Universitat Pompeu Fabra Barcelona

Department of Economics and Business

Economics Working Paper Series

Working Paper No. 1877

Centralized vs. descentralized markets: The role of connectivity

Simone Alfarano, Albert Banal-Estañol, Eva Camacho, Giulia Iori, Burcu Kapar, and Rohit Rahi

January 2024

Centralized vs Decentralized Markets: The Role of Connectivity

Simone Alfarano* Albert Banal-Estañol† Eva Camacho*
Giulia Iori‡ Burcu Kapar§ Rohit Rahi¶

January 14, 2024

Abstract

We consider a setting in which privately informed agents are located in a network and trade a risky asset with other agents with whom they are directly connected. We compare the performance, both theoretically and experimentally, of a complete network (centralized market) to incomplete networks with differing levels of connectivity (decentralized markets). We show that decentralized markets can deliver higher informational efficiency, with prices closer to fundamentals, as well as higher welfare for mean-variance investors.

Keywords: *Networks, heuristic learning, informational efficiency, experimental asset markets.*
JEL Classification: *C92, D82, G14.*

*Department of Economics, Universitat Jaume I, Castelló de la Plana, Spain.
†Department of Economics and Business, Universitat Pompeu Fabra, Barcelona, Spain; Barcelona School of Economics; Barcelona School of Management; and Department of Economics, City, University of London, UK.
‡Corresponding author: Department of Economics, Universita' Ca' Foscari, 30123 Venezia, Italy; and Department of Economics, City, University of London, Northampton Square, London EC1V 0HB, UK. Email: g.iori@city.ac.uk.
§School of Business, University of Wollongong, Dubai, UAE.
¶Department of Finance, London School of Economics, London, UK.

1 Introduction

In the classical paradigm of financial markets, all buyers can freely trade with all sellers, and all of their trades clear via a centralized market mechanism that matches demand and supply. In reality, though, many markets are decentralized and fragmented, whereby not all traders are fully "connected" to each other. Fragmentation may occur because of information or transaction costs, which limit the ability or willingness of some market participants to interact and trade with others.

Examples of decentralized markets include over-the-counter (OTC) markets in which many financial assets (e.g., foreign exchange, interest-rate and other derivatives, municipal and corporate bonds, bank loans, private equity and real estate) and most durable goods are traded. In an OTC financial market, investors trade directly with a limited set of other investors and negotiate terms privately, often facing incomplete information about the prices effective elsewhere in the market. Investors may buy or sell from dealers, thus improving overall market connectivity, but each customer typically approaches only a subset of the dealers in the market. Dealers also trade with each other to balance inventory and meet liquidity needs, but like customers, each dealer only interacts with a subset of other dealers.

Financial regulators have voiced increasingly serious concerns about (the size of) OTC markets, and the potential risks to the overall financial system that they create. Following the disruption of several OTC markets during the financial crisis of 2008 (e.g., credit derivatives, asset-backed securities, and repo agreements), many commentators and policy makers argued that decentralized trading was to blame for the inability of traders to quickly identify the prices of some assets, increasing the severity of the crisis. In response, the Group of Twenty (G20) committed to a variety of far-reaching regulatory reforms to contain the threat posed by OTC trading to the financial system, including a move to trading securities on centralized, fully connected exchanges, where other financial assets like stocks are traded (G20 (2009)).

The main argument against decentralized markets is their presumed inability to aggregate and reveal privately dispersed information sufficiently quickly. The common wisdom is that such impediments to price discovery, combined with lack of transparency and limited opportunities for exploiting gains from trade, make investors worse off.

In this paper, we show that decentralized markets can outperform centralized markets in terms of both informational efficiency and welfare. We propose a model of asset trading in a network. A centralized market is one in which the network is complete, or fully connected, wherein each trader has a link with every other trader. A decentralized market corresponds to an incomplete network, in which some of the potential links are missing.[1] Our model

[1]Networks are a natural tool to model decentralized trade and intermediation. In decentralized markets,

features common values and asymmetric information. With fully rational agents, this setup would lead to full revelation of private information and very little trade. We assume instead, following Pederson (2022), that some agents do not update their beliefs while others use a heuristic updating rule. More specifically, informed agents, who have either positive or negative information about the value of the asset, do not update. Uninformed agents, in contrast, update by repeatedly averaging their own beliefs with those of their neighbors in the network.

We show that a decentralized market, corresponding to an incomplete network such as the star, core-periphery network, or random network with some missing links, is more informationally efficient than a centralized market, corresponding to a fully connected network. Furthermore, decentralization lowers the variance of payoffs which, under the assumption of mean-variance preferences, translates into higher welfare. These results continue to hold, at least in the short term, even when all agents engage in heuristic learning.

The intuition is as follows. In a setting with noisy private information, some agents are misinformed about the asset payoff. Even if these agents are very few in number compared to other agents who have accurate information or are uninformed, they can exert a disproportionate influence on prices as well as on the variance of payoffs. The reason is that agents, naturally, seek trading opportunities that present high perceived gains from trade, and these are precisely those opportunities that involve agents with "extreme" beliefs. A complete network gives agents access to counterparties with the full spectrum of beliefs, including (extreme) beliefs that are incorrect. Instead, an incomplete network precludes trades with some agents, and especially with misinformed agents who are in a minority. This leads to fewer trades with misinformed agents, pushing prices closer to the fundamental, and reducing the variance of payoffs.

We provide experimental evidence in support of the theoretical results. Experimental methods are particularly suitable for testing the performance of decentralized markets, as they can incorporate some of the inherent complexities of such markets. Field work is limited in this task due to the unobservability of crucial market variables, such as the fundamental value of financial assets and the distribution of information among traders. The experimental approach allows us to exogenously control traders' private information and trading partners, while informational efficiency and payoffs can be directly measured.

stable buyer-seller trading relationships are usually formed. These relationship can be described by a graph or network, in which the nodes correspond to traders and the edges represent potential (or realized) trading relationships between pairs. When the network is complete, every possible trading opportunity is present and therefore there is no constraint on trading patterns. In an incomplete network, some traders are unable to trade with each other. This implies either a loss of trading opportunities or the need for an intermediary between traders who are not directly linked to each other.

The information structure in our experiments mimics that of the model. Subjects can buy and/or sell an asset via a personalized limit order book, in which they can see, and match, the bids and asks of the traders they are linked to in the trading network. This design feature, used in reality in some foreign exchange markets, allows us to change the connectivity of the market, spanning from fully connected centralized markets to decentralized markets with different levels of connectivity, without changing the price formation mechanism.[2] We find that, despite having lower trading volume, decentralized markets deliver both higher informational efficiency and higher welfare (lower variance of payoffs).

In addition to offering support for the predictions of the model, the experiments provide a number of insights into trading behavior. While agents tend to trade in the direction that their private information suggests, as in the model, this is much more pronounced in centralized markets. In addition, in centralized markets, correctly informed agents trade faster (in the direction that is indicated by their private information), and thus benefit further from prices that are further away from fundamentals. These are the channels through which centralization leads to higher payoff differentials across agents, and hence higher welfare ex ante. Interestingly, the primary determinant of the payoff of an agent is her private information, not the number of links she has in the network.

In sum, our paper contributes to the literature, which we review in the next section, by showing that limited connectivity may not lead to inferior performance. In fact, fewer connections can result in prices that better reflect fundamentals, and payoffs that are less variable, thus making traders better off. We first put forward a novel mechanism through which restrictions on trading opportunities can increase informational efficiency and welfare. We then provide experimental evidence corroborating the results of the theoretical model.

2 Literature Review

2.1 Theory

There is a growing literature on trading in networks; see, for example, Kranton and Minehart (2001), Gale and Kariv (2007), Gofman (2014), Condorelli et al. (2017) and Babus and Kondor (2018), all of which feature agents with private (possibly correlated) values. With the exception of Babus and Kondor (2018), these papers do not study informational efficiency. In Babus and Kondor (2018), prices are generally not constrained informationally efficient if the network is incomplete, but full revelation is obtained in the common values limit. They

[2]Several platforms in the foreign exchange spot market use a similar arrangement, or quasi-centralized limit order book structure, including Reuters, EBS, and Hotspot FX, which together facilitate a mean turnover in excess of $0.6 trillion each day (Gould et al. (2017)).

do not analyze the effect of connectivity on informational efficiency or welfare as we do in this paper.

Instead of Bayesian learning as in Babus and Kondor (2018), we assume that learning is heuristic as in Pederson (2022), which builds on the repeated linear updating model of DeGroot (1974). In Pederson (2022) beliefs evolve independently of trades and prices.[3] Agents have common values and are either "hardheaded" and do not update their beliefs[4] (after some point), or "naive" and update by computing an average of their own belief and those of a subset of other agents. The beliefs of a naive agent, assuming that she is influenced directly or indirectly by one or more hardheaded agents, converge to a convex combination of hardheaded views. Other papers that employ DeGroot learning include DeMarzo et al. (2003), who provide a rationale for this type of learning as a behavioral heuristic, and Golub and Jackson (2010). Experimental evidence in support of DeGroot learning can be found in Chandrasekhar et al. (2020).

There are other papers that study asset trading by agents who learn through communication in a social network (Colla and Mele (2010), Ozsoylev and Walden (2011), Han and Yang (2013)). In contrast to our setup, trading in these models is in a centralized market.

In this paper we view a decentralized market as an incomplete network. There is a large literature that takes a different approach, modeling decentralization as pairwise trading with random matching. Models that feature asymmetric information and learning include Wolinsky (1990) and Blouin and Serrano (2001), where equilibrium is not fully revealing, and Duffie and Manso (2007) and Golosov et al. (2014), where there is full revelation.

Our result that decentralized markets can lead to higher informational efficiency and welfare complements other possible advantages of decentralization that have been identified in the literature. Malamud and Rostek (2017), Chen and Duffie (2021) and Rostek and Yoon (2021) model decentralized markets as multiple exchanges on which imperfectly competitive and symmetrically informed agents trade. They find that multiple exchanges can lead to a more efficient allocation of risk compared to a single exchange. Glode and Opp (2017) show that trading via a chain of intermediaries can improve efficiency by reducing the information asymmetry between direct counterparties.

2.2 Experimental Work

A broad experimental literature studies market efficiency in the presence of asymmetric information (in centralized markets). Mirroring the theoretical work, this literature has in-

[3]Beliefs can be calculated independently of trades and prices in Babus and Kondor (2018) as well.

[4]Another model in which some agents do not update their beliefs, or update them slowly, is Eyster et al. (2019). Unlike Pederson (2022), other agents in this model use Bayesian updating.

corporated information asymmetries in two principal ways. The first is to provide some (but not all) traders with perfect information. Early studies in this tradition indicate that markets have a strong tendency to disseminate private information (Plott and Sunder (1982), Sunder (1992)). The alternative modeling choice is to provide all traders with heterogeneous information. When private information is imperfect, experiments have shown that information dissemination may be slow (Plott and Sunder (1988), Forsythe and Lundholm (1990), Lundholm (1991)). Later studies have confirmed that information dispersed in the market is only imperfectly incorporated into experimental prices (Biais et al. (2005), Hanson et al. (2006), Veiga and Vorsatz (2010), Corgnet et al. (2015, 2023)).

Bossaerts et al. (2014) compare the predictions of competitive and strategic rational expectations models. Their results support the latter in the dynamics and the former in the final prices. Filippin and Mantovani (2023) find that prices do not vary with risk aversion and are close to the risk-neutral benchmark. Ruiz-Buforn et al. (2021a,b) show that the disclosure of incorrect public information can drive prices far from fundamentals, even when correct private information available in the market is sufficient to offset misleading public information. This is consistent with one of the results that we obtain in this paper.

Some papers in the literature share a few design features with ours. Page and Siemroth (2021) use an information structure and trading mechanism that are similar to those in our experiments. They quantify the extent to which prices reflect private or public information in centralized markets. The objectives of our paper are different — we do not focus on measuring informational (in)efficiency per se but on comparing it across markets with different levels of connectivity.

Cason and Friedman (2008) compare the performance of the continuous double auction (CDA) trading mechanism, which we use in this paper, with the uniform price double auction (UPDA), the single call market (SCM), and the multiple call market (MCM). They show that while allocative efficiency is higher in CDA and MCM sessions, mean prices deviate the least from the competitive equilibrium prediction in the SCM. This suggests that an increase in transaction opportunities afforded by some market arrangements does not necessarily lead to higher informational efficiency. Our results, using a different approach, are in line with this evidence. Other papers that compare alternative trading mechanisms include Schnitzlein (1996), Theissen (2000) and Attanasi et al. (2016).

Halim et al. (2019) study incentives to acquire costly information in a setting where agents learn from their neighbors in a social communication network. They show that information sharing increases trading volume. While information aggregation improves with the density of the information network, prices are actually less informative. This is because fewer signals are purchased as a result of the free riding incentive provided by the information sharing mechanism. The crucial difference of their setup in comparison to ours is the role of the network. In their experimental design, the network affects learning but has no impact on trading opportunities. The asset market itself is centralized. In our setting, the network affects learning as well, but of greater importance is how it constrains trading behavior.

Going beyond the study of centralized markets, as in the papers cited above, Asparouhova and Bossaerts (2017) investigate the efficiency properties of OTC markets which involve bilateral exchanges with transaction prices known only to the trading counterparties. They find that, while prices in the OTC market fluctuate within a narrow band around the rational expectations equilibrium benchmark, prices in the centralized market are closer to this benchmark. Thus the OTC market is somewhat worse than the centralized market in terms of informational efficiency. But welfare is higher in the OTC market because revelation of information in the centralized market destroys risk-sharing opportunities. The main differences between our paper and theirs are twofold. First, trading in our setting is primarily driven by heterogeneity in beliefs, while in theirs it is heterogeneity in endowments. Second, we study the effects of different degrees of market connectivity, while they compare bilateral exchanges with centralized trading.

3 Theory

We now introduce the model. Our goal is to construct a model of asset trading in a network, where trades are motivated by differences in information, rather than by heterogeneous values (or noise trade) or differences in endowments or risk aversion. It is well-known that if agents have rational expectations, then trading cannot be sustained by differences in information alone (Milgrom and Stokey (1982)). In Babus and Kondor (2018), where agents with heterogeneous values engage in bilateral trading in a network, there is no trade in the common values limit of the model, for any network.

Instead of learning from prices and Bayesian updating, we propose a model in which belief formation is as in Pederson (2022). In particular, we assume that informed agents (whose private information may or may not be correct ex post) do not update their beliefs, while uninformed agents follow a heuristic updating rule that involves repeatedly averaging their previous beliefs with those of their neighbors (see the literature review in Section 2.1 for further discussion of heuristic learning).

After presenting the model, we analyze the evolution of beliefs and prices in the complete network and the star network. We then compare the performance of these two networks, in terms of both informational efficiency and welfare. Finally, we discuss the assumptions of the model and possible extensions.

3.1 A Model of Trading in a Network

3.1.1 Main features

The model features a single risky asset with random payoff V that takes value V_H or V_L ($V_H > V_L$) with equal probability. There are N agents, each of whom receives two signals prior to trade. Each signal s_i takes value V_H or V_L, and satisfies

$$\text{Prob}(V = V_H | s_i = V_H) = \text{Prob}(V = V_L | s_i = V_L) = q, \tag{1}$$

where $q \in (1/2, 1)$. The signals $\{s_i\}_{i=1}^{2N}$ are independent conditional on V. Let N_H be the number of agents who receive two high signals, N_L the number of agents who receive two low signals, and N_U the number of agents who receive conflicting signals. We refer to these agents as *optimists* (or H agents), *pessimists* (or L agents), and *uninformed agents* (or U agents), respectively. The labeling of agents who receive conflicting signals as "uninformed" is due to the following result, which says that conflicting signals are equivalent to no signal at all (all proofs are in the Appendix):

Lemma 3.1 (Conflicting signals) *The probability distribution of V conditional on two conflicting signals is the same as the unconditional distribution, i.e. Prob$(V = V_H | s_1 = V_H, s_2 = V_L) =$ Prob$(V = V_H) = 1/2$.*

The conditional distribution of the asset payoff for optimists or pessimists is not the same as the unconditional distribution. Accordingly, we refer to these agents collectively as *informed agents*. We use the symbols N_H, N_L and N_U to also denote the index sets of H, L and U agents, respectively. Thus, for example, agent $i \in N_U$ is an uninformed agent. We assume that there is at least one agent of each type, and that there are more "correctly informed agents" than "misinformed agents", i.e. $N_H > N_L$ if $V = V_H$, and $N_H < N_L$ if $V = V_L$.[5]

Agents are linked to each other through a network. For tractability we adopt a discrete-time framework, with dates $t = 0, 1, 2 \ldots$, along which beliefs and prices evolve. We now discuss how we model beliefs, and then turn to trades and prices.

3.1.2 Beliefs

The belief of an agent at any date is the value that she assigns to the asset at that date. We assume that optimists value the asset at V_H, and pessimists value the asset at V_L, at all

[5] Given the signal-generating mechanism, this assumption is likely to be satisfied. Conditional on $V = V_H$, the expected number of optimists, pessimists and uninformed agents is q^2N, $(1-q)^2N$ and $2q(1-q)N$, respectively. If $q = 0.8$ and $N = 15$, these numbers are 9.6, 0.6 and 4.8.

dates. Thus optimists and pessimists do not update their beliefs. We choose their beliefs to be V_H and V_L respectively for convenience; the analysis is unaffected if the belief of optimists is $\mathbb{E}(V|s_1 = s_2 = V_H)$, and that of pessimists is $\mathbb{E}(V|s_1 = s_2 = V_L)$.[6] We denote the belief of an uninformed agent $i \in N_U$ at date t by $V_U^i(t)$. We assume that initially all uninformed agents have the same belief, given by $V_U^i(0) = (V_H + V_L)/2$ for all i. This is motivated by Lemma 3.1, though we do not assume Bayesian updating. After date 0, each uninformed agent updates her belief by averaging her own belief with the beliefs of each of her neighbors in the previous period. We will make this precise below. The setup for belief formation that we have described follows Pederson (2022). In Pederson's terminology, informed agents, both optimists and pessimists, are "fanatics", while uninformed agents are "naive".

We order the agents so that the uninformed (indexed by U) come first, followed by optimists (indexed by H), and then by pessimists (indexed by L). The evolution of beliefs of all agents is determined by an $N \times N$ adjacency matrix A, which is a nonnegative matrix with all row sums equal to one. The vector of beliefs at date t, for $t = 1, 2, \ldots$, is given by A times the vector of beliefs at date $t - 1$. The adjacency matrix takes the form

$$A = \begin{pmatrix} A_U & A_H & A_L \\ 0 & I & 0 \\ 0 & 0 & I \end{pmatrix}, \tag{2}$$

where A_U is $N_U \times N_U$, A_H is $N_U \times N_H$, and A_L is $N_U \times N_L$. The matrix A defines a "listening structure" in the language of DeMarzo et al. (2003); the ij'th element of A is the weight that agent i puts on the belief of agent j. An uninformed agent listens to herself as well as to all her neighbors in the network, and puts equal weight on her own belief and that of each of her neighbors. An informed agent listens only to herself, and thus puts a weight equal to one on her own belief. We assume that any uninformed agent is influenced by at least one informed agent, either directly or indirectly through other agents in the network.

In order to generate dispersion in beliefs, some restriction on learning is necessary. If informed agents learn as well, all beliefs converge to a consensus belief (DeMarzo et al. (2003)). In fact, if informed agents put equal weight on their own beliefs and those of their neighbors (just like the uninformed), and the network is complete, it takes only one round of updating to reach a consensus belief. It is just the equally weighted average of all initial beliefs. Later we will allow for learning by informed agents.

Since informed agents do not update their beliefs, we can focus on just the beliefs of uninformed agents. Let $\tilde{V}_U(t) = (V_U^i(t))_{i \in N_U}$ be the vector of beliefs of uninformed agents

[6]The assumption that optimists and pessimists do not update their beliefs is more persuasive the higher is q. For $q = 0.8$, $\text{Prob}(V = V_H|s_1 = s_2 = V_H) = \text{Prob}(V = V_L|s_1 = s_2 = V_L) = 0.94$.

at date t. Also, let $V_U^i(\infty) := \lim_{t\to\infty} V_U^i(t)$, the limiting belief of uninformed agent i, and $\tilde{V}_U(\infty) := (V_U^i(\infty))_{i\in N_U}$, the corresponding limiting belief vector. Let $\mathbf{1}_m$ be an m-vector of ones. Applying Proposition 1 in Pederson (2022), we obtain the following result:

Proposition 3.2 (General network: Beliefs) *The beliefs of uninformed agents for $t \geq 1$ are given by*

$$\tilde{V}_U(t) = A_U^t \mathbf{1}_{N_U} \frac{V_H + V_L}{2} + \sum_{k=0}^{t-1} A_U^k (A_H \mathbf{1}_{N_H} V_H + A_L \mathbf{1}_{N_L} V_L), \tag{3}$$

and

$$\tilde{V}_U(\infty) := \lim_{t\to\infty} \tilde{V}_U(t) = (I - A_U)^{-1}(A_H \mathbf{1}_{N_H} V_H + A_L \mathbf{1}_{N_L} V_L). \tag{4}$$

Thus the belief of an uninformed agent converges to a convex combination of V_H and V_L, not necessarily the same for all uninformed agents.

