Analytical Handle for ZF Reception in Distributed Massive MIMO

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***Abstract*—This paper considers distributed massive MIMO networks where a large number of antennas, either collocated or geographically scattered over a region, communicate with mobile users. This communication is impaired by interference from similar transmissions in adjacent regions and by noise. Focusing on zero-forcing (ZF) reception, we derive simple expressions that very accurately approximate the instantaneous signal-to-interference-plus-noise ratio (SINR) and the ergodic spectral efficiency of an arbitrary user. These expressions enable shortcutting any assessment of the network-level performance, either analytical or simulation-based.**

***Index Terms*—Distributed Antenna Systems, Massive MIMO, Cell-free, Zero Forcing.**

I. Introduction

Massive multiple-input multiple-output (MIMO) provides powerful means to cope with the challenge of achieving very high spectral efficiencies in wireless networks [1–3]. However, the deployment of very large numbers of colocated antennas at base station (BS) sites presents practical challenges, particularly in micro BSs and at microwave frequencies [4]. One approach to unshackle these challenges is to scatter the antennas connected to each BS over the entire cell. Such distributed approach to massive MIMO can be further extended across multiple cells via BS cooperation. Then, the cell boundaries within a certain region effectively vanish, justifying the recently coined denomination as "cell-free" massive MIMO [5–8]. This distributed topology creates a scenario that is more complex to analyse than its co-located counterpart. The downlink of distributed massive MIMO with conjugate beamforming is studied in [5] and found to significantly outperform non-cooperative small-cell networks. The uplink of distributed massive MIMO is studied in [9] under the assumption of perfect channel state information (CSI).

In this paper, we consider the uplink of a distributed massive MIMO network where a large number of receive antennas cooperate to serve a much smaller number of transmitting users. To overcome the assumption of perfect CSI, this cooperation is limited to a set of geographically neighboring antennas that forms a cooperating *cluster*. As such, the reception of in-cluster signals is impaired by out-of-cluster interference. This setting is rather general, admitting the interpretation of distributed massive MIMO with a clustered structure but also that of a conventional network where BSs cooperate forming clusters of sufficiently many antennas. The estimation of the channels coefficients is assisted by the transmission of uplink pilots. With zero-forcing (ZF) reception within each cluster, we derive a simple yet very accurate closed-form approximation for the instantaneous SINR of an arbitrary user under distance-dependent path loss, shadowing and Rayleigh fading. Based on subsequent characterizations of the characteristic function (CF) and probability density function (PDF) of the received SINR, we further express the spectral efficiency of an arbitrary user. We establish the transmit power level above which the spectral efficiency saturates because of out-of-cluster interference [10]. Numerical examples illustrate the accuracy of our analysis and illustrate the performance gains offered by distributed massive MIMO.

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II. System Model

We consider a cluster of N distributed antennas jointly detecting the transmissions from $K \ll N$ in-cluster users in the presence of interference from $\bar{K}$ out-of-cluster users. The transmitting users and the receiving antennas are arbitrarily located. The $\mathcal{C}^{N\times 1}$ received vector at the cluster of interest is

$$\mathbf{y} = \mathbf{H}\mathbf{s} + \bar{\mathbf{H}}\bar{\mathbf{s}} + \mathbf{n}, \tag{1}$$

