-up they propose. On the other hand, Marín (1993) imposes a constraint only on uninformed traders and hence also does not face the non-existence problem we stress in this paper.

Another issue is the robustness of the non-existence result to the introduction of new assets. Indeed, a typical reason for non-existence of REE is a low number of prices compared to the number of signals in the economy. The issue is a bit more complicated in the case of constraints. It is no longer the case that, generically, you can add an additional asset and recover existence. In particular, if you introduce an asset whose payoffs are strongly correlated with the existing asset, then there will still exist a positive measure region of the state space where a trader will be constrained in his holdings of *both* assets and exactly the same argument will apply to prove non-existence. On the other hand, it is always possible to introduce the right kind of asset to have fully revealing prices on the whole state space. For instance, in the previous example with  $n=0$ , introducing an asset with payoff equal to  $-f$  will restore the equilibrium. It is also worthwhile noticing that the explicit introduction of noise traders or the enlargement of the uncertainty space to hope for partially revealing prices (as opposed to fully or non revealing prices) will not restore the existence of the equilibrium. Indeed, it is easy to see that the equilibrium will fail to exist both in classical noisy rational expectations set-ups — such as the Grossman and Stiglitz (1980) economy— and in more recent rational expectations models without noise traders — such as the Marín and Rahi (1996) economy— as soon as we impose, say, short

sales constraints on the informed traders and require the equilibrium price to satisfy our measurability *Condition B.1*.

A final issue is whether the inexistence of the equilibrium will prevail under other price setting mechanisms and learning hypotheses. In particular, one could argue that in dealer markets, where all transactions go through a dealer who sets bid and ask prices before knowing the nature of the demand of the trader calling him (order to buy or to sell), the problem of non-existence of the equilibrium will not arise. Diamond and Verrechia (1987) is a paper that falls in this category where the inexistence of the equilibrium issue does not arise even though some traders, including informed traders, may face a short sales constraint. On the other hand, we believe that our result crucially depends on the assumption of agents having rational expectations and that some alternative learning hypotheses might be immune to the non-existence of the equilibrium problem. For instance, the equilibrium will not fail to exist in economies where agents learn from past prices only, as in Hellwig (1982). No doubt, investigating the impact of constraints under different market arrangements and learning hypotheses is a promising area for further research.

#### **4. CONCLUDING REMARKS.**

We have shown that the measurability condition imposed in the “standard” definition of a REE for economies with asymmetrically informed agents facing some type of trading constraint is not sufficient to rule out equilibrium prices that contain information not possessed by any *active* trader in the market. We have also shown that the appropriate strengthening of the measurability condition yields non existence of the REE in a robust manner. Although our approach in this paper has been minimalist in the sense that we have only dealt with simple examples, the analysis suggests that the non-existence result may arise as soon as we have trading constraints that affect rational expectations agents with private information and that can be binding on a non-negligible subset of the state space.

This result has interesting implications both for the understanding of financial markets and for economic modeling. Since trading constraints usually arise as a result of financial regulation and, in general, regulation measures do not discriminate between informed and uninformed agents, the analysis in this paper sets the basis to think of

trading constraints as a cause rather than a solution for market breakdowns. Regarding economic modeling, the result in this paper not only adds more doubts on the REE concept but also points at serious problems researchers will face when, for instance, trying to solve security design problems while taking into account the two key ingredients that are short-sales constraints and asymmetric information.

There are several extensions and new problems to address in future research. One extension involves identifying the necessary and sufficient conditions for the equilibrium to fail to exist in a general equilibrium model with a general specification of the state space, preferences, endowments, and asset structure. Another extension would focus on identifying the precise way in which the basic principle of this paper can end up producing market breakdowns in real security markets. One reason why our result on non-existence may not translate into actual market breakdowns could be the existence of a market for information where constrained traders could sell their information. Hence, we believe that explicitly modeling such a market is an interesting topic for further research. A final issue we are interested in addressing is the robustness of this non-existence result to different market microstructures and learning hypotheses.

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