3.1.3 Trades and prices

Beliefs evolve independently of trades and prices, just as in Pederson (2022). However, unlike Pederson (2022), where trading is Walrasian, we assume that agents arrive sequentially and one unit is traded at each date, similar to the trading environment in Glosten and Milgrom (1985). At each time t. $t = 1, 2, \ldots$, one agent is chosen at random and she trades with one of her neighbors. The neighbor she chooses to trade with is the one whose belief is furthest away from her own belief, thus maximizing the gains from trade. In the case of ties, she chooses her trading partner at random. Each trade involves the exchange of one unit of the asset, with the buyer being the agent who values the asset more. There are no short-sale constraints. Once the agent chosen to trade at t has traded, she returns to the pool of all agents and may be chosen again in the future. The probability of an agent of a particular type being chosen to trade is equal to the proportion of agents of that type in the population. Thus an H (resp. L, U) agent is chosen with probability N_H/N (resp. N_L/N, N_U/N).

We assume that any agent who is chosen to trade extracts a fraction $\alpha \in (0, 1]$ of the surplus (the difference between her belief and that of her counterparty).[7] Let p_{XY} be the price that arises when X is chosen to trade and she trades with Y (where X and Y are H, L or U). We have $p_{XY} = (1 - \alpha)V_X + \alpha V_Y$, where V_X and V_Y are the beliefs of X and Y respectively. We use this pricing rule even when the beliefs of the two agents are the same. This is motivated by the fact that there would be gains from trade if the two agents differ slightly in their endowments or risk aversion, and any split of these gains would imply a price

[7] We exclude $\alpha = 0$ since this is a degenerate case where the agent chosen to trade is indifferent between not trading and trading with any of her neighbors.

close to the common belief. We can think of the price as the outcome of Nash bargaining, where α is the bargaining power of the agent chosen to trade. If $\alpha = 1/2$, both parties to the trade have the same bargaining power and they split the surplus equally. If $\alpha = 1$, the price is equal to the belief of the counterparty that the chosen agent trades with. This case describes a competitive market in which each agent quotes a price that is equal to her own belief, so that any agent chosen to trade faces a competitive bid-ask spread, and trades are executed at the best bid or ask price. If the network is complete, all agents face the same bid-ask spread.

Trades between two informed agents result in the same price regardless of the network and the date, given by

$$p_{HH} = V_H, \tag{5}$$

$$p_{LL} = V_L, \tag{6}$$

$$p_{HL} = (1-\alpha)V_H + \alpha V_L = V_H - \alpha(V_H - V_L), \tag{7}$$

$$p_{LH} = \alpha V_H + (1-\alpha)V_L = V_L + \alpha(V_H - V_L). \tag{8}$$

For a pairing involving an uninformed agent, the price depends on the network and date, and in general also on the position of the uninformed agent in the network.

From now on, we focus on the analysis of the case where $V = V_H$. Then the optimists are correctly informed, the pessimists are misinformed, and there are more optimists than pessimists. The case of $V = V_L$ is completely analogous, involving only a relabeling.

3.2 Network Structures: Beliefs and Prices

In this section we analyze the evolution of beliefs and prices in the complete network and the star, which represent the two extremes of a centralized market and a maximally decentralized market. In the complete network, every agent has a link with every other agent. In the star, there is a central agent or center who is chosen at random. The central agent has a link with all other agents, while a noncentral agent has only one link, with the center. In the next section, we will compare the performance of these two network structures.

We show in the Online Appendix that the results for the star, and for the comparison of the star with the complete network, generalize to a core-periphery network defined as follows. There are M agents in the core and the remaining $N - M$ in the periphery. An agent in the core has a link with every other agent in the core; thus the core is a complete subnetwork. In addition, each agent in the core has a link with $K := (N - M)/M$ agents in the periphery; this is the "clientele" of this core agent. All agents in the core have a clientele of the same size, and these clienteles do not overlap. An agent in the periphery has only one link, with the core agent whose clientele she belongs to.

3.2.1 Complete network

Recall that each uninformed agent puts equal weight on her own belief and the beliefs of each of her neighbors. For the complete network, this means that every element of A_U, A_H and A_L is $1/N$. We denote the belief of agent $i \in N_U$ in the complete network by $\overset{c}{V}{}_U^i$. Applying Proposition 3.2, we obtain the following result:

Proposition 3.3 (Complete network: Beliefs) *In the complete network, all uninformed agents have the same beliefs: $\overset{c}{V}{}_U^i(t) = \overset{c}{V}_U(t)$, for all $i \in N_U$, $t \geq 0$, where*

$$\overset{c}{V}_U(t) := \bar{V} - \left(\frac{N_U}{N}\right)^t \frac{N_H - N_L}{N_H + N_L} \frac{V_H - V_L}{2},$$

and

$$\bar{V} := \frac{N_H V_H + N_L V_L}{N_H + N_L}.$$

The belief $\overset{c}{V}_U(t)$ is strictly increasing in t, and $\lim_{t\to\infty} \overset{c}{V}_U(t) = \bar{V}$.

Thus the beliefs of uninformed agents get closer to the true value V_H over time, converging to the belief of the "representative informed agent", $\bar{V}$.

Let $\overset{c}{p}_t$ denote the price in the complete network at t, $t \geq 1$, and $\overset{c}{p}_\infty$ the limiting price. For $t \geq 1$, the belief of the uninformed, $\overset{c}{V}_U(t)$, is closer to V_H than to V_L. Hence, when U is chosen to trade she always trades with L. When H is chosen she trades with L, and when L is chosen she trades with H. The next proposition characterizes the expected price.

Proposition 3.4 (Complete network: Prices) *In the complete network, $\mathbb{E}(\overset{c}{p}_t)$ is strictly increasing in t for $t \geq 1$, and converges to*

$$\mathbb{E}(\overset{c}{p}_\infty) = \bar{V} - \alpha \frac{V_H - V_L}{N(N_H + N_L)} \Big[N(N_H - N_L) + N_L N_U\Big]. \quad (9)$$

The expected price gets closer to the true value of the asset over time, but remains below $\bar{V}$, the limiting average belief of all agents, at every date. Note that all trades involve L. This is the reason for the inefficiency of the complete network, in both the informational and welfare sense, as we shall show later.

3.2.2 Star network

There are three possible stars, with H, L, or U at the center, which we refer to as the H-star, the L-star and the U-star, respectively. The star is an H-star with probability N_H/N, an L-star with probability N_L/N, and a U-star with probability N_U/N.

The beliefs of an uninformed agent in the star depend on whether she is at the center or not, and in the latter case they also depend on the type of agent at the center. We denote the belief of the uninformed at date t in the H-star and L-star by $\overset{\star}{V}{}_U^H(t)$ and $\overset{\star}{V}{}_U^L(t)$, respectively. In the U-star, we denote the belief of the uninformed agent at the center by $\overset{\star}{V}{}_{\breve{U}}^U(t)$, and that of any other uninformed agent by $\overset{\star}{V}{}_U^U(t)$.

Proposition 3.5 (Star network: Beliefs) *In the star network, the beliefs of uninformed agents, for $t \geq 0$, are as follows:*

i. In the H-star,

$$\overset{\star}{V}{}_U^H(t) = V_H - \frac{1}{2^{t+1}}(V_H - V_L),$$

which is strictly increasing in t, and $\lim_{t\to\infty} \overset{\star}{V}{}_U^H(t) = V_H$.

ii. In the L-star,

$$\overset{\star}{V}{}_U^L(t) = V_L + \frac{1}{2^{t+1}}(V_H - V_L),$$

which is strictly decreasing in t, and $\lim_{t\to\infty} \overset{\star}{V}{}_U^L(t) = V_L$.

iii. In the U-star, both $\overset{\star}{V}{}_{\breve{U}}^U(t)$ and $\overset{\star}{V}{}_U^U(t)$ are increasing in t, and converge to $\bar{V}$ as $t \to \infty$.

iv. The expected belief of an uninformed agent is strictly increasing in t, and converges to $\bar{V}$ as $t \to \infty$.

If H or L is at the center, uninformed agents repeatedly average their beliefs with the unchanging beliefs of the center, converging monotonically to that unchanging belief. The expected belief of an uninformed agent, where the expectation is taken over the H-star, L-star and U-star, increases over time and converges to $\bar{V}$, just as in the complete network (see Proposition 3.3).

Next, we turn to prices in the star, which we denote by $\overset{\star}{p}$. We first calculate the expected price in each of the three possible stars, the H-star, the L-star and the U-star. Then we average these expected prices, using the probabilities N_H/N, N_L/N and N_U/N, to obtain the overall expected price in the star. In each of the three stars, the price is random and depends on whether the agent chosen to trade is at the center, and if she is not at the center, what her type is. If the agent chosen to trade is at the center, she trades in the same way as in the complete network, since she is linked to all other agents. However, if the chosen agent is not at the center, she has no choice but to trade with the center.

Proposition 3.6 (Star network: Prices) *In the star network, $\mathbb{E}(\mathring{p}_t)$ is strictly increasing in t for $t \geq 1$, and converges to*

$$\mathbb{E}(\mathring{p}_\infty) = \bar{V} - \alpha \frac{V_H - V_L}{N^2(N_H + N_L)} \left[N(N_H - N_L) + N_L N_U \right]. \tag{10}$$

Like the complete network, the star network becomes more informationally efficient over time. Even though the expected belief of an uninformed agent converges to $\bar{V}$ in both networks, the limiting expected price is not the same.

3.3 Comparison of Network Performance

In this section we use the results of Sections 3.2.1 and 3.2.2 to assess the performance of the complete network relative to the star, in terms of informational efficiency and welfare.

3.3.1 Informational efficiency

Consider a network with price process $\{p_t\}$. Let

$$\mathcal{I}_t := -|p_t - V|. \tag{11}$$

Our measure of *informational efficiency* at date t is $\mathbb{E}(\mathcal{I}_t)$. Given that our analysis is for the case in which the realized value of V is V_H, we must have $p_t \leq V_H$, so that $\mathcal{I}_t = p_t - V_H$. Hence informational efficiency at date t is

$$\mathbb{E}(\mathcal{I}_t) = \mathbb{E}(p_t) - V_H.$$

Thus a higher expected price at a given date means higher informational efficiency at that date.

We will also have occasion to consider informational efficiency averaged over a period of time, or equivalently, averaged over a number of transactions. Let

$$\mathcal{I} := \frac{1}{T} \sum_{t=1}^{T} \mathcal{I}_t. \tag{12}$$

We measure *average informational efficiency* over T transactions by $\mathbb{E}(\mathcal{I})$, which is given by

$$\mathbb{E}(\mathcal{I}) = \frac{1}{T}\sum_{t=1}^{T}[\mathbb{E}(p_t)] - V_H.$$

Now we compare the star and the complete network in terms of informational efficiency:

Proposition 3.7 (Star vs complete network: Informational efficiency)

i. The star network is asymptotically more informationally efficient than the complete network, i.e. $\mathbb{E}(\overset{\star}{p}_\infty) > \mathbb{E}((\overset{c}{p}_\infty)$.

ii. Suppose $\alpha \geq 1/2$. *Then the star network is more informationally efficient than the complete network at every date, i.e.* $\mathbb{E}(\overset{\star}{p}_t) > \mathbb{E}(\overset{c}{p}_t)$ *for all* $t \geq 1$.

Part (i) is immediate from (9) and (10). Part (ii) is a stronger result that holds under the assumption that an agent chosen to trade has at least as much bargaining power as any counterparty ($\alpha \geq 1/2$). In the complete network, the misinformed L agents have a bigger influence on the price. This is because whenever H or U is chosen to trade, she prefers to trade with L. In the star, on the other hand, if H or U is chosen and she is not at the center, she must trade with the center, even if the center is not L. This is partially offset by the case in which L is chosen to trade and she is not at the center; she must then trade with the center even if it is occupied by L or U. But the probability of L being chosen to trade is lower than that of H. This turns out to be sufficient for the expected price in the star to be higher than in the complete network.

3.3.2 Welfare

We compare welfare across networks for a static economy in which trades are based on the limiting beliefs of agents as $t \to \infty$ (later we will define welfare for a dynamic economy, and use that for welfare comparisons). We assume that agents have mean-variance preferences, with risk aversion r that is low enough to ensure that any agent is willing to buy or sell one unit when chosen to trade. We continue to assume, as above, that the realized value of V is V_H. Then, if the transaction price is p, the payoff of the buyer is $V_H - p$ and that of the seller is $p - V_H$. The price p is random and depends on the agent chosen to trade, as well as on the counterparty that this agent trades with.

We measure welfare by

$$W := \mathbb{E}(\tilde{\Pi}) - r\mathrm{Var}(\tilde{\Pi}),$$

where $\tilde{\Pi}$ is the ex ante payoff of an agent before trading begins and before she knows what her type is (H, L or U). We can think of $\tilde{\Pi}$ as being equal to the random value Π or $-\Pi$, depending on whether the agent is the buyer or the seller. These values are equiprobable since every transaction involves a buyer and a seller. Therefore, $\mathbb{E}(\tilde{\Pi}) = 0$, and

$$\operatorname{Var}(\tilde{\Pi}) = \mathbb{E}(\tilde{\Pi}^2) = \frac{1}{2}\mathbb{E}(\Pi^2) + \frac{1}{2}\mathbb{E}[(-\Pi)^2] = \mathbb{E}(\Pi^2). \tag{13}$$

Since the expected payoff is zero for any network, welfare is higher for a particular network if and only if $\operatorname{Var}(\tilde{\Pi})$ is lower.

We have the following result comparing welfare in the star and the complete network:

Proposition 3.8 (Star vs complete network: Welfare) *Consider the static limiting economy as $t \to \infty$. Suppose $\alpha \geq 1/2$ and $N_H + N_U \geq 3N_L$. Then welfare in the star is higher than in the complete network.*

The star network restricts trade with pessimists who have extreme beliefs, and this effect is stronger when there are fewer pessimists, and hence a lower probability of a pessimist being at the center of the star. Under the conditions in the proposition, the result is a lower variance of the payoff, and hence higher welfare. This is the only reason for the better performance of the star since, in the limiting economy, average beliefs are the same in both networks.

3.4 Discussion

A number of modeling choices we have made were dictated by our objective of building a tractable framework. First, we restrict agents to arrive sequentially and trade at a discrete set of dates. This means that the model cannot generate results on trading volume or the speed of trading since one unit of the asset is traded at every date regardless of the network structure. Second, we analyze simple symmetric networks, in which randomness is limited to the choice of agent at the center of the star. Real-world financial networks are not symmetric and have a much greater degree of randomness. Third, the model takes an extreme view of agent behavior; in particular, a subset of agents do not update their beliefs. Fourth, our welfare analysis is for a single-period economy.

The first point can be partially addressed as follows. Our informational efficiency result for the star is in fact stronger than that stated in Proposition 3.7. As can be seen from the proof of the proposition, we show that $\mathbb{E}(\overset{\star}{p}_t) > \mathbb{E}(\overset{c}{p}_{t'})$ for all $t, t' \geq 1$. Thus the lowest expected price in the star (across all dates) is higher than the highest expected price in the complete network.[8] It follows that, even if there are more transactions in the complete

[8] In the Online Appendix, we show that the same is true for the core-periphery network.

network over a given period of time, the final price is on average lower than in the star, and hence informational efficiency is lower as well.

In order to address the other three issues flagged above, we need to rely on numerical simulations, which we present in Section 4.1.

4 Extensions of the Model

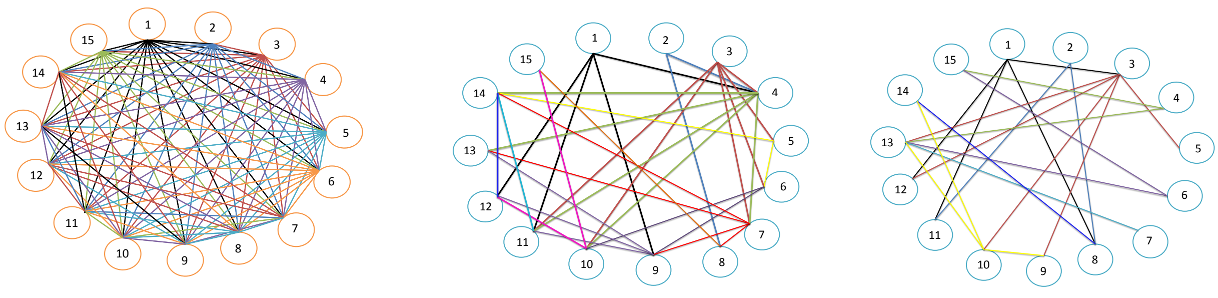
In Section 4.1, we present simulations that extend the theoretical results in several directions (albeit only for a given parameter constellation). First, we show that the higher informational efficiency of the star, relative to the complete network, generalizes to more realistic random networks with varying levels of connectivity. We show in particular that the complete network is less efficient than a "medium connected" random network, which is in turn less efficient than a "low connected" random network, which is in turn less efficient than the star network. In this sense, the star network is an extreme version of a low connected network. Second, we introduce learning by informed agents. We show that in the short run, the ranking of networks by informational efficiency remains the same as in the case of no learning by the informed, but this ranking is reversed in the long run. Finally, we calculate welfare for a dynamic economy over a finite horizon and show that higher welfare and higher informational efficiency go hand in hand.

In Section 4.2, we provide analytical results as well as simulations for the case where agents receive a public signal in addition to their private information. We show that the price converges to the true value of the asset if the public signal is correct, but to the wrong value if the public signal is incorrect. We also show that the ranking of networks by informational efficiency is the same as in the model with no public signal, provided the public signal is incorrect. For a correct public signal, however, the ranking is reversed.

4.1 Simulations

For the simulations we set the two values of the asset payoff V to $V_H = 10$ and $V_L = 0$, and we take V_H to be the realization of V, as in the preceding analysis. We set the number of agents to $N = 15$, of which there are $N_H = 9$ optimists, $N_L = 1$ pessimists, and $N_U = 5$ uninformed agents. These numbers are consistent with the expected number of each type that would arise with 15 traders and a probability $q = 0.8$ of a signal being correct (see footnote 5). Finally, we set the bargaining power of the agent chosen to trade to $\alpha = 1/2$; thus the two trading parties have the same bargaining power and split the surplus equally.

We consider random networks in which there is a probability ϕ of any edge between nodes being activated, independent across edges. We use two different values of this probability: $\phi = 0.33$ for a "medium connected" network or "MC", and $\phi = 0.2$ for a "low connected network" or "LC").[9] The complete network corresponds to $\phi = 1$; we also refer to it as a "fully connected" network or "FC". We generate an MC and an LC, which remain fixed for all the simulations. These networks are shown in Figure 1, together with FC. The numbers represent the nodes to which traders are randomly assigned in each simulation run.



(a) Fully connected (FC) (b) Medium connected (MC) (c) Low connected (LC)

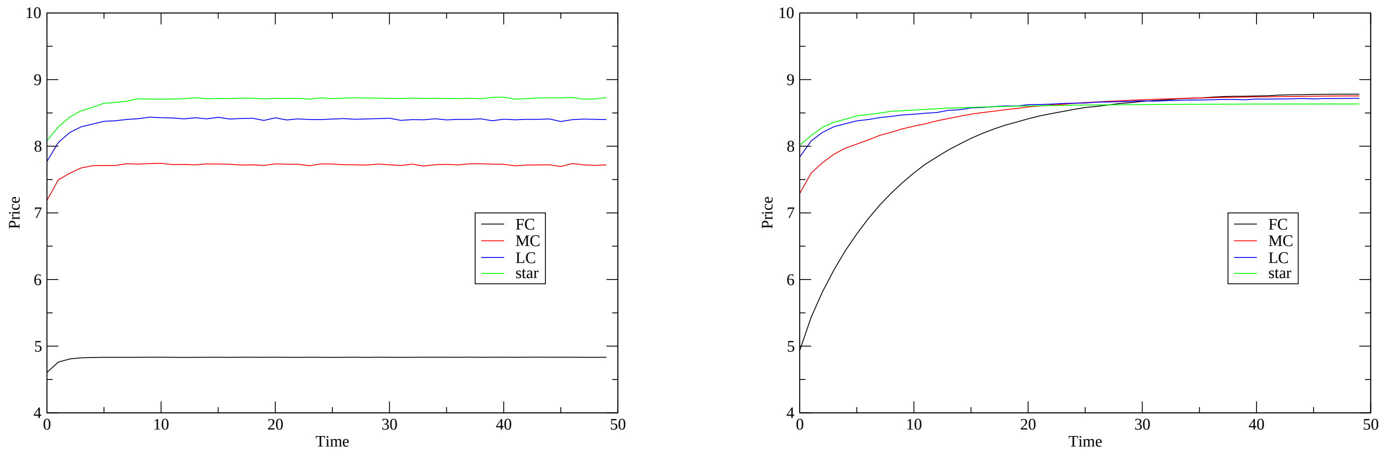
Figure 1: Networks with three levels of connectivity

We extend the model to allow for learning by informed agents as follows. For a given network, consider an informed agent (H or L) with $n-1$ neighbors. Let $\hat{V}_t$ be her belief at date t, and $\check{V}_t$ the equally weighted average of the date t beliefs of her neighbors. Then her belief at date $t+1$ is given by $\hat{V}_{t+1} = \kappa \hat{V}_t + (1-\kappa)\check{V}_t$, where κ is a scalar between 0 and 1. The case of no learning by the informed, as in the baseline model, corresponds to $\kappa = 1$. If $\kappa = 1/n$, learning by this informed agent, who has $n-1$ neighbors, is just as for the uninformed. In the simulations we set $\kappa = 0.9$.