where $\mathbf{s} = (s_1, \ldots, s_K)^{\mathrm{T}}$ and $\bar{\mathbf{s}} = (\bar{s}_1, \ldots, \bar{s}_{\bar{K}})^{\mathrm{T}}$ are vectors containing data symbols from the K in-cluster users and the $\bar{K}$ out-of-cluster users, respectively. These symbols are independent and normalized such that $\mathbb{E}[\mathbf{s}\mathbf{s}^H] = \mathbf{I}$ and $\mathbb{E}[\bar{\mathbf{s}}\bar{\mathbf{s}}^H] = \mathbf{I}$. The vector $\mathbf{n}$ represents additive white Gaussian noise satisfying $\mathbb{E}[\mathbf{n}\mathbf{n}^H] = \sigma^2\mathbf{I}$. The channels connecting the users with the receive antennas are subject to path loss, shadowing and Rayleigh fading. It follows that the entries of the $\mathcal{C}^{N\times K}$ aggregate channel matrix $\mathbf{H}$ are independent yet nonidentically distributed, $H_{nk} \sim \mathcal{CN}(0, P_{nk})$ with P_{nk} denoting the power received at antenna n from in-cluster user k. Similarly, the $\mathcal{C}^{N\times\bar{K}}$ channel matrix $\bar{\mathbf{H}}$ features independent and nonidentically distributed entries, $H_{n\bar{k}} \sim \mathcal{CN}(0, P_{n\bar{k}})$ with $P_{n\bar{k}}$ the power received at antenna n from out-of-cluster user $\bar{k}$.

A. Channel estimation

Let L be the fading coherence in symbols and $\tilde{T}$ be the number of symbols reserved for pilot transmissions per coherence interval. The K in-cluster users transmit orthogonal sequences of T pilots each for the total of $\tilde{T} = KT$. During the transmission of pilots by user k, the $\mathcal{C}^{T\times 1}$ received sequence at antenna n can be written as

$$\mathbf{y}_n = \sqrt{P_{nk}} h_{nk}\, \mathbf{s}_{\mathrm{p}} + \mathbf{w}_n, \tag{2}$$

where h_{nk} is the Rayleigh fading, $\mathbf{s}_{\mathrm{p}} = [s_1, \ldots, s_T]^{\mathrm{T}}$ is the $\mathcal{C}^{T\times 1}$ pilot sequence and $\mathbf{w}_n = [w_{n1}, \ldots, w_{nT}]^{\mathrm{T}}$ denotes the $\mathcal{C}^{T\times 1}$ sequence of out-of-cluster interference plus noise with each symbol having variance

$$\sigma_n^2 = \sum_{k=K+1}^{K+\bar{K}} P_{nk} + \sigma^2. \tag{3}$$

At each receiving antenna, a linear minimum mean squared error (LMMSE) estimate of h_{nk} is obtained as [11]

$$\hat{h}_{nk} = \left(\frac{\sqrt{P_{nk}}}{P_{nk}\|\mathbf{s}_{\mathrm{p}}\|^2 + \sigma_n^2} \right) \mathbf{s}_{\mathrm{p}}^H \mathbf{y}_n. \tag{4}$$

Thus, h_{nk} can be written as

$$h_{nk} = \hat{h}_{nk} + e_{nk}, \tag{5}$$

where the channel estimation error e_{nk} is independent of $\hat{h}_{nk}$ as well as complex Gaussian with zero mean and variance

$$\sigma_{e_{nk}}^2 = \frac{\sigma_n^2}{P_{nk}\|\mathbf{s}_{\mathrm{p}}\|^2 + \sigma_n^2}. \tag{6}$$

The path loss and shadowing, i.e., the values of P_{nk}, are perfectly known because of the slow variation of these quantities.

B. ZF Receiver

Based on (5), we can re-express (1) as

$$\mathbf{y} = \hat{\mathbf{H}}\mathbf{s} + \underbrace{\mathbf{E}\mathbf{s} + \bar{\mathbf{H}}\bar{\mathbf{s}} + \mathbf{n}}_{\mathbf{z}}, \tag{7}$$

where the $\mathcal{C}^{N\times K}$ matrix $\hat{\mathbf{H}}$ contains independent entries $\sqrt{P_{nk}}\hat{h}_{nk}$ and the $\mathcal{C}^{N\times K}$ estimation error matrix $\mathbf{E}$ contains independent entries $\sqrt{P_{nk}} e_{nk}$. Noting that $\mathbf{z}$ is not spatially white, we proceed to whiten it by applying to (7) a filter $\mathbf{\Lambda}^{-1/2}$ where