We perform 50,000 simulation runs for each specification (four network structures, FC, MC, LC and the star, with and without learning by the informed), and plot the average price for each date t. The price at t is random because agents are randomly allocated to the nodes of the network, and one of these agents is randomly chosen to trade.

Figure 2(a) shows that our result on informational efficiency for the star generalizes to random networks. For any date t, the complete network is the least efficient, as the price is the lowest and furthest from the true value of the asset, V_H. At the other extreme, the star, which is in some sense the least connected network, performs best. In between, we have the random networks MC and LC with intermediate levels of connectivity. Notice that, even if we exogenously increase the volume of trade (the number of transactions) in the complete

[9] The average degree of a random network $G(N, \phi)$ is $\phi(N-1)$. Hence the average degree is approximately 5 for MC and 3 for LC.



(a) No learning by H and L agents ($\kappa = 1$)

(b) Learning by H and L agents ($\kappa = 0.9$)

Figure 2: Price comparison across networks

network, it still performs worst (the price in the complete network at $t = 50$ is lower than the price at any date in the other networks).

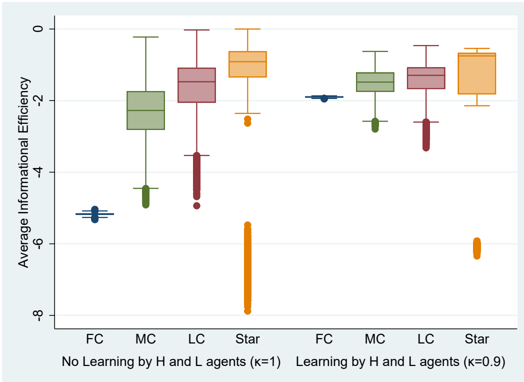
Figure 2(b) shows that if informed agents learn from others ($\kappa = 0.9$), the ranking of the four networks is the same in the short term as in the baseline case ($\kappa = 1$), but it is reversed in the long term. That is, after sufficiently many trading rounds, more connected networks do better and, in particular, the complete network does best. Further simulations that we do not report here show that this ranking reversal happens sooner if we increase the speed of learning by the informed (i.e. lower the value of κ). Note that, even in the complete network, the price converges to a value that is quite far from the expected payoff of the asset under Bayesian inference (which is also the rational expectations equilibrium price under risk neutrality).[10]

Next, we compare informational efficiency averaged across time (or equivalently, across transactions) for different networks. In Figure 3(a), we present a box plot for each network for values of $\mathcal{I}$, defined by (12), where we set the horizon $T = 50$. The box for a given network

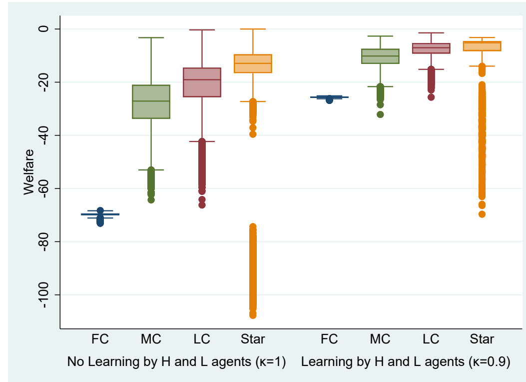
[10]The expected payoff of the asset conditional on all the information available in the market, $\tilde{I}$, is

$$\mathbb{E}(V|\tilde{I}) = 10 \cdot \text{Prob}(V = 10|\tilde{I}) + 0 \cdot \text{Prob}(V = 0|\tilde{I}) = 10 \left[1 + \left(\frac{1-q}{q}\right)^{\eta}\right]^{-1},$$

where η is the number of private signals with value 10 minus the number of private signals with value 0. In the simulations, $\eta = 16$ and $q = 0.8$, which gives us $\mathbb{E}(V|\tilde{I}) = 9.999999998 \approx V_H$.



(a) Average informational efficiency



(b) Welfare

Figure 3: Comparison of average informational efficiency and welfare across networks

represents the middle 50% of data, with the line inside the box indicating the median.[11]

Finally, we turn to welfare analysis. In Section 3, the welfare analysis is for a single-period economy. In Figure 3(b), we show box plots for welfare, measured as the negative of the standard deviation of payoffs of agents aggregated across all dates until $t = 50$. The ranking of networks by average informational efficiency is the same as the ranking by welfare.[12]

In sum, the simulations show that our results on informational efficiency are robust to more realistic random network structures, and in the short term, also to learning by informed agents. Furthermore, we calculate welfare for a dynamic economy and show that this measure is increasing in network connectivity.

4.2 Public Information

The analysis so far has focused on a setting in which the distribution of information is balanced, in the sense that some agents are confident about the information that they have, and this confidence is the same for both correctly informed and misinformed agents. More formally, it is straightforward to check that

$$\text{Prob}(V = V_H | s_1 = s_2 = V_H) = \text{Prob}(V = V_L | s_1 = s_2 = V_L) = \left[1 + \left(\frac{1-q}{q}\right)\right]^{-1}.$$

[11]The whiskers extend up to 1.5 times the interquartile range from the top/bottom of the box to the furthest datum within that distance. If there are any data beyond the whiskers (“outliers”), they are represented individually as points.

[12]The star has a long “tail”, which corresponds to the data points that arise when there is a misinformed agent at the center of the star (since $N_L = 1$, there is only one such agent).

Thus optimists and pessimists have an equally strong basis for sticking to their belief that the asset value is high or low.

This symmetry is broken if, in addition to their private signals, agents observe a public signal. We consider a public signal that takes the same form as the private signals, as described by (1). We continue to assume as before that the true value of the asset is V_H, and that on the basis of their private information alone, there are three groups consisting of N_H optimists, N_L pessimists, and N_U uninformed agents. Now we add the public signal to the information set of each agent.

If the public signal is correct, i.e. its realized value is V_H, optimists have three high signals, uninformed agents have two high signals and one low signal (which by Lemma 3.1 is equivalent to one high signal), and pessimists have two low signals and one high signal (which is equivalent to one low signal). Hence it is no longer sensible to think of both optimists and pessimists as "fanatics" in the sense of Pederson (2022). Instead, we assume that only optimists stick to their belief that the asset value is V_H, while both pessimists and uninformed agents update their beliefs as described before (we continue to refer to agents as uninformed if their private information consists of two conflicting signals, even though after accounting for public information this moniker is not accurate).

If the public signal is incorrect, optimists have in effect only one high signal, the uninformed have one low signal, and pessimists have three low signals. In this case we assume that pessimists maintain their belief that the asset payoff is V_L, while the other agents update.

Proposition 4.1 (General network, public signal: Beliefs and prices) *Consider a network in which every agent is connected directly or indirectly, through the network, to at least one optimist and one pessimist. Then we have the following results:*

i. If the public signal is correct, beliefs and prices converge to V_H as $t \to \infty$.

ii. If the public signal is incorrect, beliefs and prices converge to V_L as $t \to \infty$.

This result follows from Proposition 1 in Pederson (2022). All beliefs converge to the unchanging beliefs of the optimists in the first case, and to those of the pessimists in the second case. Hence the limiting price is V_H or V_L for any network, implying that all networks are asymptotically equivalent with regard to informational efficiency.

Over a finite horizon, however, connectivity does matter. We assume that in the case of correct public information, the initial belief of a pessimist (who effectively observes one low signal) is V_L and that of an uninformed agent (who effectively observes one high signal) is V_H. These agents update their beliefs over time while optimists (who observe three high signals) stick to V_H throughout. For the case of an incorrect public signal, these roles are reversed: the initial belief of an optimist is V_H, and that of an uninformed agent is V_L, while pessimists stick to V_L for all t. Let $\overset{\star}{p}_t^X$ be the time t price in the star with an agent of type X at the center, where X can be H, L or U.

Proposition 4.2 (Star vs complete network, public signal: Informational efficiency) *Suppose $N_H > N_L + N_U$. We have the following results:*

i. Suppose the public signal is correct. Then (a) $\mathbb{E}(\overset{\star}{p}_t^H) > \mathbb{E}(\overset{\star}{p}_t^U) > \mathbb{E}(\overset{\star}{p}_t^L)$ for all $t \geq 1$; and (b) $\mathbb{E}(\overset{c}{p}_t) > \mathbb{E}(\overset{\star}{p}_t^H)$ if t is large enough.

ii. Suppose the public signal is incorrect. Then, for all $t \geq 1$, (a) $\mathbb{E}(\overset{\star}{p}_t^H) > \mathbb{E}(\overset{c}{p}_t)$ if $\alpha \geq 1/3$; and (b) $\mathbb{E}(\overset{c}{p}_t) > \mathbb{E}(\overset{\star}{p}_t^L)$.

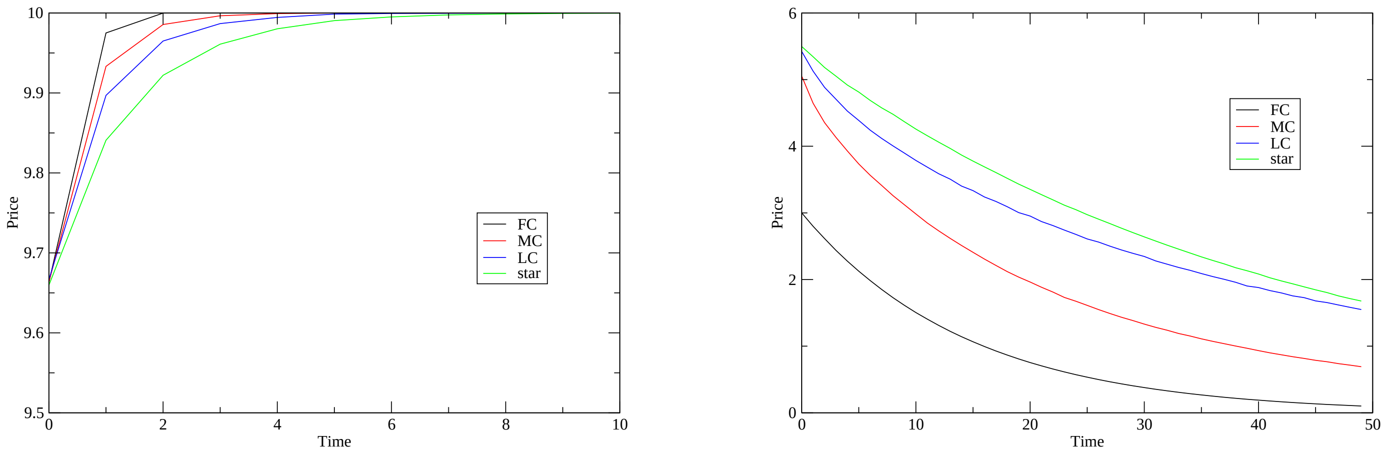
Recall that in the baseline case with no public signal, the star is more informationally efficient than the complete network. Part (i) of Proposition 4.2 shows that if there is a public signal, and it is correct, the ranking is reversed for large t (the simulations discussed below suggest that t does not have to be that large). While the expected price converges to the true value V_H for both networks, this convergence is faster in the complete network. In fact, this is true regardless of the type of agent at the center of the star.

When the public signal is incorrect, there is a preponderance of information pointing to the incorrect value V_L. Both networks converge to V_L in this case, and one would expect that convergence is again faster in the complete network. Part (ii) of Proposition 4.2 shows that this is indeed the case in comparison to the H-star, but not relative to the L-star. Whether the star is on average more informationally efficient than the complete network remains an open question, though the simulations below suggest that it is.

Figure 4 shows simulation results for the four network structures considered in Section 4.1. These are the same as in Figure 2(a), but this time with public information. As shown in Proposition 4.1, prices converge to V_H if the public signal is correct and to V_L if it is incorrect. If the public signal is incorrect, the ranking of the four networks by informational efficiency is the same as in the model with no public signal. However, if the public signal is correct, the ranking is reversed. Note that convergence is much faster in the latter case for all four networks.

5 Experimental Evidence

We present the experimental evidence as follows. We first describe the experimental design and, at the end of that section, we explain how the assumptions of the model map into the experimental setting. Next, we discuss the type of analysis we do, the variables we use, and



(a) Correct public signal

(b) Incorrect public signal

Figure 4: Price comparison across networks when there is a public signal

their descriptive statistics. We then present, in turn, results on trading volume and network performance in terms of average informational efficiency and welfare, and explain how they compare with the theoretical results. Finally, we provide additional results on the dynamics of informational efficiency as well as on trading behavior and profits.

5.1 The Experimental Design

We run a series of independent experimental "markets" with 15 traders. At the beginning of each market, each trader is endowed with 10 units of a one-market lived asset and 1000 units of experimental currency (ECU).[13] Each unit of the asset pays a dividend V at the end of the market, where V is equal to 0 or 10 ECU with equal probability. The dividend payout is randomly determined at the beginning of the market, but it is not revealed to the traders until the end of the market.

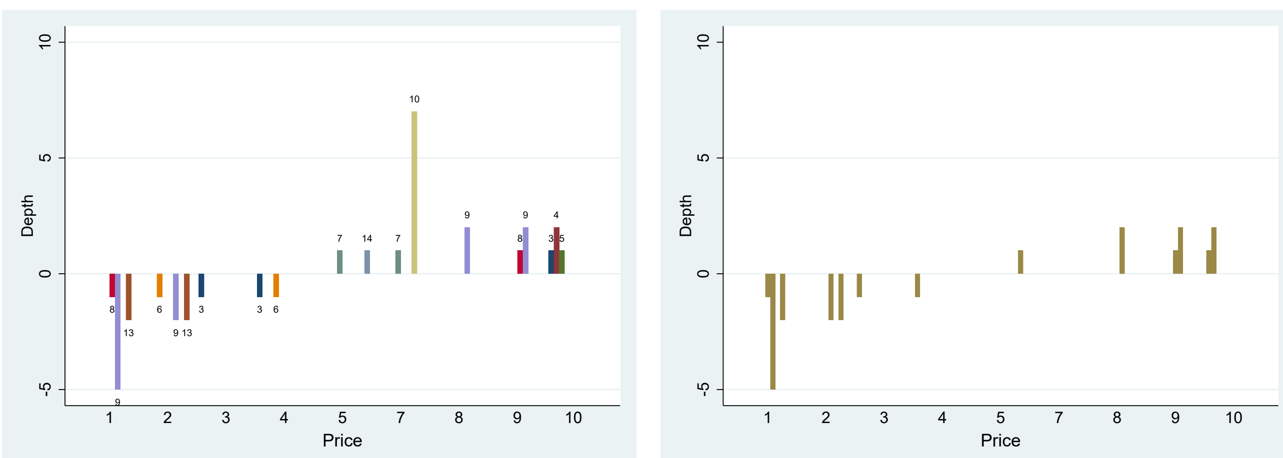
5.1.1 Connectivity and trading

Trading is in a network. We use the networks shown in Figure 1 (FC, MC, and LC), giving rise to three different treatments. The network structure in each treatment is fixed but traders are randomly assigned to the nodes of the network at the beginning of each market.

[13]The high level of initial cash ensures that traders are not cash-constrained. Trader profits will be defined in terms of gains/losses relative to this initial level of cash.

The trading protocol for each market is a 3 minute continuous double auction in which, at any point in time, traders can submit bids and asks to buy or sell one unit of the asset, or accept the outstanding offers of traders with whom they are directly connected in the network. In other words, each subject can only observe and accept quotes in her "local" limit order book and, similarly, her offers can only be observed and accepted by subjects with whom she is directly connected. Every bid, ask or transaction concerns only one unit of the asset, but a trader can buy or sell as much as desired as long as she has enough cash or units of the asset (short sales or borrowing are not allowed). Bids and asks are not restricted to be integers. Orders cannot be canceled. Agents know how many traders they are connected to (and the overall level of connectivity of the network), but not their identity, nor the identity of any trader submitting a particular bid or ask.[14]

Figure 5 shows an example of a hypothetical "global" limit order book and the resulting local limit order book for agent 7 in the medium connectivity network (Figure 1(b)), who is connected to agents 3, 4, 8, 9, 13 and 14 but not to any other agent. Negative levels of depth represent bids (willingness to buy) whereas positive levels represent asks (willingness to sell). If agent 7 accepts the buy order submitted by agent 3 at the posted price of 3.6,



(a) Global limit order book

(b) Local limit order book for subject 7

Figure 5: Limit order book

all the agents linked to agent 3 (i.e. 4, 5, 6, 10 and 11) will see the order disappearing from their limit order book. Hence, as soon as the order is removed, they can infer that it has been executed.[15]

[14] We used z-Tree software to conduct the experiments (Fischbacher (2007)). In the Online Appendix, we include part of the presentation that we did for explaining the trading software to subjects.

[15] Note that orders may in principle "cross" in the local order book of an individual agent, i.e. there may be a bid higher than an ask. Moreover, there may be, in non-fully connected networks, orders that cross in the global order book, and in the local order books of some agents.

5.1.2 Information and payoffs

Subjects receive noisy signals about the realized dividend at the beginning of the market. Signals are presented to subjects as taking values of 0 or 10. A signal of 0 (10) means that the realized dividend is 0 (10) with probability 0.8 and 10 (0) with probability 0.2. In all markets, each subject receives two private signals. All private signals in a given market (a total of 30 signals) are mutually independent conditional on the realized dividend. In some markets, subjects also receive a public signal, which has the same distribution as the private signals and is conditionally independent of them. This gives rise to three different informational environments: "No Public Information", "Correct Public Information" and "Incorrect Public Information". We compare performance across the three treatments (FC, MC and LC) in each informational environment.

Subjects' net profits in each market consist of the dividend paid out and the gains or losses generated by their trading activity. The net profit of subject i in market m is

$$\tilde{\Pi}_m^i := S_m^i V_m + (C_m^i - C_0^i), \tag{14}$$

where V_m is the realized dividend in market m, S_m^i is the number of units of the asset held by subject i at the end of the market and C_0^i and C_m^i are, respectively, her initial and final cash. Her final cash is determined by her initial cash, plus the price received for the units sold, minus the price paid for the units bought. A subject's final payoff for the session is computed as her accumulated profit in the markets run in that session, and paid in cash at the end of the session.[16]

5.1.3 Summary information

We have three treatments: full, medium and low connectivity (FC, MC and LC). We run three sessions for each treatment. For any given treatment, subjects do not receive public information in one of the sessions, while they do in the other two sessions. This gives us three informational environments for each treatment (no public information, correct public information, and incorrect public information). We run more sessions with public information so that we have enough observations for the case in which the public signal is incorrect (the probability of which is 0.2). In each session there are 15 subjects (each subject can

[16] One experimental currency unit is equivalent to 2 euro cents. The average payoff was about €20 and each session lasted around 90 minutes. Note that it is possible for a subject to make losses in a given market. To avoid the problems associated with subjects making session-level losses, we paid all subjects a participation fee of €3, which could be used to offset losses. No subject earned a negative final payoff in any session.

only participate in one session), and consists of a number of independent markets.[17] In each market, the subjects' positions in the network as well as the signals are refreshed so that the position and information of each subject is random across markets.

Table 1: Summary information across treatments

Treatments	Number of Sessions	Number of Markets	Number of Markets with No Public Info	Number of Markets with Correct Public Info	Number of Markets with Incorrect Public Info	Number of Traders per Session	Fraction of Markets with High Dividend	Avg Number of Net Correct Private Signals
FC	3	40	10	24	6	15	0.525	18.65
MC	3	44	14	22	8	15	0.409	18.81
LC	3	45	15	23	7	15	0.466	18.71

Table 1 displays the different treatments implemented, the corresponding number of sessions and markets, the number of markets for each informational environment, the number of traders in each session, the proportion of markets with the high dividend, and the market-average "number of net correct private signals" (defined as the number of correct minus the number of incorrect private signals received by the agents in the market).

5.1.4 Relationship to the model

The asset payoff and the informational environment in the experiments is exactly as in the model, as described in Section 3.1.1, with the two possible asset values set to $V_H = 10$ and $V_L = 0$, and with the probability of a signal being correct set to $q = 0.8$. The number of traders is $N = 15$. These are also the numerical values in the simulations of the model in Section 4.1. The networks FC, MC and LC (shown in Figure 1) are the same in the simulations and the experiments. We do not consider the star in the experiments as this is more a modeling convenience than a realistic financial network.

The experimental setting differs from that in the model or simulations in, essentially, one respect. The trading protocol in the experiments is a continuous double auction in which traders can post bid and ask prices at any time and can also execute trades whenever they wish. This difference is due to our objective of keeping the experimental setting close to real-world markets, but having to make compromises in the model to achieve tractability.[18]

[17] Our aim was to have 15 markets per session. However, due to a technical issue, we had to drop the data for the last few markets in two sessions, and thus ended up with 10 and 14 markets, respectively, in these sessions.

[18] In addition, agents cannot engage in borrowing or short sales in the experiments.

5.2 Analysis, Variables and Descriptive Statistics

We perform analysis at market, transaction, and subject levels. We show box plots and regression analysis results.[19] In the regressions, we first pool all the observations and then we separate them by informational environment. Standard errors are corrected for session-level clustering and reported in parentheses (***p<0.01, **p<0.05, *p<0.1, † p<0.15).

We now briefly discuss the dependent and explanatory variables of our analysis. Summary descriptive statistics for these variables are displayed in Table 2.

Table 2: Descriptive statistics

		All Observations					No Public Info	Correct Public Info	Incorrect Public Info
		Obs.	Mean	Median	Min	Max	Mean	Mean	Mean
Dependent Var.	Trading Volume	129	54.4	50	23	123	58.7	50.4	59.4
	Avg Info Efficiency	129	-2.19	-1.81	-0.07	-8.14	-2.95	-1.07	-4.46
	Welfare	129	-20.03	-15.12	-0.53	-96.90	-28.92	-9.78	-37.17
	Informational Efficiency	7,017	-2.35	-1.50	0	-10	-3.07	-1.12	-4.47
	Profit	1,935	46.51	28.3	-299.8	358.1	35.8	47.8	61.9
	Stock Bought	1,935	3.62	1	0	83	3.92	3.35	3.96
	Stock Sold	1,935	3.62	2	0	17	3.92	3.35	3.96
	Stock Held	1,935	10	9	0	93	10	10	10
Independent Var.	Medium Connectivity	129	0.341	0	0	1	0.358	0.318	0.380
	Low Connectivity	129	0.348	0	0	1	0.384	0.333	0.333
	Correct Public Information	129	0.534	1	0	1	-	-	-
	Incorrect Public Information	129	0.162	0	0	1	-	-	-
	Net Private Information	129	18.72	20	10	28	18.35	18.89	18.85
	Correctly Informed Agent	1,935	0.669	1	0	1	0.663	0.670	0.676
	Misinformed Agent	1,935	0.044	0	0	1	0.051	0.040	0.047
	Connected Agent	1,935	0.461	0	0	1	0.500	0.437	0.466

5.2.1 Market and transaction level

We consider trading volume, average informational efficiency, and welfare as dependent variables at the market level. We also analyze informational efficiency at the transaction level.

Trading volume τ_m in market m is the total number of times the asset is traded by any pair of agents in that market (each transaction involves one unit of the asset). As shown in Table 2, trading volume is lowest when there is a public signal and it is correct.

The *average informational efficiency* (averaged across transactions) in market m is

$$\mathcal{I}_m := \frac{1}{\tau_m} \sum_{j=1}^{\tau_m} \mathcal{I}_{j,m}, \tag{15}$$

[19] We explain how to interpret a box plot in the discussion of Figure 3 and in footnote 11.

where $\mathcal{I}_{j,m}$ is the *informational efficiency* for the j'th transaction in market m, defined as

$$\mathcal{I}_{j,m} := -|p_{j,m} - V_m|, \tag{16}$$

where $p_{j,m}$ is the price of the ordered transaction j in market m, and V_m is the realized dividend in market m. $\mathcal{I}_{j,m}$ is the analog of the variable $\mathcal{I}_t$ in the model, defined by (11) (note that in the model each date corresponds to a transaction). Similarly, $\mathcal{I}_m$ is the analog of the variable $\mathcal{I}$ in the model, defined by (12). As in the theoretical framework, with these measures, the highest level of efficiency is zero and, the less efficient the market or transaction is, the more negative the corresponding measure is. As we can see in Table 2, average informational efficiency is highest when there is a public signal and it is correct, and the lowest when there is a public signal but it is incorrect.

Assuming mean-variance preferences, (ex ante) *welfare* in market m, W_m, is measured by the negative of the standard deviation of the profits of the 15 participants in market m, i.e.

$$W_m = -\sqrt{\frac{1}{14}\sum_{i=1}^{15}\left(\tilde{\Pi}_m^i - \frac{1}{15}\sum_{j=1}^{15}\tilde{\Pi}_m^j\right)^2},$$

where $\tilde{\Pi}_m^i$ is the profit of agent i in market m, as defined by (14). This is the empirical equivalent of the welfare measure in the model, except that in the model we consider only payoffs in the limiting economy, while here payoffs are aggregated over the duration of the market. We can see from Table 2 that the ranking of the three informational environments is the same for welfare as for average informational efficiency.

The independent variables at the market and transaction levels include dummies for connectivity: medium and low connectivity, as opposed to full connectivity, our base category. We also include dummies for a correct public signal and an incorrect one, using no public information as the default category. For private information we use the number of net correct private signals. By Lemma 3.1, this measures the strength of the pooled private information in the market that points to the correct value of the dividend.

5.2.2 Subject level

We define the following dependent variables at the individual subject level: profits, stock bought, stock sold and stock held. Stock bought and sold count the number of units of the asset bought and sold, respectively, by the agent during the operation of the market. Stock held is the number of units of the asset held by the agent at the end of the market. The average number of units of the asset bought (which must be equal to the average number sold) varies in the same direction as trading volume across the three informational environments, as shown in Table 2. Average stock held is by definition equal to the initial endowment. The number of observations is equal to the number of markets (129) times the number of subjects in each market (15), which yields a total of 1935.

As independent variables at the subject level, we use, in addition to the market-level variables mentioned above, a dummy variable for correctly informed agent, which takes a value of 1 if the subject has received two correct private signals, a dummy variable for misinformed agent, which takes a value of 1 if the subject has received two incorrect private signals, and a dummy variable for "connected agent", which takes a value of 1 if the number of links of the subject (her degree) is above the median in that market. As the probability of receiving a correct signal is 0.8, the fraction of correctly informed traders is (approximately) 0.64.

5.3 Trading Volume

This section shows that agents in centralized markets trade more than in decentralized markets. In the next section we will see that, despite this, decentralized markets perform better in terms of average informational efficiency and welfare. The higher number of transactions in centralized markets is not sufficient for prices to better reflect private information, although all traders observe all transactions in such markets.

Figure 6 shows that trading volume decreases with lower connectivity, especially when moving from full connectivity to medium connectivity, in each of the three informational environments.[20] These results are confirmed by the regressions shown in Table 3. The

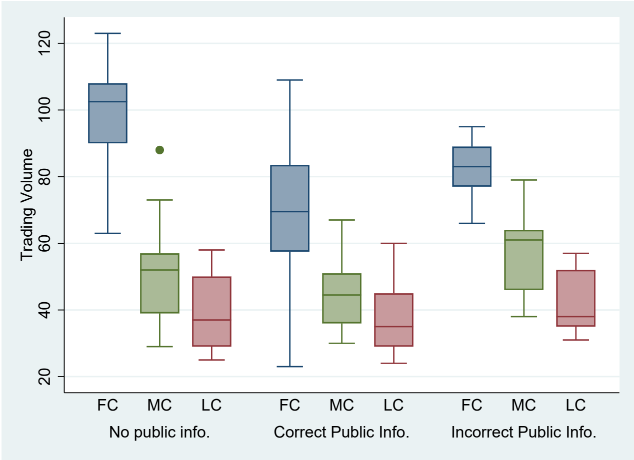


Figure 6: Trading volume

[20] A Mann-Whitney test (two sample t-test) rejects the null hypothesis of equal median (equal mean) at the 1% significance level for each pair of treatments (FC, MC and LC) in each of the three informational environments (no public information, correct public information and incorrect public information).

Table 3: Trading volume regressions

	All	No Public Info	Correct Public Info	Incorrect Public Info
Medium Connectivity	−30.419***	−48.232***	−24.506***	−22.666**
	(6.053)	(0.236)	(5.334)	(7.584)
Low Connectivity	−40.309***	−58.389***	−31.268***	−37.625***
	(5.425)	(0.174)	(3.490)	(4.909)
Correct Public Info	−11.412*			
	(5.290)			
Incorrect Public Info	−0.520			
	(5.309)			
Net Private Info	−0.491	−1.611*	0.032	−0.405
	(0.282)	(0.434)	(0.371)	(0.947)
Constant	94.234***	128.146***	68.033***	88.241***
	(9.981)	(7.993)	(8.794)	(14.148)
Observations	129	39	69	21
R-squared	0.60	0.79	0.49	0.70

coefficients of the dummies for medium and low connectivity are negative and strongly significant in all the regressions, both in the pooled sample in column 1 as well as for each informational environment in columns 2-4. The coefficient for low connectivity is more negative than the one for medium connectivity. Thus, in decentralized markets, trading volume is lower than in centralized markets, and as the connectivity of decentralized markets decreases trading volume decreases further. These results are in line with "common wisdom".

We do not observe systematic differences in trading volume across informational environments. Column 1 shows that trading volume is lower when the public signal is correct, compared to the case of no public signal. But if the public signal is incorrect, trading volume is not significantly different from the no public signal case.[21] The number of net correct private signals has a negative impact on trading volume when there is no public signal (column 2), but has no significant effect when there is a public signal (columns 3 and 4).

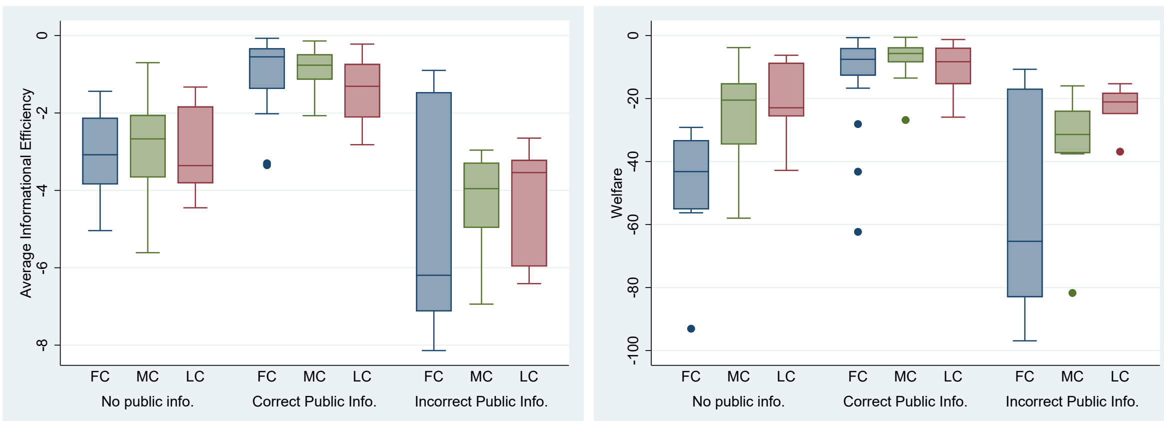
5.4 Network Performance

In this section we present the effects of connectivity (full, medium, or low) on performance, taking into account the informational environment (no public signal, correct public signal, or incorrect public signal). We show that decentralization can outperform centralization in terms of average informational efficiency and welfare, consistent with the results of the model (Sections 3 and 4), despite involving less trade (Section 5.3).

[21] While our model does not have predictions on trading volume, we can use belief dispersion, or disagreement, as a proxy, to explain these results. When there is no public signal, beliefs vary from V_L to V_H. When there is a public signal, all beliefs converge to either V_L or V_H (Proposition 4.1). Figure 4 suggests that this convergence is faster when the public signal is correct than when it is not. Thus in the model, disagreement is the lowest in the case of a correct public signal and highest when there is no public signal, which is consistent with the above results on trading volume.

5.4.1 Average informational efficiency

We first consider average informational efficiency, as measured by $\mathcal{I}_m$ (see (15)), i.e. informational efficiency averaged across all transactions in market m. Figure 7(a) shows that the fully connected network is generally not more efficient than less-connected networks. In the pooled regression in Table 4, the dummies for connectivity are not significant. Thus connectivity appears to have no impact, positive or negative, on informational efficiency. But a more nuanced picture emerges when we split the sample by informational environment.



(a) Average informational efficiency

(b) Welfare

Figure 7: Average informational efficiency and welfare

Table 4: Average informational efficiency regressions

	All	No Public Info	Correct Public Info	Incorrect Public Info
Medium Connectivity	0.175	0.369***	0.101	-0.465
	(0.189)	(0.016)	(0.218)	(0.816)
Low Connectivity	-0.106	0.034*	-0.355**	-0.455
	(0.147)	(0.012)	(0.065)	(0.815)
Correct Public Info	1.804***			
	(0.112)			
Incorrect Public Info	-1.590***			
	(0.239)			
Net Private Info	0.131***	0.199**	0.058†	0.225**
	(0.027)	(0.029)	(0.028)	(0.057)
Constant	-5.387***	-6.751***	-2.089**	-8.375***
	(0.522)	(0.536)	(0.536)	(1.038)
Observations	129	39	69	21
R-squared	0.66	0.38	0.16	0.31

In the case of no public information, average informational efficiency in decentralized markets is higher than in centralized markets, as shown by the positive and significant coefficients on the dummies for medium and low connectivity (column 2). This is in line with our theoretical results on the higher informational efficiency of incomplete networks, in particular Proposition 3.7, and the simulations which show that the medium and low connected random networks outperform the fully connected network (Figures 2 and 3(a)).

When there is a public signal and it is correct, average informational efficiency is higher in centralized markets (FC) than in highly decentralized markets (LC), in line with part (i) of Proposition 4.2 and Figure 4(a). However, in the case of an incorrect public signal, connectivity has no significant effect on average informational efficiency. Generally speaking, the effect of connectivity on average informational efficiency is less significant when there is a public signal. This is consistent with Proposition 4.1, which shows that prices converge to the same value for all networks if there is a public signal.

Results across informational environments are intuitive. Average informational efficiency is highest when the public signal is correct, and lowest when the public signal is incorrect. This can be seen in Figure 7(a), as well as in the pooled regression (column 1), where the coefficient on the dummy variable for correct public information is positive and significant, whereas that on the dummy for incorrect public information is negative and significant. These results are also consistent with Proposition 4.1, which says that the price converges to the correct value of the asset when the public signal is correct, but to the wrong value when the signal is incorrect (see also Figure 4). The intermediate performance of the no public signal case is in line with Propositions 3.4 and 3.6, which show that the expected price converges to a value between the high and low value of the dividend (see also Figure 2).

The regressions also show, both overall and in the split samples, that average informational efficiency is higher if there is more private information pointing to the true dividend. The coefficient on the net number of correct private signals is always positive and almost always significant. Comparing the magnitudes, private information is more important when there is no public signal, and especially when there is but it is incorrect.

5.4.2 Welfare

Figure 7(b) shows that welfare is generally higher in decentralized, less connected markets. This is especially the case when public information is not available or when it is incorrect, although in the latter case there is a lot more variance across markets.[22] When the public signal is correct, welfare is high and fairly similar across all levels of connectivity. This is because agents are more homogeneously informed about the true value of the asset after the release of a correct public signal, leading to a lower variance of payoffs (Figure 4(a) shows that convergence can be quite fast). Figure 7(b) also indicates that the ranking of informational

[22] A Mann-Whitney test (two sample t-test) rejects the null hypothesis of equal median (equal mean) at the 1% significance level between FC and LC when there is no public information.

environments with respect to welfare is the same as for informational efficiency, i.e. highest in the case of a correct public signal and lowest in the case of an incorrect public signal.

Table 5: Welfare regressions

	All	No Public Info	Correct Public Info	Incorrect Public Info
Medium Connectivity	11.035**	23.037***	5.056*	8.044
	(3.936)	(0.307)	(2.407)	(15.626)
Low Connectivity	13.314**	25.078***	2.444*	19.763
	(4.043)	(0.226)	(0.978)	(15.532)
Correct Public Info	19.300***			
	(3.442)			
Incorrect Public Info	-8.702			
	(5.489)			
Net Private Info	1.783***	3.181**	0.535*	2.469
	(0.421)	(0.565)	(0.225)	(1.262)
Constant	-70.742***	-105.233***	-22.319***	-93.388**
	(10.781)	(10.394)	(4.174)	(24.940)
Observations	129	39	69	21
R-squared	0.549	0.705	0.079	0.459

These observations are confirmed in the regressions shown in Table 5, where the dummies for medium and low connectivity have a positive effect which is significant, except in the case of an incorrect public signal. In the pooled regression (column 1), the dummy for correct public information has a positive, significant effect on the level of welfare whereas the dummy for incorrect public information has a negative, though not significant, effect. The regressions also show that welfare is higher if there is private information pointing to the true dividend. This effect is more pronounced in the case of no public information.

These findings are consistent with welfare results in the model, in particular Proposition 3.8 which shows that welfare is higher in the star compared to the complete network, and the simulations that show that welfare goes up as connectivity falls (Figure 3(b)).

5.5 Dynamics of Informational Efficiency

Now we provide a more granular analysis of informational efficiency. Figure 8 plots $\mathcal{I}_{j,m}$, which measures informational efficiency for the j'th ordered transaction in market m (see (16)), as a function of j, averaged across all markets with the same level of connectivity and informational environment. The figure suggests that prices gets closer over time to the true dividend in all cases, except in the fully or medium connected market with an incorrect public signal. Full convergence is not achieved for any network or informational environment.[23]

We observe that, after a given number of transactions, informational efficiency is generally higher in more decentralized markets, except in the case of a correct public signal. Still, in the

[23]Market prices are quite far from the rational expectations equilibrium price under risk neutrality, which is very close to the true dividend (see footnote 10).

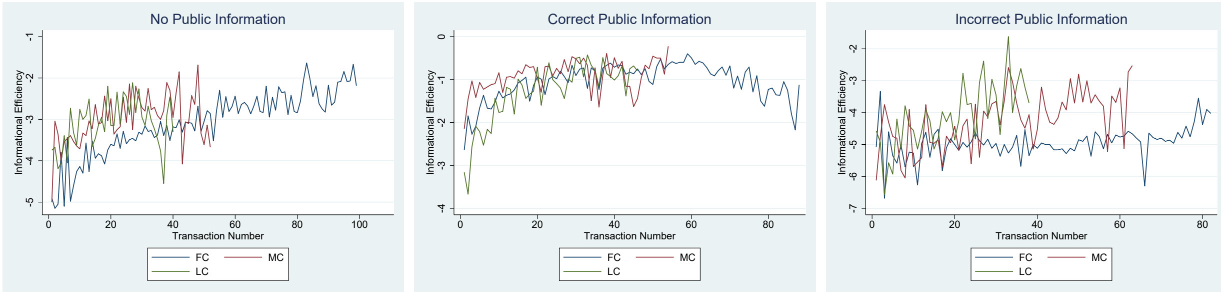


Figure 8: Informational efficiency as a function of the number of transactions

case of no public information, centralized markets reach comparable levels of informational efficiency towards the end of the market because it involves a higher number of transactions (Section 5.3). Section 5.4.1 shows, though, that on average, across all the transactions in the market, the level of informational efficiency is lower in centralized markets.

Table 6: Informational efficiency regressions for the first 40 transactions

	All	No Public Info	Correct Public Info	Incorrect Public Info
Medium Connectivity	0.456	0.983***	0.474	−0.969
	(0.251)	(0.030)	(0.309)	(0.511)
Low Connectivity	0.135	0.843***	−0.242	−0.811*
	(0.245)	(0.022)	(0.174)	(0.382)
Correct Public Info	2.016***			
	(0.172)			
Incorrect Public Info	−1.402***			
	(0.218)			
Net Private Info	0.124***	0.184*	0.057	0.221***
	(0.025)	(0.055)	(0.035)	(0.047)
Second Block	0.580***	0.831***	0.603***	0.118
	(0.082)	(0.000)	(0.044)	(0.220)
Medium Connectivity*Second Block	−0.108	−0.170**	−0.333**	0.678
	(0.107)	(0.023)	(0.115)	(0.423)
Low Connectivity*Second Block	0.238	−0.352***	0.291***	1.063**
	(0.177)	(0.019)	(0.069)	(0.289)
Constant	−6.046***	−7.597**	−2.622**	−8.469***
	(0.466)	(1.011)	(0.753)	(0.659)
Observations	4.844	1.466	2.558	820
R-squared	0.391	0.165	0.065	0.191

In Table 6 we report results for regressions of (transaction-level) informational efficiency on connectivity and information variables for the first 40 transactions, separated into two blocks of 20 transactions (the minimum number of transactions in any market is 40). The clearest results are for the case of no public information (column 2). Decentralization improves upon centralization if we consider the first 20 transactions, as the medium and low connectivity coefficients are positive and significant. This difference is reduced but not eliminated if we consider the second block of 20 transactions, as shown by the interaction terms of the second block and the connectivity coefficients. This is consistent with Figure 2(b), where we see a decline in the beneficial effect of lower connectivity as the number of transactions increases.