$$\begin{aligned}\mathbf{\Lambda} &= \mathbb{E}[\mathbf{z}\mathbf{z}^H] \\ &= \mathrm{diag}\{\Lambda_1, \ldots, \Lambda_N\},\end{aligned} \tag{8}$$

and $\Lambda_n = \sigma_n^2 + \sum_{k=1}^{K} P_{nk}\sigma_{e_{nk}}^2$ with the expectation in (8) taken over channel estimation error, out-of-cluster interference and noise. Applying the estimated $\hat{\mathbf{H}}$ to perform ZF on the whitened observations, we recover

$$\hat{\mathbf{y}} = \mathbf{s} + \left(\hat{\mathbf{H}}^H \mathbf{\Lambda} \hat{\mathbf{H}}\right)^{-1} \hat{\mathbf{H}}^H \mathbf{\Lambda}^{-1}\mathbf{z}. \tag{9}$$

III. Received SINR Approximation

In this section, we derive an accurate approximation for the received SINR of an arbitrary in-cluster user. Based on (9), the instantaneous received SINR of user k can be written as

$$\gamma_k = \frac{1}{\left[\left(\hat{\mathbf{H}}^H \mathbf{\Lambda} \hat{\mathbf{H}}\right)^{-1} \hat{\mathbf{H}}^H \mathbf{\Lambda}^{-1}\mathbf{z}\mathbf{z}^H \mathbf{\Lambda}^{-1} \hat{\mathbf{H}} \left(\hat{\mathbf{H}}^H \mathbf{\Lambda} \hat{\mathbf{H}}\right)^{-1}\right]_{kk}}, \tag{10}$$

where the denominator contains the instantaneous channel gains of both in-cluster and out-of-cluster users. We note that, if the clusters are sensibly formed, the instantaneous channel gains of the in-cluster users tend to be larger than those of the out-of-cluster users. Furthermore, the number of pilot symbols is selected such that the estimation error is small when compared to the channel estimates. Therefore, we are motivated to replace $\mathbf{z}\mathbf{z}^H$ in (10) with its expected value to obtain

$$\gamma_k \approx \frac{1}{\left[\left(\hat{\mathbf{H}}^H \mathbf{\Lambda} \hat{\mathbf{H}}\right)^{-1}\right]_{kk}}, \tag{11}$$

which can be further expressed as [12]

$$\gamma_k \approx \hat{\mathbf{h}}_k^H \mathbf{\Lambda}^{-\frac{1}{2}} (\mathbf{I} - \mathbf{M}) \mathbf{\Lambda}^{-\frac{1}{2}} \hat{\mathbf{h}}_k, \tag{12}$$

where $\hat{\mathbf{h}}_k$ is the kth column of $\hat{\mathbf{H}}$ while

$$\mathbf{M} = \hat{\mathbf{H}}_{\acute{k}} (\hat{\mathbf{H}}_{\acute{k}}^H \hat{\mathbf{H}}_{\acute{k}})^{-1} \hat{\mathbf{H}}_{\acute{k}}^H, \tag{13}$$

with $\hat{\mathbf{H}}_{\acute{k}}$ representing $\hat{\mathbf{H}}$ with $\hat{\mathbf{h}}_k$ removed. Note that $\mathbf{M}$ contains the channels from all the undesired in-cluster users. Since the randomness of small-scale fading vanishes with an increasing number of antennas [1], we next analyze $\mathbb{E}[\mathbf{M}]$ where the expectation is taken over the small-scale fading components in $\hat{\mathbf{H}}_{\acute{k}}$. Whilst not shown here due to space limitations, some simple mathematical manipulations show that $\mathbb{E}[\mathbf{M}]$ is a diagonal matrix. As such, following an approach similar to that in [9] we are motivated to approximate $\mathbf{M}$ by a simple diagonal matrix that has the same trace as $\mathbf{M}$. Note that $\mathbf{M}$ is an idempotent matrix and the trace of $\mathbf{M}$ is equal to its rank, i.e., $K-1$. We spread the trace of $\mathbf{M}$ equally along the diagonal to define a new matrix

$$\breve{\mathbf{M}} = \frac{K-1}{N}\mathbf{I}, \tag{14}$$

as an approximation to $\mathbb{E}[\mathbf{M}]$. As such, we re-express $\mathbf{M}$ as

$$\mathbf{M} = \breve{\mathbf{M}} + \mathbf{\Xi}, \tag{15}$$

where $\mathbf{\Xi}$ is an error matrix. Substituting (15) into (12) we get

$$\gamma_k \approx \hat{\mathbf{h}}_k^H \mathbf{\Lambda}^{-\frac{1}{2}} (\mathbf{I} - \breve{\mathbf{M}}) \mathbf{\Lambda}^{-\frac{1}{2}} \hat{\mathbf{h}}_k \left(1 - \Psi\right), \tag{16}$$

where

$$\Psi = \frac{\hat{\mathbf{h}}_k^H \mathbf{\Lambda}^{-\frac{1}{2}} \mathbf{\Xi} \mathbf{\Lambda}^{-\frac{1}{2}} \hat{\mathbf{h}}_k}{\hat{\mathbf{h}}_k^H \mathbf{\Lambda}^{-\frac{1}{2}} (\mathbf{I} - \breve{\mathbf{M}}) \mathbf{\Lambda}^{-\frac{1}{2}} \hat{\mathbf{h}}_k}. \tag{17}$$

In the following, we argue that the term Ψ is small in a distributed massive MIMO scenario. Note that the numerator and the denominator in Ψ are quadratic forms containing the sum of N terms. Thus, we divide both the numerator and the denominator by N and set N to be large. In doing so, the denominator of Ψ becomes

$$\frac{\hat{\mathbf{h}}_k^H \mathbf{\Lambda}^{-\frac{1}{2}}(\mathbf{I}-\check{\mathbf{M}})\mathbf{\Lambda}^{-\frac{1}{2}}\hat{\mathbf{h}}_k}{N} = \left(\frac{N-K+1}{N^2}\right)\hat{\mathbf{h}}_k^H \mathbf{\Lambda}\hat{\mathbf{h}}_k \quad (18)$$

as $\mathbf{I}-\check{\mathbf{M}} = \frac{N-K+1}{N}\mathbf{I}$. For large N, this converges to the mean by the strong law of large numbers for non-identically distributed variables [13]. Thus, for growing N,

$$\begin{aligned}\frac{\hat{\mathbf{h}}_k^H \mathbf{\Lambda}^{-\frac{1}{2}}(\mathbf{I}-\check{\mathbf{M}})\mathbf{\Lambda}^{-\frac{1}{2}}\hat{\mathbf{h}}_k}{N} &\approx \frac{\mathbb{E}\left[\hat{\mathbf{h}}_k^H \mathbf{\Lambda}\hat{\mathbf{h}}_k\right]}{N} \\ &= \frac{\text{tr}(\mathbf{P}_k \mathbf{\Lambda})}{N}, \end{aligned} \quad (19)$$

with $\mathbf{P}_k = \text{diag}\{P_{1k}, P_{2k} \ldots, P_{Nk}\}$ containing power values for user k at all receiving antennas and $\text{tr}(\cdot)$ denoting the trace of a matrix. Thus, the convergence of the denominator to a non-zero constant is guaranteed for any sequence of $\mathbf{P}_k$ matrices with finite power values.

The numerator in (17) is more complex as $\mathbf{\Xi}$ is non-Hermitian and the dependencies between entries prevent us from applying the strong law of large numbers. Therefore we relax the convergence criteria to mean-square. Whilst not shown here due to page limitations, using lengthy but straightforward calculations we can show that the expected value of the numerator's squared magnitude converges to zero for $N \to \infty$. Thus, in a mean-square sense,

$$\frac{\hat{\mathbf{h}}_k^H \mathbf{\Lambda}^{-\frac{1}{2}}\mathbf{\Xi}\mathbf{\Lambda}^{-\frac{1}{2}}\hat{\mathbf{h}}_k}{N} \to 0. \quad (20)$$