Except in the case of an incorrect public signal, the main coefficient of the second block is positive and significant, showing that informational efficiency improves over time. This is consistent with our simulation results on price dynamics (see Figures 2 and 4).

5.6 Individual Trading Behavior and Profits

In this section we describe how individual trading behavior and profits depend on information and connectivity variables, both at the individual and network levels. We first investigate the determinants of individual profits. Subsequently, we explore the sources of agent profit differentials by looking into trading behavior, both in terms of direction and speed of trade. As individual behavior and performance are highly dependent on the level of the dividend, we separate the observations by the realization of the dividend in the market.

5.6.1 Profits: information vs connectivity

Table 7 shows how profits depend on information and connectivity variables. As shown by model 1, agents' private information is the most significant and quantitatively important variable, with correctly informed agents outperforming uninformed agents, who in turn outperform misinformed agents. The individual connectivity of an agent, in contrast, does not affect profits significantly. In other words, profits flow from the misinformed to the correctly informed, and this does not depend on how many neighbors these agents have.

Model 2, which includes interaction variables of private information with network and individual connectivity, shows that the gains from correct information, as well as the losses due to incorrect information, are higher in centralized markets. Indeed, the interaction variables of private information and medium and low connectivity generally display opposing signs to the main effects. This is consistent with our previous finding that the cross-sectional variance of profits increases with market connectivity, leading to lower welfare ex ante.

5.6.2 Individual behavior: direction of trade and connectivity

We now investigate the sources of agent profit differentials by looking into individual trading behavior. We first regress the number of units of the asset bought, sold and held on information and connectivity variables. Table 8 shows that in centralized markets agents tend to trade in the direction that their private information indicates, with correctly informed agents buying when the dividend is high and selling when the dividend is low, while misinformed agents trade in the opposite direction. This is significantly less pronounced in decentralized

Table 7: Profit regressions

	Model 1		Model 2	
	Dividend 0	Dividend 10	Dividend 0	Dividend 10
Correctly Informed Agent	14.323***	15.243***	24.952**	14.361*
	(3.608)	(3.447)	(7.995)	(6.623)
Correctly Informed Agent*Medium Connectivity			-17.104*	3.053
			(8.015)	(8.035)
Correctly Informed Agent*Low Connectivity			-19.306*	-2.203
			(8.975)	(8.179)
Misinformed Agent	-44.294***	-16.559**	-44.562***	-24.770*
	(9.059)	(5.434)	(4.873)	(11.836)
Misinformed Agent*Medium Connectivity			-4.502	10.064
			(12.843)	(12.548)
Misinformed Agent*Low Connectivity			30.177**	12.892
			(12.242)	(14.167)
Connected Agent	0.713	-0.205	-2.006	-1.406
	(1.181)	(1.405)	(2.005)	(3.139)
Correctly Informed Agent*Connected Agent			4.640*	1.777
			(2.046)	(2.754)
Misinformed Agent*Connected Agent			-21.101	0.510
			(12.102)	(8.330)
Medium Connectivity	0.171	0.101	12.012*	-2.354
	(0.583)	(0.650)	(5.249)	(4.993)
Low Connectivity	0.193	0.191	12.131*	1.106
	(1.207)	(1.195)	(6.171)	(5.446)
Correct Public Info	-0.201	-0.004	-0.426	-0.090
	(0.386)	(0.025)	(0.388)	(0.117)
Incorrect Public Info	-0.074	0.010	0.571	0.104
	(0.493)	(0.059)	(0.607)	(0.271)
Net Private Info	-0.779***	-0.518***	-0.762***	-0.528***
	(0.126)	(0.108)	(0.132)	(0.108)
Constant	6.709**	100.235***	-0.533	101.350***
	(2.294)	(0.978)	(4.969)	(2.901)
Observations	1,035	900	1,035	900
R-squared	0.203	0.114	0.225	0.117

markets. The interaction variables between correct information and medium/low connectivity always have the opposite sign to those of the standalone variable of correct information, and the coefficients are highly significant. In fact the interacted effects almost reverse the main effect, and thus correctly informed agents in decentralized markets trade in a similar way to other agents. The coefficients for misinformed agents are not significant, possibly because they are very few in number. Overall, these findings are consistent with the results of the previous section, where we saw that decentralization lowers the payoff differential between agents who have correct information and those who do not.

5.6.3 Individual behavior: speed of trading and connectivity

Figure 9 compares the speed of trading by correctly informed agents to that of other agents, most of whom are uninformed. Focusing on the case of no public information, we compute the average distribution of buy and sell volume of the individual traders over time. Each

Table 8: Regressions for stock bought, sold and held

	Stock Bought		Stock Sold		Stock Held	
	Dividend 0	Dividend 10	Dividend 0	Dividend 10	Dividend 0	Dividend 10
Correctly Informed Agent	-5.733***	3.985***	2.353***	-2.579***	-8.086***	6.563***
	(1.374)	(0.559)	(0.510)	(0.110)	(1.831)	(0.586)
Correctly Informed Agent*Medium Connectivity	6.028***	-3.365***	-2.544***	2.131***	8.572***	-5.496***
	(1.376)	(0.740)	(0.666)	(0.546)	(1.923)	(0.804)
Correctly Informed Agent*Low Connectivity	5.119***	-2.333**	-2.080**	2.957**	7.199***	-5.290***
	(1.458)	(0.727)	(0.831)	(1.102)	(2.112)	(1.111)
Misinformed Agent	9.182*	-1.026	0.404	1.703**	8.778**	-2.729
	(4.776)	(1.077)	(1.456)	(0.734)	(3.389)	(1.799)
Misinformed Agent*Medium Connectivity	-3.919	-1.972	-2.723	2.226	-1.196	-4.198
	(5.183)	(1.563)	(1.666)	(2.862)	(3.913)	(2.530)
Misinformed Agent*Low Connectivity	-7.986	-1.590	-1.354	3.108	-6.632	-4.697
	(5.733)	(1.531)	(1.667)	(3.613)	(4.746)	(3.798)
Connected Agent	0.431	-0.397	-0.373	-0.306	0.804	-0.091
	(0.361)	(0.745)	(0.686)	(0.523)	(1.012)	(0.468)
Correctly Informed Agent*Connected Agent	0.248	-0.482	-0.112	0.204	0.360	-0.685
	(0.494)	(0.567)	(0.753)	(0.962)	(1.164)	(1.017)
Misinformed Agent*Connected Agent	-1.703	3.028**	2.364**	-5.318	-4.066	8.347**
	(3.166)	(1.117)	(1.008)	(3.617)	(3.411)	(3.305)
Medium Connectivity	-6.696***	1.011	-0.584	-2.806***	-6.112***	3.816***
	(1.185)	(0.760)	(0.346)	(0.450)	(1.248)	(0.534)
Low Connectivity	-6.689***	-0.285	-1.458*	-4.109***	-5.231***	3.824***
	(1.184)	(0.626)	(0.639)	(0.576)	(1.419)	(0.644)
Correct Public Info	-0.143	-1.406***	-0.361	-1.402***	0.218**	-0.004
	(0.299)	(0.339)	(0.293)	(0.336)	(0.069)	(0.051)
Incorrect Public Info	-0.153	-0.317	0.064	-0.453	-0.216	0.136
	(0.800)	(0.287)	(0.537)	(0.309)	(0.285)	(0.159)
Net Private Info	0.009	-0.057	-0.066	0.013	0.075	-0.070*
	(0.045)	(0.031)	(0.036)	(0.021)	(0.044)	(0.036)
Constant	8.959***	4.365***	5.429***	7.288***	13.530***	7.077***
	(1.159)	(0.729)	(0.780)	(0.548)	(0.947)	(0.707)
Observations	1,035	900	1,035	900	1,035	900
R-squared	0.096	0.086	0.144	0.118	0.076	0.066

figure corresponds to a given network (FC, MC or LC), a given value of the dividend ($D = 0$ or 10), and either buy or sell volume. For example, the figure in the top row and second column shows the proportion of buy volume over time in the fully connected network with a dividend of 10. We take a market with these characteristics and a correctly informed trader in that market. We calculate the buy volume of this trader up to t relative to her total buy volume in the market. We then average these values across all correctly informed traders in markets with the same characteristics (fully connected with a high dividend), and plot this point in the figure. The value at date t on the graph for "other traders" in the same figure is analogous.

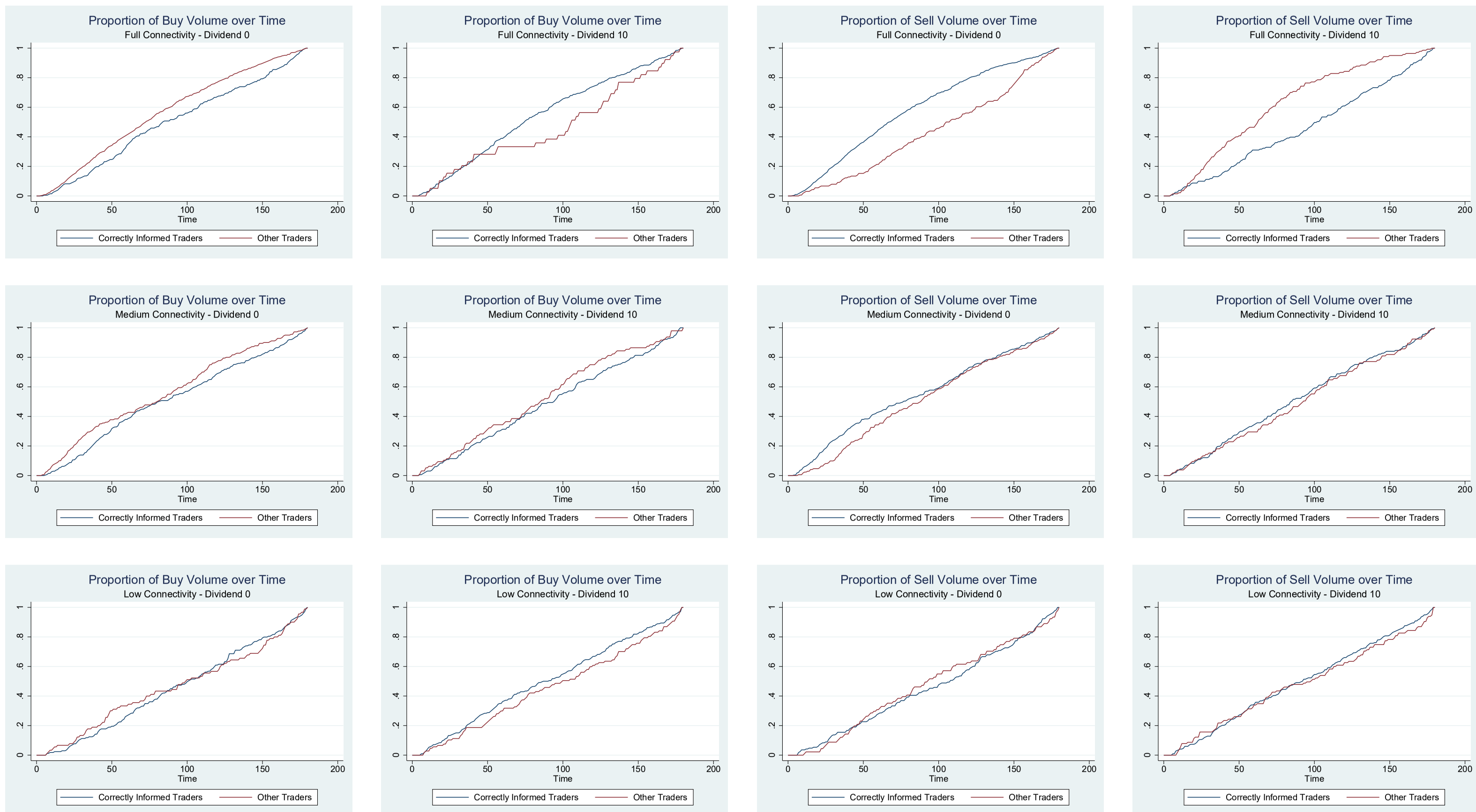


Figure 9: Speed of trading

Figure 9 shows that in centralized markets (row 1), correctly informed traders buy more slowly than other buyers when the dividend is 0, and faster when the dividend is 10. Also, they sell faster than other sellers when the dividend is 0, and more slowly when the dividend is 10. These effects are much less apparent in decentralized markets (rows 2 and 3). All these results are confirmed by Kolmogorov-Smirnov tests checking for equality of distributions (see Table 9). The distributions of correctly informed agents (buyers and sellers) are significantly different from those of other agents in fully connected markets, but the difference is not significant in medium and low connected markets.

Table 9: KS test statistics for equal distributions

	Buyer		Seller	
	Dividend 0	Dividend 10	Dividend 0	Dividend 10
Full Connectivity	0.1436**	0.2210***	0.2597***	0.2980***
Medium Connectivity	0.1105	0.1271	0.0940	0.0660
Low Connectivity	0.0939	0.1105	0.0939	0.0718

Test statistics for two-sample Kolmogorov-Smirnov test checking the equality of the distribution of transactions of correctly informed traders and other traders.

To summarize, while agents who have the same information sometimes trade in opposite directions, agents do tend to trade in the direction that their private information suggests. This is especially the case in fully connected markets. Quite naturally, correctly informed agents reap higher profits in centralized markets because they trade more in the direction which turns out to be the right one ex post, while misinformed agents lose more as they trade more in the opposite direction. In addition, in fully connected markets, correctly informed trade faster (in the right direction), and thus benefit from prices that are further away from fundamentals. Decentralization reduces payoff differentials across agents and thus increases welfare ex ante, consistent with the results we obtained in the previous section.

6 Conclusion

In light of recent proposals to change the organization of asset markets, an assessment of the performance of decentralized markets deserves more attention. In this paper we seek to isolate and understand the role of connectivity. We find that fewer connections do not necessarily imply poorer performance. Despite lower trading volume, decentralized markets can be more informationally efficient, and make mean-variance investors better off, when compared to centralized markets.

We provide a simple model of networked trading with noisy private information. The model, in the tradition of DeGroot (1974), Pederson (2022), and others, features heuristic updating, or rule-of-thumb learning. We assume that informed agents (whose private information may or may not be correct ex post) do not update their beliefs, while uninformed agents use a heuristic updating rule, whereby they repeatedly average their own beliefs with those of their neighbors in the network. Beliefs evolve independently of trades and prices, just as in Pederson (2022). However, unlike Pederson (2022), where trading is Walrasian, we assume that agents arrive sequentially and one unit is traded at each date, similar to the trading environment in Glosten and Milgrom (1985). Our proposed model assumes, intuitively, we believe, that when it is an agent's turn to trade, she chooses to trade with a neighbor whose belief is furthest away from her own belief, thus maximizing gains from trade.

We show that, under these conditions, incomplete networks, such as the star and the core-periphery network, perform better than the complete network, in terms of both informational efficiency and welfare. In the complete network, misinformed agents with "extreme" views have a disproportionate impact on trading activity, as transactions with these agents result in higher perceived gains from trade for both buyer and seller. Fewer links in the network lead to fewer trades with misinformed agents (who are less numerous than correctly informed agents), pushing prices closer to the fundamental, and reducing the variance of payoffs.

We perform numerical simulations that extend the theoretical results in several directions. First, we show that the higher informational efficiency of the star, relative to the complete network, generalizes to more realistic random networks with varying levels of connectivity. Second, we introduce learning by informed agents. We show that in the short run, the ranking of networks by informational efficiency remains the same as in the case of no learning by the informed, but this ranking is reversed in the long run. Finally, we show that higher welfare and higher informational efficiency go hand in hand.

We extend the model to allow for public information. We show that if there is a public signal, and it is correct, the ranking of networks in the no public information case may be reversed. While the expected price converges to the true value for any network, this convergence is faster in the complete network. When the public signal is incorrect, there is a preponderance of information pointing to the incorrect value. As a result, the expected price converges to the wrong value for any network, and this convergence is again faster in the complete network. Simulations show that incomplete networks are more informationally efficient than the complete network in this informational environment.

The experimental evidence provides support for the theoretical results. In the case of no public information, average informational efficiency in decentralized markets is higher than in centralized markets. When there is a public signal and it is correct, average informational efficiency is higher in centralized markets than in highly decentralized markets. However, in the case of an incorrect public signal, connectivity has no significant effect on average informational efficiency. Generally speaking, the effect of connectivity on average informational efficiency is less significant when there is a public signal.

In addition to corroborating the results of the model on average informational efficiency and welfare, the experiments allow us to provide a number of further insights. Although informational efficiency, for a given number of transactions, is higher in decentralized markets, centralized markets reach comparable levels of informational efficiency towards the end of the market due to a higher number of transactions. Still, correctly informed agents earn higher profits in centralized markets, as compared to decentralized markets, because they trade more and faster in the direction which turns out to be the right one ex post, while misinformed agents lose more as they trade more in the opposite direction. Decentralization reduces payoff differences and thus increases welfare for mean-variance investors.

The results of our experiments not only confirm the main predictions of the theoretical model in terms of average informational efficiency and welfare, but they also provide support for the DeGroot learning model as a useful tool in understanding price formation in asset markets with dispersed information. Indeed, trading behavior and the dynamic informational efficiency results in the experiments are broadly consistent with those of the theoretical model. We show, for instance, that in the absence of public information, informational efficiency improves over time, but prices remain far from the true dividend. This provides support for the rule-of-thumb learning we assume in the model, which predicts that the price converges to an intermediate value between the two possible values of the dividend, as opposed to Bayesian learning, which predicts full revelation, very little trade, and a price very close to the true dividend. In this sense, our results provide experimental evidence in support of the DeGroot learning model, supplementing the evidence reported in Chandrasekhar et al. (2020).

Despite the assumption that one unit is traded at every date regardless of the network structure, our model is a useful vehicle for understanding the consequences of different levels of trading volume across networks. Indeed, we can calculate the effect on prices and informational efficiency of an exogenous increase in the volume of trade for a given network. But our model (or any other model that we are aware of) does not explain why we observe more trade in centralized markets. A model of networked markets that can address the effects of connectivity not only on market performance, but also on trading volume, remains a challenge for future research.

Acknowledgments: This research was supported by funding from the British Academy under the Small Research Grant scheme (grant nr. SG132471), Generalitat Valenciana under the project AICO/2021/005, Ministerio de Economía y Competitividad and Feder (PID2021-

122403NB-I00), Severo Ochoa program (CEX2019-000915-S), and ICREA under the ICREA Academia program.

A Appendix: Proofs

Before presenting the proofs of the propositions in the paper, we state the following useful result which is immediate from (3):

Lemma A.1 (General network: Evolution of beliefs) *For $t \geq 1$,*

$$\tilde{V}_U(t+1) - \tilde{V}_U(t) = A_U^t v, \tag{17}$$

where v is the $1 \times N_U$ vector given by

$$v := (A_U - I)\mathbf{1}_{N_U}\frac{V_H + V_L}{2} + (A_H \mathbf{1}_{N_H} V_H + A_L \mathbf{1}_{N_L} V_L). \tag{18}$$

Proof of Lemma 3.1 The proof involves repeated application of Bayes' rule. First, we observe that

$$\begin{aligned}
\text{Prob}(V = V_H | s_1 = V_H, s_2 = V_L) &= \frac{\text{Prob}(s_2 = V_L | V = V_H, s_1 = V_H)\text{Prob}(V = V_H | s_1 = V_H)}{\text{Prob}(s_2 = V_L | s_1 = V_H)} \\
&= \frac{\text{Prob}(s_2 = V_L | V = V_H)}{\text{Prob}(s_2 = V_L | s_1 = V_H)} q.
\end{aligned}$$

We have

$$\begin{aligned}
\text{Prob}(s_2 = V_L | s_1 = V_H) &= \text{Prob}(s_2 = V_L, V = V_L | s_1 = V_H) + \text{Prob}(s_2 = V_L, V = V_H | s_1 = V_H) \\
&= \text{Prob}(s_2 = V_L | V = V_L, s_1 = V_H)\text{Prob}(V = V_L | s_1 = V_H) \\
&\quad + \text{Prob}(s_2 = V_L | V = V_H, s_1 = V_H)\text{Prob}(V = V_H | s_1 = V_H) \\
&= (1-q)\text{Prob}(s_2 = V_L | V = V_L) + q\text{Prob}(s_2 = V_L | V = V_H)\big].
\end{aligned}$$

Also,

$$\begin{aligned}
\text{Prob}(s_2 = V_L | V = V_L) &= \frac{\text{Prob}(V = V_L | s_2 = V_L)\text{Prob}(s_2 = V_L)}{\text{Prob}(V = V_L)} \\
&= 2q\text{Prob}(s_2 = V_L),
\end{aligned}$$

and

$$
\begin{aligned}
\text{Prob}(s_2 = V_L | V = V_H) &= \frac{\text{Prob}(V = V_H | s_2 = V_L)\text{Prob}(s_2 = V_L)}{\text{Prob}(V = V_H)} \\
&= 2(1-q)\text{Prob}(s_2 = V_L).
\end{aligned}
$$

Therefore,

$$
\begin{aligned}
\text{Prob}(V = V_H | s_1 = V_H, s_2 = V_L) &= \frac{\text{Prob}(s_2 = V_L | V = V_H)}{\text{Prob}(s_2 = V_L | s_1 = V_H)} q \\
&= \frac{\text{Prob}(s_2 = V_L | V = V_H)}{(1-q)\text{Prob}(s_2 = V_L | V = V_L) + q\text{Prob}(s_2 = V_L | V = V_H)} q \\
&= \frac{1}{2}.
\end{aligned}
$$

This completes the proof. $\square$

Proof of Proposition 3.3 Recall that $\mathbf{1}_m$ denotes an m-vector of ones. It is easy to check that, for an integer $n \geq 1$,

$$
A_U^n = \frac{1}{N_U} \left(\frac{N_U}{N} \right)^n \mathbf{1}_{N_U} \mathbf{1}_{N_U}^\top. \tag{19}
$$

Using this fact, we see from (3) that

$$
\tilde{\overset{c}{V}}_U(t) = \frac{1}{N_U} \left(\frac{N_U}{N} \right)^t \mathbf{1}_{N_U} \mathbf{1}_{N_U}^\top \mathbf{1}_{N_U} \frac{V_H + V_L}{2} + \left[I + \frac{1}{N_U} \sum_{k=1}^{t-1} \left(\frac{N_U}{N} \right)^k \mathbf{1}_{N_U} \mathbf{1}_{N_U}^\top \right] \frac{N_H V_H + N_L V_L}{N} \mathbf{1}_{N_U},
$$

for $t \geq 1$. Hence $\overset{c}{V}{}^i_U(t)$ is the same for all i. We denote the common value by $\overset{c}{V}_U(t)$. It is given by

$$
\begin{aligned}
\overset{c}{V}_U(t) &= \left(\frac{N_U}{N} \right)^t \frac{V_H + V_L}{2} + \left[1 + \sum_{k=1}^{t-1} \left(\frac{N_U}{N} \right)^k \right] \frac{N_H V_H + N_L V_L}{N} \\
&= \left(\frac{N_U}{N} \right)^t \frac{V_H + V_L}{2} + \left[\frac{1 - \left(\frac{N_U}{N} \right)^t}{1 - \frac{N_U}{N}} \right] \frac{N_H V_H + N_L V_L}{N} \\
&= \left(\frac{N_U}{N} \right)^t \frac{V_H + V_L}{2} + \left[1 - \left(\frac{N_U}{N} \right)^t \right] \frac{N_H V_H + N_L V_L}{N_H + N_L} \qquad (20) \\
&= \frac{N_H V_H + N_L V_L}{N_H + N_L} - \left(\frac{N_U}{N} \right)^t \frac{(N_H - N_L)(V_H - V_L)}{2(N_H + N_L)} \\
&= \bar{V} - \left(\frac{N_U}{N} \right)^t \frac{N_H - N_L}{N_H + N_L} \frac{V_H - V_L}{2}. \qquad (21)
\end{aligned}
$$

This establishes the result for $t \geq 1$. From (20), it is easy to check that it holds for $t = 0$ as well. It is immediate from (21) that $\overset{c}{V}_U(t)$ is strictly increasing in t and converges to $\bar{V}$ as t goes to infinity. $\square$

Proof of Proposition 3.4 There are only three possible pairings, HL, LH and UL, that arise when H, L or U is chosen to trade. Using (7) and (8), we have

$$\begin{aligned}\mathbb{E}(\overset{c}{p}_t) &= \frac{N_H}{N}p_{HL} + \frac{N_L}{N}p_{LH} + \frac{N_U}{N}\overset{c}{p}_{UL}(t) \\ &= \frac{1}{N}\Big[(N_H+N_L)\bar{V} - \alpha(N_H-N_L)(V_H-V_L) + N_U[\alpha V_L + (1-\alpha)\overset{c}{V}_U(t)]\Big].\end{aligned}$$

From Proposition 3.3, $\overset{c}{V}_U(t)$ is strictly increasing in t and $\lim_{t\to\infty}\overset{c}{V}_U(t) = \bar{V}$. Therefore, $\mathbb{E}(\overset{c}{p}_t)$ is strictly increasing in t and the limiting value is given by

$$\begin{aligned}\mathbb{E}(\overset{c}{p}_\infty) &= \bar{V} - \frac{\alpha}{N}\Big[(N_H-N_L)(V_H-V_L) + N_U(\bar{V}-V_L)\Big] \\ &= \bar{V} - \alpha\frac{V_H-V_L}{N(N_H+N_L)}\Big[(N_H+N_L)(N_H-N_L) + N_H N_U\Big] \\ &= \bar{V} - \alpha\frac{V_H-V_L}{N(N_H+N_L)}\Big[N(N_H-N_L) + N_L N_U\Big].\end{aligned}$$

This completes the proof. $\square$

Proof of Proposition 3.5 *Proof of (i) and (ii):* Consider the H-star. Starting from $\overset{\star}{V}{}^H_U(0) = (V_H+V_L)/2$, and repeatedly updating using $\overset{\star}{V}{}^H_U(t+1) = [\overset{\star}{V}{}^H_U(t) + V_H]/2$, we obtain

$$\begin{aligned}\overset{\star}{V}{}^H_U(t) &= \frac{1}{2}\left[\frac{1}{2^t}V_L + \left(1 + \frac{1}{2} + \frac{1}{2^2} + \ldots + \frac{1}{2^t}\right)V_H\right] \\ &= \frac{1}{2}\left[\frac{1}{2^t}V_L + 2\left(1 - \frac{1}{2^{t+1}}\right)V_H\right],\end{aligned}$$

which gives us the desired result for the H-star. The proof for the L-star is analogous.

Proof of (iii): Consider the U-star. Recall that in the adjacency matrix A, given by (2), we order the agents so that the uninformed agents come first. Without loss of generality, we place agent 1 at the center of the U-star. Then,

$$A_U = \begin{bmatrix} \frac{1}{N} & \frac{1}{N} & \frac{1}{N} & \ldots & \frac{1}{N} \\ \frac{1}{2} & \frac{1}{2} & 0 & \ldots & 0 \\ \frac{1}{2} & 0 & \frac{1}{2} & \ldots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \frac{1}{2} & 0 & 0 & \ldots & \frac{1}{2} \end{bmatrix} = \begin{bmatrix} \frac{1}{N} & \frac{1}{N}\mathbf{1}^\top_{N_U-1} \\ \frac{1}{2}\mathbf{1}_{N_U-1} & \frac{1}{2}I_{N_U-1} \end{bmatrix},$$

while the matrices A_H and A_L have $1/N$ in each entry for the first row, and 0 in all other entries. We denote by $\mathbb{I}_{N_U-1}$ the $(N_U-1)\times(N_U-1)$ matrix all of whose elements are one. We have

$$A_U^2 = \begin{bmatrix} \frac{1}{N^2}+\frac{N_U-1}{2N} & \left(\frac{1}{N^2}+\frac{1}{2N}\right)\mathbf{1}_{N_U-1}^{\top} \\ \left(\frac{1}{2N}+\frac{1}{4}\right)\mathbf{1}_{N_U-1} & \frac{1}{2N}\mathbb{I}_{N_U-1}+\frac{1}{4}I_{N_U-1} \end{bmatrix}.$$

Note that every element of A_U^2 is strictly positive, which implies that the same is true for higher powers of A_U. It follows that every element of the first column of A_U^t is strictly positive for all $t \geq 1$.

Next, we apply Lemma A.1 to the U-star. The first element of v, defined by (18), is

$$\begin{aligned} v_1 &= \left[\frac{N_U}{N}-1\right]\frac{V_H+V_L}{2}+\frac{N_HV_H+N_LV_L}{N} \\ &= \frac{1}{2N}\Big[2(N_HV_H+N_LV_L)-(N_H+N_L)(V_H+V_L)\Big] \\ &= \frac{1}{2N}(N_H-N_L)(V_H-V_L), \end{aligned} \tag{22}$$

which is strictly positive. All other elements of v are zero. Since every element of the first column of A^t is strictly positive for $t \geq 1$, we have $\overset{\star}{V}{}^{i}_{U}(t+1)-\overset{\star}{V}{}^{i}_{U}(t) > 0$ for all $i \in N_U$, $t \geq 1$. For $t=0$, we see from direct computation that $\overset{\star}{V}{}^{U}_{\breve{U}}(1) > \overset{\star}{V}{}^{U}_{\breve{U}}(0) = (V_H+V_L)/2$, and $\overset{\star}{V}{}^{U}_{U}(1) = \overset{\star}{V}{}^{U}_{U}(0) = (V_H+V_L)/2$.

Limiting beliefs can be derived from (4). We have

$$\overset{\star}{V}{}^{U}_{U}(\infty) - \left[\frac{1}{2}\overset{\star}{V}{}^{U}_{\breve{U}}(\infty)+\frac{1}{2}\overset{\star}{V}{}^{U}_{U}(\infty)\right] = 0,$$

implying that $\overset{\star}{V}{}^{U}_{\breve{U}}(\infty) = \overset{\star}{V}{}^{U}_{U}(\infty)$, i.e. the limiting beliefs of all uninformed agents are the same, whether or not they are at the center. Using (4) once again,

$$\overset{\star}{V}{}^{U}_{\breve{U}}(\infty) - \frac{1}{N}\sum_{j\in N_U}\overset{\star}{V}{}^{U}_{\breve{U}}(\infty) = \frac{N_HV_H+N_LV_L}{N},$$

or

$$\left[1-\frac{N_U}{N}\right]\overset{\star}{V}{}^{U}_{\breve{U}}(\infty) = \frac{N_HV_H+N_LV_L}{N},$$

so that $\overset{\star}{V}{}^{U}_{\breve{U}}(\infty) = \overset{\star}{V}{}^{U}_{U}(\infty) = \bar{V}$.

Proof of (iv): Using the results in parts (i) and (ii), the expected belief of an uninformed agent, conditional on H or L being at the center, is

$$\frac{N_H}{N_H+N_L}\overset{\star}{V}{}^{H}_{U}(t)+\frac{N_L}{N_H+N_L}\overset{\star}{V}{}^{L}_{U}(t) = \bar{V}-\frac{(N_H-N_L)(V_H-V_L)}{2^{t+1}(N_H+N_L)}, \tag{23}$$

which is strictly increasing in t and converges to $\bar{V}$. Combining this with part (iii) gives us the desired result. $\square$

Proof of Proposition 3.6 We introduce an additional fictitious round of trading at date 0, before any learning has taken place. In this round we assume that in the U-star, the uninformed agent at the center trades with L with probability 1, even though her belief is equidistant from the beliefs of L and H. We will show that $\mathbb{E}(\mathring{p}_t)$ is strictly increasing in t for $t \geq 0$. For the purposes of Proposition 3.6, the fictitious trading round at date 0 is irrelevant, since we only need to show monotonicity for $t \geq 1$. But the stronger monotonicity result that we establish, which holds for all $t \geq 0$, will be useful for proving Proposition 3.7.

We denote prices in the H-star, L-star and U-star by $\mathring{p}^H$, $\mathring{p}^L$ and $\mathring{p}^U$, respectively. These prices are random and depend on the trading parties, and when the trade involves an uninformed agent, they depend on the date as well. Let $\mathring{p}^Z_{XY}$ be the price in the Z-star when X is chosen to trade and she trades with Y, and at least one of X or Y is uninformed. We denote an uninformed agent by $\breve{U}$ or U depending on whether this agent is at the center or not. When both trading parties are informed, prices do not depend on who is at the center, and are given by (5)–(8). We begin by computing expected prices in each of the three stars (for each of these stars, the agent at the center is chosen to trade with probability $1/N$).

$$
\begin{aligned}
\mathbb{E}(\mathring{p}^H_t) &= \frac{1}{N} p_{HL} + \left(1 - \frac{1}{N}\right)\left[\frac{N_H - 1}{N-1} p_{HH} + \frac{N_L}{N-1} p_{LH} + \frac{N_U}{N-1} \mathring{p}^H_{UH}(t)\right] \\
&= \frac{1}{N}\Big[V_H - \alpha(V_H - V_L) + (N_H - 1)V_H + N_L V_L + \alpha N_L (V_H - V_L) + N_U[\alpha V_H + (1-\alpha)\mathring{V}^H_U(t)]\Big] \\
&= \frac{1}{N}\Big[N_H V_H + N_L V_L + \alpha(N_L - 1)(V_H - V_L) + N_U[\alpha V_H + (1-\alpha)\mathring{V}^H_U(t)]\Big], \\
\mathbb{E}(\mathring{p}^L_t) &= \frac{1}{N} p_{LH} + \left(1 - \frac{1}{N}\right)\left[\frac{N_H}{N-1} p_{HL} + \frac{N_L - 1}{N-1} p_{LL} + \frac{N_U}{N-1} \mathring{p}^L_{UL}(t)\right] \\
&= \frac{1}{N}\Big[V_L + \alpha(V_H - V_L) + N_H V_H - \alpha N_H (V_H - V_L) + (N_L - 1)V_L + N_U[\alpha V_L + (1-\alpha)\mathring{V}^L_U(t)]\Big] \\
&= \frac{1}{N}\Big[N_H V_H + N_L V_L - \alpha(N_H - 1)(V_H - V_L) + N_U[\alpha V_L + (1-\alpha)\mathring{V}^L_U(t)]\Big], \\
\mathbb{E}(\mathring{p}^U_t) &= \frac{1}{N} \mathring{p}^U_{\breve{U}L}(t) + \left(1 - \frac{1}{N}\right)\left[\frac{N_H}{N-1} \mathring{p}^U_{H\breve{U}} + \frac{N_L}{N-1} \mathring{p}^U_{L\breve{U}} + \frac{N_U - 1}{N-1} \mathring{p}^U_{U\breve{U}}\right] \\
&= \frac{1}{N}\Big[\alpha V_L + (1-\alpha)\mathring{V}^U_{\breve{U}}(t) + N_H[\alpha \mathring{V}^U_{\breve{U}}(t) + (1-\alpha)V_H] + N_L[\alpha \mathring{V}^U_{\breve{U}}(t) + (1-\alpha)V_L] \\
&\qquad + (N_U - 1)[\alpha \mathring{V}^U_{\breve{U}}(t) + (1-\alpha)\mathring{V}^U_U(t)]\Big] \\
&= \frac{1}{N}\Big[\alpha V_L + (1-\alpha)(N_H V_H + N_L V_L) + [1 + \alpha(N-2)]\mathring{V}^U_{\breve{U}}(t) + (1-\alpha)(N_U - 1)\mathring{V}^U_U(t)\Big].
\end{aligned}
$$

Therefore,

$$\begin{aligned}\mathbb{E}(\mathring{p}_t) &= \frac{N_H}{N}\mathbb{E}(\mathring{p}_t^H) + \frac{N_L}{N}\mathbb{E}(\mathring{p}_t^L) + \frac{N_U}{N}\mathbb{E}(\mathring{p}_t^U) \\ &= \frac{1}{N^2}\Big[N(N_H V_H + N_L V_L) - \alpha(N_H - N_L)(V_H - V_L) + \alpha N_U V_L \\ &\quad + (1-\alpha)N_U\Big[N_H \mathring{V}_U^H(t) + N_L \mathring{V}_U^L(t)\Big] \\ &\quad + N_U[1+\alpha(N-2)]\mathring{V}_{\breve{U}}^U(t) + (1-\alpha)N_U(N_U-1)\mathring{V}_U^U(t)\Big]. \end{aligned} \tag{24}$$

From (23), $N_H \mathring{V}_U^H(t) + N_L \mathring{V}_U^L(t)$ is strictly increasing in t, for $t \geq 0$, and from Proposition 3.5 (iii), both $\mathring{V}_{\breve{U}}^U(t)$ and $\mathring{V}_U^U(t)$ are increasing in t, for $t \geq 0$. It follows that $\mathbb{E}(\mathring{p}_t)$ is strictly increasing in t, for $t \geq 0$.

In order to calculate the limiting expected price, we note that $\mathring{V}_U^H(\infty) = V_H$, $\mathring{V}_U^L(\infty) = V_L$, and $\mathring{V}_{\breve{U}}^U(\infty) = \mathring{V}_U^U(\infty) = \bar{V}$, from Proposition 3.5. Substituting into (24), we obtain

$$\begin{aligned}\mathbb{E}(\mathring{p}_\infty) &= \frac{1}{N^2}\Big[N(N_H+N_L)\bar{V} - \alpha(N_H-N_L)(V_H-V_L) + \alpha N_U V_L \\ &\quad + (1-\alpha)N_U(N_H+N_L)\bar{V} + N_U\Big[\alpha(N-1) + (1-\alpha)N_U\Big]\bar{V}\Big] \\ &= \bar{V} + \frac{\alpha}{N^2}\Big[-(N_H-N_L)(V_H-V_L) + N_U V_L - N_U\bar{V}\Big] \\ &= \bar{V} - \frac{\alpha}{N^2(N_H+N_L)}\Big[(N_H+N_L)(N_H-N_L)(V_H-V_L) - N_U(N_H+N_L)V_L \\ &\quad + N_U(N_H V_H + N_L V_L)\Big] \\ &= \bar{V} - \alpha\frac{V_H - V_L}{N^2(N_H+N_L)}\Big[N(N_H-N_L) + N_L N_U\Big]. \end{aligned}$$

This completes the proof. $\square$

Proof of Proposition 3.7 The result in part (i) is immediate from (9) and (10). For part (ii), we first calculate the expected price in the fictitious trading round at date 0 (see the beginning of the proof of Proposition 3.6). Since $\mathring{V}_U^H(0) = \mathring{V}_U^L(0) = \mathring{V}_{\breve{U}}^U(0) = \mathring{V}_U^U(0) = (V_H + V_L)/2$, we have, from (24):

$$\begin{aligned}\mathbb{E}(\mathring{p}_0) &= \frac{1}{N^2}\left[N(N_H V_H + N_L V_L) - \alpha(N_H-N_L)(V_H-V_L) + \alpha N_U V_L + N_U(N-\alpha)\frac{V_H+V_L}{2}\right] \\ &= \frac{1}{2N^2}\Big[N\Big[2N_H V_H + 2N_L V_L + N_U(V_H+V_L)\Big] - 2\alpha(N_H-N_L)(V_H-V_L) - \alpha N_U(V_H-V_L)\Big] \\ &= \frac{V_H+V_L}{2} + \frac{(N_H-N_L)(V_H-V_L)}{2N} - \alpha\frac{V_H-V_L}{2N^2}\Big[2(N_H-N_L) + N_U\Big]. \end{aligned}$$

Comparing this expression to (9), we have

$$\begin{aligned}
\mathbb{E}(\overset{\star}{p}_0) - \mathbb{E}(\overset{c}{p}_\infty) &= \frac{V_H + V_L}{2} - \bar{V} + \frac{(N_H - N_L)(V_H - V_L)}{2N} + \alpha \frac{V_H - V_L}{2N^2(N_H + N_L)} \\
&\quad \cdot \Big[2N^2(N_H - N_L) + 2NN_L N_U - 2(N_H + N_L)(N_H - N_L) - N_U(N_H + N_L)\Big] \\
&= -\frac{N_U}{2N(N_H + N_L)}(N_H - N_L)(V_H - V_L) + \alpha \frac{V_H - V_L}{2N^2(N_H + N_L)} \\
&\quad \cdot \Big[2N^2(N_H - N_L) + 2NN_L N_U - 2(N_H + N_L)(N_H - N_L) - N_U(N_H + N_L)\Big],
\end{aligned}$$

which is increasing in α. Therefore, in order to prove that $\mathbb{E}(\overset{\star}{p}_0) - \mathbb{E}(\overset{c}{p}_\infty) > 0$ for $\alpha \geq 1/2$, it suffices to show that $\mathbb{E}(\overset{\star}{p}_0) - \mathbb{E}(\overset{c}{p}_\infty) > 0$ when $\alpha = 1/2$. We say that $F \propto G$ if F and G have the same sign ($F = cG$, for some $c > 0$). We have

$$\begin{aligned}
[\mathbb{E}(\overset{\star}{p}_0) - \mathbb{E}(\overset{c}{p}_\infty)]\Big|_{\alpha=1/2} &\propto (N_H - N_L)\Big[-2NN_U + 2N^2 - 2(N_H + N_L)\Big] + 2NN_L N_U - N_U(N_H + N_L) \\
&= 2(N-1)(N_H + N_L)(N_H - N_L) + 2NN_L N_U - N_U(N_H + N_L) \\
&> 0.
\end{aligned}$$

In the proof of Proposition 3.6 we showed that $\mathbb{E}(\overset{\star}{p}_t)$ is strictly increasing in t, for all $t \geq 0$. Also, from Proposition 3.4, $\mathbb{E}(\overset{c}{p}_t)$ is strictly increasing in t for $t \geq 1$. Therefore, we have the following chain of inequalities:

$$\mathbb{E}(\overset{\star}{p}_t) > \mathbb{E}(\overset{\star}{p}_0) > \mathbb{E}(\overset{c}{p}_\infty) > \mathbb{E}(\overset{c}{p}_t), \qquad \forall t \geq 1.$$