Using (19) and (20) we can now claim that, for large N, the numerator of Ψ converges to zero while the denominator converges to a finite constant. Hence, we conclude that

$$0 < |\Psi| \ll 1, \quad (21)$$

from which we finally approximate (14), for $K \ll N$, as

$$\begin{aligned}\gamma_k &\approx \hat{\mathbf{h}}_k^H \mathbf{\Lambda}^{-\frac{1}{2}}(\mathbf{I}-\check{\mathbf{M}})\mathbf{\Lambda}^{-\frac{1}{2}}\hat{\mathbf{h}}_k \\ &= \frac{N-K+1}{N}\,\hat{\mathbf{h}}_k^H \mathbf{\Lambda}^{-1}\hat{\mathbf{h}}_k. \end{aligned} \quad (22)$$

The expression in (22) provides a very accurate approximation for the received SINR of an arbitrary in-cluster user in a distributed massive MIMO scenario, with channel estimation errors and out-of-cluster interference included.

IV. Spectral efficiency Analysis

In this section, we derive new expressions for the spectral efficiency of an arbitrary in-cluster user. Considering a Gaussian distribution for the interference-plus-noise, the spectral efficiency of user k can be written as

$$R_k = (1-\alpha)\int_0^\infty \log_2(1+\xi)\, f_{\gamma_k}(\xi)\, d\xi \quad (23)$$

where $\alpha = \tilde{T}/L$ and $f_{\gamma_k}(\cdot)$ is the PDF of γ_k, which can be written in terms of the CF $\phi_{\gamma_k}(\cdot)$ as

$$f_{\gamma_k}(\xi) = \frac{1}{2\pi}\int_{-\infty}^{\infty} \phi_{\gamma_k}(t)\, e^{-jt\xi}\, dt. \quad (24)$$

Based on (22), the CF of γ_k is

$$\begin{aligned}\phi_{\gamma_k}(t) &= \mathbb{E}\left[e^{jt\gamma_k}\right] \\ &\approx \mathbb{E}\left[e^{jtC\,\hat{\mathbf{h}}_k^H \mathbf{\Lambda}^{-1}\hat{\mathbf{h}}_k}\right], \end{aligned} \quad (25)$$

where $C = \frac{N-K+1}{N}$ and the expectation is over the entries of $\hat{\mathbf{h}}_k = [h_{1k}, \cdots, h_{Nk}]^{\mathrm{T}}$. As the channel estimates from user k to each receiving antenna are independent and Gaussian, the joint PDF of the channel gains in $\hat{\mathbf{h}}_k$ can be written as

$$\begin{aligned}f(\hat{\mathbf{h}}_k) &= \prod_{n=1}^{N} f(\hat{h}_{nk}) \\ &= \frac{1}{\pi^N \det(\mathbf{G_k})} e^{-\hat{\mathbf{h}}_k^H \mathbf{G}_k^{-1}\hat{\mathbf{h}}_k} \end{aligned} \quad (26)$$

where $\mathbf{G}_k = \mathbb{E}\left[\hat{\mathbf{h}}_k \hat{\mathbf{h}}_k^H\right]$. Substituting (26) into (25) we can write the CF as in (27), at the top of next page, where $d\hat{h}_{nk} = d\Re(\hat{h}_{nk})d\Im(\hat{h}_{nk})$. The multi-fold integral in (27) can be solved using the identity in [14, Lemma 02], giving

$$\phi_{\gamma_k}(t) \approx \frac{1}{\det\left(\mathbf{I} - jtC\mathbf{\Lambda}^{-1}\mathbf{G}_k\right)} \quad (28)$$

$$= \frac{1}{\prod_{n=1}^{N}(1 - jtC\vartheta_{nk})} \quad (29)$$

with

$$\vartheta_{nk} = \frac{P_{nk}\|\mathbf{s}_{\mathrm{p}}\|^2}{(P_{nk}\|\mathbf{s}_{\mathrm{p}}\|^2 + \sigma_n^2)\Lambda_n} \quad (30)$$

denoting the nth diagonal entry of $\mathbf{\Lambda}^{-1}\mathbf{G}_k$. Since $\phi_{\gamma_k}(0) = 1$, the CF produces a valid PDF after inversion [15].