This establishes the result. Note that there is no trading at date 0. The expected price in the fictitious trading round at date 0, $\mathbb{E}(\overset{\star}{p}_0)$, serves as a lower bound for $\mathbb{E}(\overset{\star}{p}_t)$ for $t \geq 1$. Since $\mathbb{E}(\overset{c}{p}_\infty)$ is an upper bound for $\mathbb{E}(\overset{c}{p}_t)$ for $t \geq 1$, it suffices to show that $\mathbb{E}(\overset{\star}{p}_0) > \mathbb{E}(\overset{c}{p}_\infty)$. The lowest expected price in the star (across time) is higher than the highest expected price in the complete network. $\square$

Proof of Proposition 3.8 The ex ante payoff of an agent is $\tilde{\Pi}$ and the payoff for the buyer is Π (see Section 3.3.2). We use Π_{XY} to denote the value of Π when the agent chosen to trade is X and she trades with Y. The corresponding value for the seller $-\Pi_{XY}$. When both X and Y are informed, Π_{XY} does not depend on the network or date. Using (5)–(8), we have

$$\Pi_{HH} = V_H - p_{HH} = 0, \tag{25}$$

$$\Pi_{LL} = V_H - p_{LH} = V_H - V_L, \tag{26}$$

$$\Pi_{HL} = V_H - p_{HL} = \alpha(V_H - V_L), \tag{27}$$
$$\Pi_{LH} = V_H - p_{LH} = (1-\alpha)(V_H - V_L). \tag{28}$$

For the complete network, we denote the ex ante payoff by $\tilde{\Pi}^c$, and the payoff of the buyer by $\overset{c}{\Pi}$. The possible pairings are HL, LH and UL. The values of Π for HL and LH are given above. For the pairing UL, we write the payoff as $\overset{c}{\Pi}_{UL}$. It is given by

$$\overset{c}{\Pi}_{UL} = V_H - [\alpha V_L + (1-\alpha)\bar{V}] = \frac{\alpha N_H + N_L}{N_H + N_L}(V_H - V_L),$$

so that, using (13),

$$\begin{aligned}\mathrm{Var}(\tilde{\Pi}^c) = \mathbb{E}(\overset{c}{\Pi}{}^2) &= \frac{N_H}{N}(\Pi_{HL})^2 + \frac{N_L}{N}(\Pi_{LH})^2 + \frac{N_U}{N}(\overset{c}{\Pi}_{UL})^2 \\ &= \frac{(V_H - V_L)^2}{N}\left[\alpha^2 N_H + (1-\alpha)^2 N_L + N_U\left(\frac{\alpha N_H + N_L}{N_H + N_L}\right)^2\right]. \end{aligned} \tag{29}$$

For the star network, we use the notation $\tilde{\Pi}^*$ for the ex ante payoff, and $\overset{\star}{\Pi}$ for the payoff of the buyer. Let $\overset{\star}{\Pi}{}^Z_{XY}$ be the value of $\overset{\star}{\Pi}$ in the Z-star when the agent chosen to trade is X and she trades with Y, and at least one of X or Y is uninformed. We have

$$\overset{\star}{\Pi}{}^H_{UH} = 0, \tag{30}$$
$$\overset{\star}{\Pi}{}^L_{UL} = V_H - V_L, \tag{31}$$
$$\overset{\star}{\Pi}{}^U_{UL} = V_H - [\alpha V_L + (1-\alpha)\bar{V}] = \frac{\alpha N_H + N_L}{N_H + N_L}(V_H - V_L), \tag{32}$$
$$\overset{\star}{\Pi}{}^U_{HU} = V_H - [(1-\alpha)V_H + \alpha\bar{V}] = \frac{\alpha N_L}{N_H + N_L}(V_H - V_L), \tag{33}$$
$$\overset{\star}{\Pi}{}^U_{LU} = V_H - [(1-\alpha)V_L + \alpha\bar{V}] = \frac{(1-\alpha)N_H + N_L}{N_H + N_L}(V_H - V_L), \tag{34}$$
$$\overset{\star}{\Pi}{}^U_{UU} = V_H - \bar{V} = \frac{N_L}{N_H + N_L}(V_H - V_L). \tag{35}$$

Now we calculate the variance of the ex ante payoff:

$$\begin{aligned}\mathrm{Var}(\tilde{\Pi}^*) = \mathbb{E}(\overset{\star}{\Pi}{}^2) = &= \left[\frac{N_H}{N}\frac{1}{N} + \frac{N_L}{N}\left(1 - \frac{1}{N}\right)\frac{N_H}{N-1}\right]\Pi^2_{HL} + \left[\frac{N_L}{N}\frac{1}{N} + \frac{N_H}{N}\left(1 - \frac{1}{N}\right)\frac{N_L}{N-1}\right]\Pi^2_{LH} \\ &+ \frac{N_H}{N}\left(1 - \frac{1}{N}\right)\frac{N_H - 1}{N-1}\Pi^2_{HH} + \frac{N_L}{N}\left(1 - \frac{1}{N}\right)\frac{N_L - 1}{N-1}\Pi^2_{LL} \\ &+ \frac{N_H}{N}\left(1 - \frac{1}{N}\right)\frac{N_U}{N-1}(\overset{\star}{\Pi}{}^H_{UH})^2 + \frac{N_L}{N}\left(1 - \frac{1}{N}\right)\frac{N_U}{N-1}(\overset{\star}{\Pi}{}^L_{UL})^2 \\ &+ \frac{N_U}{N}\frac{1}{N}(\overset{\star}{\Pi}{}^U_{UL})^2 + \frac{N_U}{N}\left(1 - \frac{1}{N}\right)\frac{N_H}{N-1}(\overset{\star}{\Pi}{}^U_{HU})^2 + \end{aligned}$$

$$
\begin{aligned}
&+ \frac{N_U}{N}\left(1-\frac{1}{N}\right)\frac{N_L}{N-1}(\overset{\star}{\Pi}{}^U_{LU})^2 + \frac{N_U}{N}\left(1-\frac{1}{N}\right)\frac{N_U-1}{N-1}(\overset{\star}{\Pi}{}^U_{UU})^2 \\
= &\frac{(V_H-V_L)^2}{N^2}\Bigg[\alpha^2 N_H(N_L+1) + (1-\alpha)^2 N_L(N_H+1) + N_L(N_L+N_U-1) \\
&+ N_U\left(\frac{\alpha N_H + N_L}{N_H+N_L}\right)^2 + N_H N_U\left(\frac{\alpha N_L}{N_H+N_L}\right)^2 \\
&+ N_L N_U\left(\frac{(1-\alpha)N_H+N_L}{N_H+N_L}\right)^2 + N_U(N_U-1)\left(\frac{N_L}{N_H+N_L}\right)^2\Bigg].
\end{aligned}
$$

Comparing this expression with (29), we have

$$
\begin{aligned}
\mathrm{Var}(\tilde{\Pi}^c) - \mathrm{Var}(\tilde{\Pi}^*) = &\frac{(V_H-V_L)^2}{N^2}\Bigg[\alpha^2 N_H(N-N_L-1) + (1-\alpha)^2 N_L(N-N_H-1) \\
&- N_L(N_L+N_U-1) + N_U(N-1)\left(\frac{\alpha N_H+N_L}{N_H+N_L}\right)^2 - N_H N_U\left(\frac{\alpha N_L}{N_H+N_L}\right)^2 \\
&- N_L N_U\left(\frac{(1-\alpha)N_H+N_L}{N_H+N_L}\right)^2 - N_U(N_U-1)\left(\frac{N_L}{N_H+N_L}\right)^2\Bigg].
\end{aligned}
$$

It is straightforward to check that this expression is increasing in α for $\alpha \geq 1/2$. Therefore, in order to prove that $\mathrm{Var}(\tilde{\Pi}^c) - \mathrm{Var}(\tilde{\Pi}^*) > 0$ for $\alpha \geq 1/2$, it suffices to show that it is positive for $\alpha = 1/2$. We have

$$
\begin{aligned}
\left.[\mathrm{Var}(\tilde{\Pi}^c) - \mathrm{Var}(\tilde{\Pi}^*)]\right|_{\alpha=1/2} \propto\ & N_H(N-N_L-1) + N_L(N-N_H-1) \\
&- 4N_L(N_L+N_U-1) + N_U(N-N_L-1)\left(1+\frac{N_L}{N_H+N_L}\right)^2 \\
&- N_U[N_H + 4(N_U-1)]\left(\frac{N_L}{N_H+N_L}\right)^2 \\
=\ & (N_H+N_U)(N_H-N_L) + (N_H+N_U-3N_L)(N_L+N_U-1) \\
&+ \frac{2N_L N_U(N-N_L-1)}{N_H+N_L} - 3N_U(N_U-1)\left(\frac{N_L}{N_H+N_L}\right)^2 \\
=\ & (N_H+N_U)(N_H-N_L) + (N_H+N_U-3N_L)(N_L+N_U-1) \\
&+ \frac{2N_L N_U(N_H+N_U-1)(N_H-N_L+2N_L)}{(N_H+N_L)^2} \\
&- 3N_U(N_U-1)\left(\frac{N_L}{N_H+N_L}\right)^2,
\end{aligned}
$$

which is positive if $N_H + N_U \geq 3N_L$. $\square$

Proof of Proposition 4.2 *Proof of (i):* Suppose the public signal is correct. Then optimists do not update, while all other agents do.

Consider the complete network. We reorder agents so that pessimists come first, followed by uninformed agents, and then optimists. The adjacency matrix takes the form

$$\begin{pmatrix} A_{L,U} & A_H \\ 0 & I \end{pmatrix},$$

where $A_{L,U}$ is the $(N_L + N_U) \times (N_L + N_U)$ matrix that describes the listening structure among pessimists and uninformed agents, while A_H specifies the weights that pessimists and uninformed agents put on the beliefs of optimists. Every element of $A_{L,U}$ and A_H is $1/N$. Let $\tilde{V}_{L,U}(t)$ be the belief vector for pessimists and uninformed agents. Then, by Proposition 1 of Pederson (2022), we have the following expression, analogous to (3):

$$\tilde{\overset{c}{V}}_{L,U}(t) = A_{L,U}^t \begin{pmatrix} \mathbf{1}_{N_L} V_L \\ \mathbf{1}_{N_U} V_H \end{pmatrix} + \sum_{k=0}^{t-1} A_{L,U}^k \mathbf{1}_{N_L+N_U} \frac{N_H V_H}{N}.$$

Equation (19) applies to powers of $A_{L,U}$ with $N_L + N_U$ replacing N_U. Thus we have

$$\begin{aligned} \tilde{\overset{c}{V}}_{L,U}(t) = {} & \frac{1}{N_L + N_U} \left(\frac{N_L + N_U}{N} \right)^t \mathbf{1}_{N_L+N_U} \mathbf{1}_{N_L+N_U}^\top \begin{pmatrix} \mathbf{1}_{N_L} V_L \\ \mathbf{1}_{N_U} V_H \end{pmatrix} \\ & + \left[I + \frac{1}{N_L + N_U} \sum_{k=1}^{t-1} \left(\frac{N_L + N_U}{N} \right)^k \mathbf{1}_{N_L+N_U} \mathbf{1}_{N_L+N_U}^\top \right] \mathbf{1}_{N_L+N_U} \frac{N_H V_H}{N}, \end{aligned}$$

for $t \geq 1$. Hence all elements of the vector $\tilde{\overset{c}{V}}_{L,U}(t)$ are the same. We denote the common value by $\overset{c}{V}_{L,U}(t) = \overset{c}{V}_L(t) = \overset{c}{V}_U(t)$. It is given by

$$\begin{aligned} \overset{c}{V}_{L,U}(t) &= \left(\frac{N_L + N_U}{N} \right)^t \frac{N_L V_L + N_U V_H}{N_L + N_U} + \left[1 + \sum_{k=1}^{t-1} \left(\frac{N_L + N_U}{N} \right)^k \right] \frac{N_H V_H}{N} \\ &= \left(\frac{N_L + N_U}{N} \right)^t \frac{N_L V_L + N_U V_H}{N_L + N_U} + \left[1 - \left(\frac{N_L + N_U}{N} \right)^t \right] V_H \\ &= V_H - \left(\frac{N_L + N_U}{N} \right)^t \frac{N_L (V_H - V_L)}{N_L + N_U}. \end{aligned} \tag{36}$$

If H is chosen to trade, she trades with L or U, who have the same beliefs, and if L or U is chosen to trade, she trades with H, giving rise to the following prices:

$$\begin{aligned} \overset{c}{p}_{HL}(t) = \overset{c}{p}_{HU}(t) = (1 - \alpha) V_H + \alpha \overset{c}{V}_{L,U}(t) = V_H - \alpha [V_H - \overset{c}{V}_{L,U}(t)], \\ \overset{c}{p}_{LH}(t) = \overset{c}{p}_{LH}(t) = \alpha V_H + (1 - \alpha) \overset{c}{V}_{L,U}(t) = V_H - (1 - \alpha) [V_H - \overset{c}{V}_{L,U}(t)]. \end{aligned}$$

Therefore,

$$\begin{aligned}
\mathbb{E}(\overset{c}{p}_t) &= \frac{N_H}{N}\overset{c}{p}_{HL}(t) + \frac{N_L + N_U}{N}\overset{c}{p}_{LH}(t) \\
&= V_H - \frac{V_H - V_{L,U}(t)}{N}\Big[\alpha N_H + (1-\alpha)(N_L + N_U)\Big]. \qquad (37)
\end{aligned}$$

Now consider the H-star. Optimists do not update their beliefs, including the optimist at the center. Since uninformed agents have the same beliefs as the center, they do not update either. Only pessimists update. Starting from $\overset{\star}{V}{}_L^H(0) = V_L$, and repeatedly updating using $\overset{\star}{V}{}_L^H(t) = [\overset{\star}{V}{}_L^H(t-1) + V_H]/2$, we obtain

$$\begin{aligned}
\overset{\star}{V}{}_L^H(t) &= \frac{1}{2^t}V_L + \frac{1}{2}\left(1 + \frac{1}{2} + \frac{1}{2^2} + \ldots + \frac{1}{2^{t-1}}\right)V_H \\
&= \frac{1}{2^t}V_L + \left(1 - \frac{1}{2^t}\right)V_H \\
&= V_H - \frac{1}{2^t}(V_H - V_L). \qquad (38)
\end{aligned}$$

If the center is chosen to trade, she trades with L. If the agent chosen to trade is not at the center, she must trade with the center. Prices for different pairings are

$$\begin{aligned}
\overset{\star}{p}{}^H_{HL}(t) &= (1-\alpha)V_H + \alpha\overset{\star}{V}{}_L^H(t) = V_H - \alpha[V_H - \overset{\star}{V}{}_L^H(t)], \\
\overset{\star}{p}{}^H_{LH}(t) &= \alpha V_H + (1-\alpha)\overset{\star}{V}{}_L^H(t) = V_H - (1-\alpha)[V_H - \overset{\star}{V}{}_L^H(t)], \\
\overset{\star}{p}{}^H_{HH} &= \overset{\star}{p}{}^H_{UH} = V_H.
\end{aligned}$$

Therefore,

$$\begin{aligned}
\mathbb{E}(\overset{\star}{p}{}^H_t) &= \frac{1}{N}\overset{\star}{p}{}^H_{HL}(t) + \left(1 - \frac{1}{N}\right)\left[\frac{N_H - 1}{N-1}\overset{\star}{p}{}^H_{HH} + \frac{N_L}{N-1}\overset{\star}{p}{}^H_{LH}(t) + \frac{N_U}{N-1}\overset{\star}{p}{}^H_{UH}\right] \\
&= V_H - \frac{V_H - \overset{\star}{V}{}_L^H(t)}{N}\Big[\alpha + (1-\alpha)N_L\Big]. \qquad (39)
\end{aligned}$$

Now we compare prices in the complete network and in the H-star, using (36)–(39):

$$\mathbb{E}(\overset{c}{p}_t) - \mathbb{E}(\overset{\star}{p}{}^H_t) = \frac{V_H - V_L}{2^t N}[\alpha + (1-\alpha)N_L] - \left(\frac{N_L + N_U}{N}\right)^t \frac{N_L(V_H - V_L)}{N(N_L + N_U)}\Big[\alpha N_H + (1-\alpha)(N_L + N_U)\Big],$$

which is decreasing in α since $N_H > N_L + N_U$. Hence,

$$\mathbb{E}(\overset{c}{p}_t) - \mathbb{E}(\overset{\star}{p}{}^H_t) \geq [\mathbb{E}(\overset{c}{p}_t) - \mathbb{E}(\overset{\star}{p}{}^H_t)]\Big|_{\alpha=1}$$

$$\propto 1 - \left[\frac{2(N_L + N_U)}{N}\right]^t \frac{N_H N_L}{N_L + N_U},$$

which is positive for sufficiently large t (note that $N_H > N_L + N_U$ implies that $2(N_L + N_U) < N$).

Next, we compare the U-star to the H-star. The initial belief of U agents is V_H. We denote the U at the center by $\breve{U}$ to distinguish this agent from a U who is not at the center. In the first updating round at $t = 1$, the belief of the center goes down from $\overset{\star}{V}{}^U_{\breve{U}}(0) = V_H$ to

$$\overset{\star}{V}{}^U_{\breve{U}}(1) = \frac{(N_H + N_U)V_H + N_L V_L}{N} = V_H - \frac{N_L}{N}(V_H - V_L).$$

For $t \geq 1$, $\overset{\star}{V}{}^U_{\breve{U}}(t)$ increases monotonically to V_H. Since $\overset{\star}{V}{}^U_L(1) = \overset{\star}{V}{}^H_L(1) = (V_H + V_L)/2$, we have, for $t \geq 1$,

$$\overset{\star}{V}{}^U_L(t) \leq \overset{\star}{V}{}^H_L(t) = V_H - [V_H - \overset{\star}{V}{}^H_L(t)].$$

Clearly, $\overset{\star}{V}{}^U_U(t) \leq V_H$.

For now we restrict ourselves to $t \geq 2$. We will check the case of $t = 1$ later.

$$\begin{aligned}
\overset{\star}{V}{}^U_{\breve{U}}(t) &= \frac{1}{N}\Big[N_H V_H + N_L \overset{\star}{V}{}^U_L(t-1) + (N_U - 1)\overset{\star}{V}{}^U_U(t-1) + \overset{\star}{V}{}^U_{\breve{U}}(t-1)\Big] \\
&< \frac{1}{N}\Big[(N_H + N_U - 1)V_H + N_L \overset{\star}{V}{}^H_L(t-1) + \overset{\star}{V}{}^U_{\breve{U}}(t)\Big],
\end{aligned}$$

so that, using (38),

$$\begin{aligned}
\overset{\star}{V}{}^U_{\breve{U}}(t) &< \frac{1}{N-1}\Big[(N_H + N_U - 1)V_H + N_L \overset{\star}{V}{}^H_L(t-1)\Big] \\
&= V_H - \frac{N_L}{N-1}[V_H - \overset{\star}{V}{}^H_L(t-1)] \\
&= V_H - 2\frac{N_L}{N-1}[V_H - \overset{\star}{V}{}^H_L(t)].
\end{aligned}$$

Prices for different pairings satisfy

$$\begin{aligned}
\overset{\star}{p}{}^U_{\breve{U}H}(t) &= \alpha V_H + (1-\alpha)\overset{\star}{V}{}^U_{\breve{U}}(t) < V_H - 2(1-\alpha)\frac{N_L}{N-1}[V_H - \overset{\star}{V}{}^H_L(t)], \\
\overset{\star}{p}{}^U_{H\breve{U}}(t) &= (1-\alpha)V_H + \alpha \overset{\star}{V}{}^U_{\breve{U}}(t) < V_H - 2\alpha\frac{N_L}{N-1}[V_H - \overset{\star}{V}{}^H_L(t)], \\
\overset{\star}{p}{}^U_{L\breve{U}}(t) &= (1-\alpha)\overset{\star}{V}{}^U_L + \alpha \overset{\star}{V}{}^U_{\breve{U}}(t) < V_H - \left[(1-\alpha) + 2\alpha\frac{N_L}{N-1}\right][V_H - \overset{\star}{V}{}^H_L(t)], \\
\overset{\star}{p}{}^U_{U\breve{U}}(t) &= (1-\alpha)\overset{\star}{V}{}^U_U + \alpha \overset{\star}{V}{}^U_{\breve{U}}(t) < V_H - 2\alpha\frac{N_L}{N-1}[V_H - \overset{\star}{V}{}^H_L(t)].
\end{aligned}$$

Note that if the center is chosen trade, she may choose to trade with L or with H, depending on the parameters, at a price which is less than or equal to $\overset{\star}{p}^U_{\breve{U}H}$. The expected price in the U-star satisfies

$$\mathbb{E}(\overset{\star}{p}^U_t) \leq \frac{1}{N}\overset{\star}{p}^U_{\breve{U}H}(t) + \left(1 - \frac{1}{N}\right)\left[\frac{N_H}{N-1}\overset{\star}{p}^U_{H\breve{U}}(t) + \frac{N_L}{N-1}\overset{\star}{p}^U_{L\breve{U}}(t) + \frac{N_U - 1}{N-1}\overset{\star}{p}^U_{U\breve{U}}(t)\right]$$
$$< V_H - \frac{N_L}{N(N-1)}\Big[2\alpha(N-1) + (1-\alpha)(N+1)\Big][V_H - \overset{\star}{V}^H_L(t)].$$

Therefore, using (39),

$$\mathbb{E}(\overset{\star}{p}^H_t) - \mathbb{E}(\overset{\star}{p}^U_t) > \frac{V_H - \overset{\star}{V}^H_L(t)}{N(N-1)}\Big[N_L\Big[2\alpha(N-1) + (1-\alpha)(N+1)\Big] - (N-1)\Big[\alpha + (1-\alpha)N_L\Big]\Big],$$

which is positive. This establishes the desired result for $t \geq 2$.