For the special case of fully distributed antennas with distinct received SINRs, i.e., $\varphi_n \neq \varphi_\ell \forall\ n \neq \ell$, we can apply partial fraction decomposition and re-express (28) as

$$\phi_{\gamma_k}(t) \approx \left[\prod_{n=1}^{N}\varphi_n\right]\sum_{n=1}^{N}\frac{\eta_{nk}}{(\varphi_n - jt)} \quad (31)$$

where $1/\varphi_n = C\vartheta_{nk}$ and $1/\eta_{nk} = \prod_{l\neq n}^{N}(\varphi_\ell - \varphi_n)$. Plugging (31) into (24) and solving the resulting integral via [16, Eq. 7, 3.382], we obtain the approximate expression

$$f_{\gamma_k}(\xi) \approx \left[\prod_{n=1}^{N}\varphi_n\right]\sum_{n=1}^{N}\eta_{nk}\, e^{-\xi\varphi_n}. \quad (32)$$

Substituting (32) into (23) and solving the resulting integral by means of [16, Eq. 2, 4.337], we derive the spectral efficiency of user k as

$$R_k \approx \left[(1-\alpha)\prod_{n=1}^{N}\frac{\varphi_n}{\log_e 2}\right]\sum_{n=1}^{N} -\frac{\eta_{nk}}{\varphi_n} e^{\varphi_n}\,\mathtt{Ei}(-\varphi_n), \quad (33)$$

where $\mathtt{Ei}(-\varphi_n) = -\int_{\varphi_n}^{\infty}\frac{e^{-x}}{x}dx$ is the exponential integral function. Since $\mathtt{Ei}(\cdot)$ is easily computable, (33) is more computationally efficient than the numerical integration in (23). Moreover, it is very accurate in a fully distributed massive MIMO network.

$$\phi_{\gamma_k}(t) \approx \frac{1}{\pi^N \det(\mathbf{G_k})} \int_{-\infty}^{\infty} \cdots \int_{-\infty}^{\infty} e^{-\hat{\mathbf{h}}_k^H (\mathbf{G}_k^{-1} - jtC\mathbf{\Lambda}^{-1})\hat{\mathbf{h}}_k} \, d\hat{h}_{1k} d\hat{h}_{2k} \dots d\hat{h}_{Nk} \tag{27}$$

Conversely, for colocated antennas with identical received SINRs, i.e., $\vartheta_{nk} = \vartheta_{\ell k} \forall\ n \neq \ell$, we can simplify (28) as

$$\phi_{\gamma_k}(t) = \frac{1}{(1 - jtC\vartheta_k)^N} \tag{34}$$

where $\vartheta_k = \vartheta_{1k} = \ldots = \vartheta_{Nk}$. Substituting (34) into (24) and solving the resulting integral results in

$$f_{\gamma_k}(\xi) \approx \frac{\xi^{N-1} e^{-\xi/C\vartheta_k}}{(N-1)!(C\vartheta_k)^N}. \tag{35}$$

Plugging (35) into (23) and solving the resulting integral using [17] we derive the spectral efficiency of user k as

$$R_k \approx (1 - \alpha)\left[\mathtt{Ei}(\varphi_k) e^{\varphi_k} \zeta + \tau\right] \tag{36}$$

where

$$\zeta = \log_2(e) \sum_{n=0}^{N-1} \frac{(-1)^{N-n-1}}{(N-n-1)!} \tag{37}$$

and

$$\tau = \log_2(e) \sum_{n=0}^{N-1} \frac{(-1)^{N-n-1}}{(N-n-1)!} \sum_{\ell=1}^{n} \frac{1}{\ell} \sum_{r=0}^{\ell-1} \frac{\varphi_k^r}{r!}. \tag{38}$$

Eq. (36) very closely approximates the spectral efficiency of an in-cluster user in a colocated massive MIMO network. This provides a computationally efficient alternative to [18, Eq. 13], derived for a colocated MIMO network with perfect CSI.

For the general case with sets of equal received SINRs, Jensen's inequality provides an alternative expression for R_k. As $\log_2(1+x)$ is a concave function of x,

$$R_k \leq (1 - \alpha) \log_2\left(1 + \mathbb{E}[\gamma_k]\right) \tag{39}$$

whose right-hand side is approached as N grows large. Inserting the expected value of (22) into (39) results in the closed-form expression

$$R_k \approx (1 - \alpha) \log_2(1 + C\Theta) \tag{40}$$

where $\Theta = \sum_{n=1}^{N} \frac{P_{nk}^2 T}{(P_{nk}T + \sigma_n^2)\Lambda_n}$.

V. Numerical Results

In this section, we present numerical examples to confirm the accuracy of our new approximations on the received SINR and the spectral efficiency. We consider a cooperating cluster consisting of 7 hexagonal cells, each 1 km in radius. Users are uniformly located across the network with 2 users in each cell, i.e., $K = 14$. Considering interference from the users in the immediate outer ring of cells, we set $\bar{K} = 24$. Each cell features an antenna array at its center and the distributed massive MIMO network is formed through cooperation across cells. There is log-normal shadow fading with a standard deviation of 8 dB and the path loss exponent is 4.

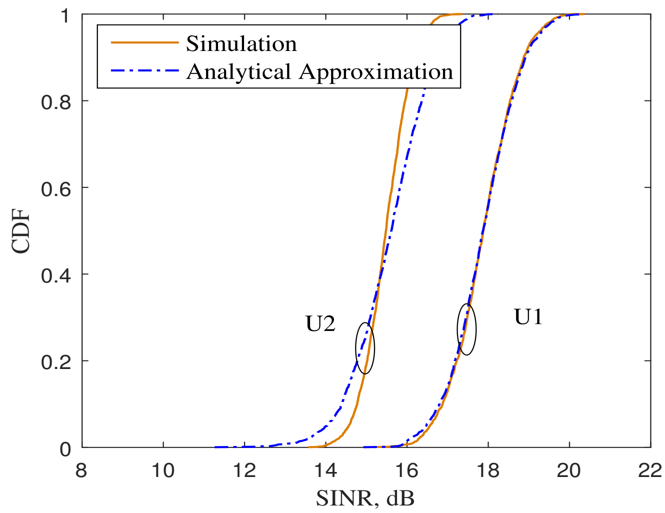

Fig. 1. Approximated and simulated instantaneous received SINR CDFs for distributed massive MIMO.

In Fig. 1, we consider 20 antennas at each base station such that $N = 140$. We plot the CDFs of the approximate received SINR in (22) alongside those of the exact SINR in (10) simulated for two users, U1 and U2, located at the cluster center and the cluster edge respectively. A sequence of $T = 20$ training symbols is utilized to estimate the fading during each coherence interval $L = 1000$. The figure shows that our SINR approximation accurately predicts the exact values for U1, located at the cluster center. Due to the asymmetric power profile experienced by U2, located at the cluster edge, its CDFs are slightly lose at the tails. However, the mean of the SINR is accurately preserved for both users.

In Fig. 2, we plot the spectral efficiency of an arbitrary user, located towards the cluster center, vs the average received SINR for the same cooperating cluster of 7 cells. We consider four different scenarios by varying the number of antennas at each base station as 4, 10, 20 and 40, such that the ratio between the number of antennas and users is 2:1, 5:1, 10:1 and 20:1, respectively. When the number of antennas is large, our approximate spectral efficiency results, generated using (40), very accurately predicts the values generated via Monte Carlo simulation throughout the full range of SINRs. Even when the ratio between the number of antennas and users is as small as 2:1, the gap between the analytical approximation and the simulation is modest. For ratios of 10:1 or higher, the gap essentially vanishes. We also observe how out-of-cluster interference gives rise to a saturation regime at high SINRs where further increases in transmit power do not noticeably improve the spectral efficiency.

The foregoing plots were generated for one snapshot of user locations. Next, in Fig. 3, we consider the spectral efficiency of

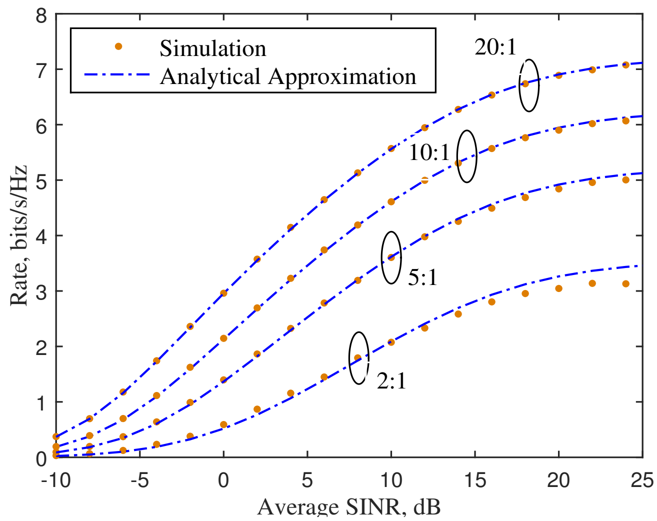

Fig. 2. Spectral efficiency of an in-cluster user versus average received SINR for different number of base station antennas.

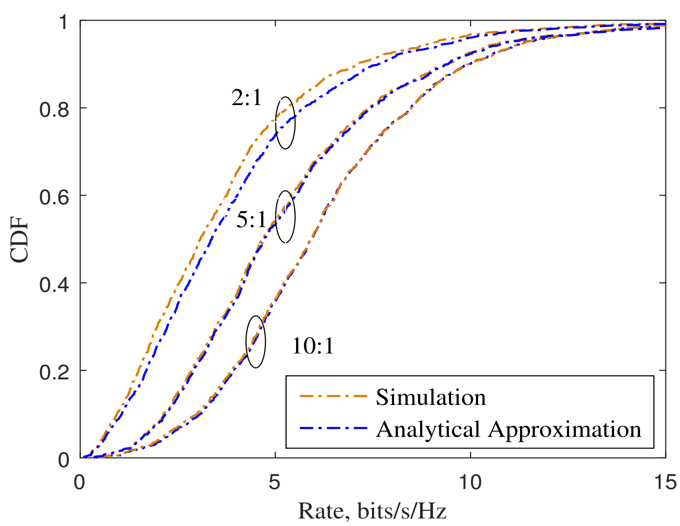

Fig. 3. Approximated and simulated spectral efficiency CDFs for different numbers of base station antennas.

distributed massive MIMO averaged over the user locations. We numerically generate the spectral efficiency CDF of an arbitrary in-cluster user by considering 10^6 snapshots, in each of which both in-cluster and out-of-cluster users are uniformly located throughout the network with two users in each cell. The transmit powers are such that the average received SINR of the strongest in-cluster user is 10 dB. We consider three different scenarios by varying the number of antennas per base station as 4, 10, and 20, such that the ratio between the number of antennas and users is 2:1, 5:1, and 10:1, respectively. The agreement with our analytical approximations is satisfactory for a ratio as low as 2:1 and excellent for higher ratios.

VI. Conclusion

The performance of a distributed massive MIMO network with a clustered structure was analyzed with channel estimation errors and out-of-cluster interference taken into account. For ZF reception, a new closed-form expression was derived that accurately approximates the instantaneous SINR of an arbitrary in-cluster user. New expressions were also derived for the spectral efficiency when the antennas are fully or partially distributed. Extensive numerical examples were used to illustrate the accuracy of our approximations. We expect the obtained expressions to facilitate performance assessments over large networks.

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