For $t = 1$, we can calculate all the relevant prices. From (38) and (39),

$$\mathbb{E}(\overset{\star}{p}^H_1) = V_H - \frac{V_H - V_L}{2N}\Big[\alpha + (1-\alpha)N_L\Big].$$

In the U-star,

$$\overset{\star}{p}^U_{\breve{U}H}(1) = V_H - (1-\alpha)\frac{N_L}{N}(V_H - V_L),$$
$$\overset{\star}{p}^U_{H\breve{U}}(1) = V_H - \alpha\frac{N_L}{N}(V_H - V_L),$$
$$\overset{\star}{p}^U_{L\breve{U}}(1) = V_H - (1-\alpha)\frac{V_H - V_L}{2} - \alpha\frac{N_L}{N}(V_H - V_L),$$
$$\overset{\star}{p}^U_{U\breve{U}}(1) = V_H - \alpha\frac{N_L}{N}(V_H - V_L),$$

so that

$$\mathbb{E}(\overset{\star}{p}^U_1) \leq V_H - \frac{N_L(V_H - V_L)}{2N^2}\Big[2\alpha(N-1) + (1-\alpha)(N+2)\Big].$$

Therefore,

$$\mathbb{E}(\overset{\star}{p}^H_1) - \mathbb{E}(\overset{\star}{p}^U_1) \geq \frac{V_H - V_L}{2N^2}\Big[N_L\Big[2\alpha(N-1) + (1-\alpha)(N+2)\Big] - N\Big[\alpha + (1-\alpha)N_L\Big]\Big],$$

which is positive.

Finally, we compare the L-star to the U-star. We denote the L at the center by $\breve{L}$ to distinguish this agent from an L who is not at the center. At $t = 1$, we have $\overset{\star}{V}^L_{\breve{L}}(1) = \overset{\star}{V}^U_{\breve{U}}(1) = [(N_H + N_U)V_H + N_L V_L]/N$, $\overset{\star}{V}^L_U(1) = (V_H + V_L)/2 < \overset{\star}{V}^U_U(1) = V_H$, and $\overset{\star}{V}^L_L(1) = V_L < \overset{\star}{V}^U_L(1) = (V_H + V_L)/2$. Hence, for all $t \geq 1$, $\overset{\star}{V}^L_{\breve{L}}(t) \leq \overset{\star}{V}^U_{\breve{U}}(t)$, $\overset{\star}{V}^L_U(t) < \overset{\star}{V}^U_U(t)$, and

$\overset{\star}{V}{}_L^L(t) < \overset{\star}{V}{}_L^U(t)$. If a noncentral agent is chosen to trade, she must trade with the center, resulting in a price that is lower in the L-star. If the center is chosen to trade, she always trades with L in the L-star, but may trade with L or with H in the U-star. Either way, the price in the L-star is lower. It follows that $\mathbb{E}(\overset{\star}{p}{}_t^L) < \mathbb{E}(\overset{\star}{p}{}_t^U)$, for all $t \geq 1$.

Proof of (ii): Suppose the public signal is incorrect. Then pessimists do not update, while all other agents do. In the complete network, using the analysis and notation analogous to that in part (i), we get

$$\overset{c}{V}_{H,U}(t) = V_L + \left(\frac{N_H + N_U}{N}\right)^t \frac{N_H(V_H - V_L)}{N_H + N_U}, \tag{40}$$

$$\mathbb{E}(\overset{c}{p}_t) = V_L + \frac{\overset{c}{V}_{H,U}(t) - V_L}{N}\Big[\alpha N_L + (1-\alpha)(N_H + N_U)\Big]. \tag{41}$$

The L-star is analogous to the H-star in part (i). We obtain

$$\overset{\star}{V}{}_H^L(t) = V_L + \frac{1}{2^t}(V_H - V_L),$$
$$\mathbb{E}(\overset{\star}{p}{}_t^L) = V_L + \frac{\overset{\star}{V}{}_H^L(t) - V_L}{N}\Big[\alpha + (1-\alpha)N_H\Big].$$

Therefore,

$$\mathbb{E}(\overset{c}{p}_t) - \mathbb{E}(\overset{\star}{p}{}_t^L) = \frac{V_H - V_L}{2^t N^2}\left[2\left(\frac{2(N_H + N_U)}{N}\right)^{t-1} N_H\Big[\alpha N_L + (1-\alpha)(N_H + N_U)\Big] - N[\alpha + (1-\alpha)N_H]\right].$$

Since $N_H > N_L + N_U$, we have $2(N_H + N_U) > 2N_H > N$, and hence $\mathbb{E}(\overset{c}{p}_t) - \mathbb{E}(\overset{\star}{p}{}_t^L) > 0$.

Now consider the H-star. The beliefs of H agents converge monotonically to V_L. The H agent at the center, who we identify by the notation $\breve{H}$, converges faster to V_L than other H agents. For $t \geq 1$, we have $\overset{\star}{V}{}_H^H(t) > \overset{\star}{V}{}_{\breve{H}}^H(t) \geq \overset{c}{V}_H(t)$, and $\overset{\star}{V}{}_U^H(t) > V_L$. Using (40), the prices for different pairings satisfy

$$\overset{\star}{p}{}_{\breve{H}L}^H(t) = \alpha V_L + (1-\alpha)\overset{\star}{V}{}_{\breve{H}}^H(t) \geq \alpha V_L + (1-\alpha)\overset{c}{V}_H(t) = V_L + (1-\alpha)[\overset{c}{V}_H(t) - V_L],$$
$$\overset{\star}{p}{}_{L\breve{H}}^H(t) = (1-\alpha)V_L + \alpha\overset{\star}{V}{}_{\breve{H}}^H(t) \geq (1-\alpha)V_L + \alpha\overset{c}{V}_H(t) = V_L + \alpha[\overset{c}{V}_H(t) - V_L],$$
$$\overset{\star}{p}{}_{U\breve{H}}^H(t) = (1-\alpha)\overset{\star}{V}{}_U^H(t) + \alpha\overset{\star}{V}{}_{\breve{H}}^H(t) > (1-\alpha)V_L + \alpha\overset{c}{V}_H(t) = V_L + \alpha[\overset{c}{V}_H(t) - V_L],$$
$$\overset{\star}{p}{}_{H\breve{H}}^H(t) = (1-\alpha)\overset{\star}{V}{}_H^H(t) + \alpha\overset{\star}{V}{}_{\breve{H}}^H(t) > \overset{c}{V}_H(t) = V_L + [\overset{c}{V}_H(t) - V_L].$$

Therefore,

$$\mathbb{E}(\overset{\star}{p}{}_t^H) = \frac{1}{N}\overset{\star}{p}{}_{\breve{H}L}^H(t) + \left(1 - \frac{1}{N}\right)\left[\frac{N_H - 1}{N-1}\overset{\star}{p}{}_{H\breve{H}}^H(t) + \frac{N_L}{N-1}\overset{\star}{p}{}_{L\breve{H}}^H(t) + \frac{N_U}{N-1}\overset{\star}{p}{}_{U\breve{H}}^H(t)\right]$$

$$> V_L + \frac{\overset{c}{V}_H(t) - V_L}{N}\Big[N_H + \alpha(N_L + N_U - 1)\Big].$$

Comparing with (41), we get

$$\begin{aligned}\mathbb{E}(\overset{\star}{p}{}_t^H) - \mathbb{E}(\overset{c}{p}_t) &> \frac{\overset{c}{V}_H(t) - V_L}{N}\Big[N_H + \alpha(N_L + N_U - 1) - [\alpha N_L + (1-\alpha)(N_H + N_U)]\Big] \\ &= \frac{\overset{c}{V}_H(t) - V_L}{N}\Big[\alpha(N_H - 1) + (2\alpha - 1)N_U\Big].\end{aligned}$$

Since $N_H > N_L + N_U \geq N_U + 1$, and $\alpha \geq 1/3$,

$$\mathbb{E}(\overset{\star}{p}{}_t^H) - \mathbb{E}(\overset{c}{p}_t) > \frac{\overset{c}{V}_H(t) - V_L}{N}(3\alpha - 1)N_U \geq 0.$$

This completes the proof. $\square$

References

Asparouhova, E. and Bossaerts, P. (2017). Experiments on percolation of information in dark markets. *Economic Journal*, 127(605):518–544.

Attanasi, G., Centorrino, S., and Moscati, I. (2016). Over-the-counter markets vs. double auctions: A comparative experimental study. *Journal of Behavioral and Experimental Economics*, 63:22–35.

Babus, A. and Kondor, P. (2018). Trading and information diffusion in over-the-counter markets. *Econometrica*, 86(5):1727–1769.

Biais, B., Hilton, D., Mazurier, K., and Pouget, S. (2005). Judgemental over-confidence, self-monitoring and trading performance in an experimental financial market. *Review of Economic Studies*, 72:287–312.

Blouin, M. R. and Serrano, R. (2001). A decentralized market with common values uncertainty: Non-steady states. *Review of Economic Studies*, 68:323–346.

Bossaerts, P., Frydman, C., and Ledyard, J. (2014). The speed of information revelation and eventual price quality in markets with insiders: Comparing two theories. *Review of Finance*, 18:1–22.

Cason, T. N. and Friedman, D. (2008). A comparison of market institutions. In Plott, C., , and Smith, V., editors, *The Handbook of Experimental Economic Results*, pages 264–272. North-Holland.

Chandrasekhar, A. G., Larreguy, H., and Xandri, J. P. (2020). Testing models of social learning on networks: Evidence from two experiments. *Econometrica*, 88(1):1–32.

Chen, D. and Duffie, D. (2021). Market fragmentation. *American Economic Review*, 111(7):2247–2274.

Colla, P. and Mele, A. (2010). Information linkages and correlated trading. *Review of Financial Studies*, 23:203–246.

Condorelli, D., Galeotti, A., and Renou, L. (2017). Bilateral trading in networks. *Review of Economic Studies*, 84:82–105.

Corgnet, B., Deck, C., DeSantis, M., Hampton, K., and Kimbrough, E. (2023). When do security markets aggregate dispersed information? *Management Science*, 69(6):3697–3729.

Corgnet, B., DeSantis, M., and Porter, D. (2015). Revisiting information aggregation in asset markets: Reflective learning & market efficiency. Working Paper, Chapman University.

DeGroot, M. H. (1974). Reaching a consensus. *Journal of the American Statistical Association*, 69(345):118–121.

DeMarzo, P., Vayanos, D., and Zwiebel, J. (2003). Persuasion bias, social influence, and unidimensional opinions. *Quarterly Journal of Economics*, 118(3):909–968.

Duffie, D. and Manso, G. (2007). Information percolation in large markets. *American Economic Review*, 97(2):203–209.

Eyster, E., Rabin, M., and Vayanos, D. (2019). Financial markets where traders neglect the informational content of prices. *Journal of Finance*, 74(1):371–399.

Filippin, A. and Mantovani, M. (2023). Risk aversion and information aggregation in binary-asset markets. *Quantitative Economics*, 14:753–798.

Fischbacher, U. (2007). z-Tree: Zurich toolbox for ready-made economic experiments. *Experimental Economics*, 10(2):171–178.

Forsythe, R. and Lundholm, R. (1990). Information aggregation in an experimental market. *Econometrica*, 58(2):309–347.

G20 (2009). Leaders' statement: The Pittsburgh summit. September 24–25.

Gale, D. M. and Kariv, S. (2007). Financial networks. *American Economic Review*, 92(2):99–103.

Glode, V. and Opp, C. (2017). Asymmetric information and intermediation chains. *American Economic Review*, 106(9):2699–2721.

Glosten, L. and Milgrom, P. (1985). Bid, ask and transaction prices in a specialist market with heterogeneously informed traders. *Journal of Financial Economics*, 13(1):71–100.

Gofman, M. (2014). A network-based analysis of over-the-counter markets. Working Paper.

Golosov, M., Lorenzoni, G., and Tsyvinski, A. (2014). Decentralized trading with private information. *Econometrica*, 82(3):1055–1091.

Golub, B. and Jackson, M. O. (2010). Naïve learning in social networks and the wisdom of crowds. *American Economic Journal: Microeconomics*, 2(1):112–149.

Gould, M. D., Mason, A. P., and Howison, S. D. (2017). Quasi-centralized limit order books. *Quantitative Finance*, 17(6):831–853.

Halim, E., Riyanto, Y. E., and Roy, N. (2019). Costly information acquisition, social networks and asset prices: Experimental evidence. *Journal of Finance*, 74(4):1975–2010.

Han, B. and Yang, L. (2013). Social networks, information acquisition, and asset prices. *Management Science*, 59(6):1444–1457.

Hanson, R., Oprea, R., and Porter, D. (2006). Information aggregation and manipulation in an experimental market. *Journal of Economic Behavior & Organization*, 60:449–459.

Kranton, R. E. and Minehart, D. F. (2001). A theory of buyer-seller networks. *American Economic Review*, 91:485–508.

Lundholm, R. (1991). What affects the efficiency of a market? Some answers from the laboratory. *Accounting Review*, 66(3):486–515.

Malamud, S. and Rostek, M. (2017). Decentralized exchange. *American Economic Review*, 107(11):3320–3362.

Milgrom, P. and Stokey, N. (1982). Information, trade and common knowledge. *Journal of Economic Theory*, 26:17–27.

Ozsoylev, H. N. and Walden, J. (2011). Asset pricing in large information networks. *Journal of Economic Theory*, 146:2252–2280.

Page, L. and Siemroth, C. (2021). How much information is incorporated in financial asset prices? Experimental evidence. *Review of Financial Studies*, 34(9):4412–4449.

Pederson, L. H. (2022). Game on: Social networks and markets. *Journal of Financial Economics*, 146:1097–1119.

Plott, C. R. and Sunder, S. (1982). Efficiency of experimental security markets with insider information: An application of rational-expectations models. *Journal of Political Economy*, 90(4):663–698.

Plott, C. R. and Sunder, S. (1988). Rational expectations and the aggregation of diverse information in laboratory security markets. *Econometrica*, 56(5):1085–1118.

Rostek, M. and Yoon, J. H. (2021). Exchange design and efficiency. *Econometrica*, 89(6):2887–2928.

Ruiz-Buforn, A., Camacho-Cuena, E., Morone, A., and Alfarano, S. (2021a). Overweighting of public information in financial markets: A lesson from the lab. *Journal of Banking and Finance*, 133:106298.

Ruiz-Buforn, A., Camacho-Cuena, E., Morone, A., and Alfarano, S. (2021b). Single vs. multiple disclosures in an experimental asset market with information acquisition. *European Journal of Finance*, 28:1–27.

Schnitzlein, C. R. (1996). Call and continuous trading mechanisms under asymmetric information: An experimental investigation. *Journal of Finance*, 51(2):613–636.

Sunder, S. (1992). Market for information: Experimental evidence. *Econometrica*, 60(3):667–695.

Theissen, E. (2000). Market structure, informational efficiency and liquidity: An experimental comparison of auction and dealer markets. *Journal of Financial Markets*, 3:333–363.

Veiga, H. and Vorsatz, M. (2010). Information aggregation in experimental asset markets in the presence of a manipulator. *Experimental Economics*, 13:379–398.

Wolinsky, A. (1990). Information revelation in a market with pairwise meetings. *Econometrica*, 58(1):1–23.

Online Appendix

Supplemental Material for “Centralized vs Decentralized Markets:
The Role of Connectivity”

B2 Introduction

This Online Appendix contains additional results (Section B3) as well as details of instructions that subjects were given in the experiments (Sections B4 and B5).

B3 Core-Periphery Network

In this section we provide some results for the core-periphery network as described in Section 3.2 of the paper (for convenience, we repeat this description below). The numbers for equations, lemmas and propositions correspond to those in the main paper, unless they are specific to the Online Appendix, in which case the numbers are prefixed by "B".

B3.1 Model and Results

We consider a core-periphery network with M agents in the core and the remaining $N - M$ in the periphery. An agent in the core has a link with every other agent in the core; thus the core is a complete subnetwork. In addition, each agent in the core has a link with $K := (N - M)/M$ agents in the periphery; this is the "clientele" of this core agent. All agents in the core have a clientele of the same size, and these clienteles do not overlap. An agent in the periphery has only one link, to the core agent whose clientele she belongs to.

We assume that there are agents of all three types in the core as well as in each clientele, and within each of these their relative proportions are the same as in the entire network. We use a "hat" for the number of agents of a given type in the core, and a "check" for the number in each clientele. Thus the number of optimists in the core is $\hat{N}_H := (N_H/N)M \geq 1$, the number of optimists in each clientele is $\check{N}_H := (N_H/N)K \geq 1$, and similarly for pessimists and uninformed agents. We denote the belief of an uninformed agent at time t by $\overset{\diamond}{V}_U(t)$, $\overset{\diamond}{V}{}^H_U(t)$, $\overset{\diamond}{V}{}^L_U(t)$ or $\overset{\diamond}{V}{}^U_U(t)$, depending on whether she is in the core, in the clientele of an optimist, in the clientele of a pessimist, or in the clientele of an uninformed agent.

Proposition B3.1 (Core-periphery network: Beliefs) *In the core-periphery network, the beliefs of uninformed agents, for $t \geq 0$, are as follows:*

i. The belief of an uninformed agent in the periphery linked to an informed agent in the core is the same as the corresponding belief in the star, i.e. $\overset{\diamond}{V}{}^H_U(t) = \overset{\star}{V}{}^H_U(t)$ and $\overset{\diamond}{V}{}^L_U(t) = \overset{\star}{V}{}^L_U(t)$.

ii. Both $\overset{\diamond}{V}_U(t)$ and $\overset{\diamond}{V}{}^U_U(t)$ are increasing in t, and converge to $\bar{V}$ as $t \to \infty$.

iii. The expected belief of an uninformed agent is strictly increasing in t, and converges to $\bar{V}$ as $t \to \infty$.

All proofs are in the Section B3.2. We denote the price at time t in the core-periphery network by $\mathring{p}_t$.

Proposition B3.2 (Core-periphery network: Prices) *In the core-periphery network, $\mathbb{E}(\mathring{p}_t)$ is strictly increasing in t for $t \geq 1$, and converges to*

$$\mathbb{E}(\mathring{p}_\infty) = \bar{V} - \alpha \frac{M(V_H - V_L)}{N^2(N_H + N_L)} \big[N(N_H - N_L) + N_L N_U \big]. \tag{B42}$$

Comparing (10) and (B42), we see that the limiting expected price in the core-periphery network is the same as in the star if we set $M = 1$. However, this is just a formal relationship as $M = 1$ is not admissible, given our assumption that there are agents of all three types in the core.

The results on informational efficiency and welfare are the same as in the star:

Proposition B3.3 (Core-periphery vs complete network: Informational efficiency)

i. The core-periphery network is asymptotically more informationally efficient than the complete network, i.e. $\mathbb{E}(\mathring{p}_\infty) > \mathbb{E}(\overset{c}{p}_\infty)$.

ii. Suppose $\alpha \geq 1/2$. Then the core-periphery network is more informationally efficient than the complete network at every date, i.e. $\mathbb{E}(\mathring{p}_t) > \mathbb{E}(\overset{c}{p}_t)$ for all $t \geq 1$.

Proposition B3.4 (Core-periphery vs complete network: Welfare) *Consider the static limiting economy as $t \to \infty$. Suppose $\alpha \geq 1/2$ and $N_H + N_U \geq 3N_L$. Then welfare in the core-periphery network is higher than in the complete network.*

B3.2 Proofs

Proof of Proposition B3.1 We put agents in the following order: uninformed agents in the core, uninformed agents in clientele $1, 2, \ldots$, optimists in the core, optimists in clientele $1, 2, \ldots$, pessimists in the core, and pessimists in clientele $1, 2, \ldots$. The clienteles are also ordered in the same way: first the clienteles of uninformed agents in the core, then the clienteles of optimists in the core, and finally the clienteles of pessimists in the core. We have

$$
A_U = \begin{bmatrix}
\frac{1}{M+K} & \frac{1}{M+K} & \cdots & \frac{1}{M+K} & \frac{1}{M+K}\mathbf{1}_{\check{N}_U}^{\top} & 0 & \cdots & 0 & 0 & 0 & \cdots & 0 \\
\frac{1}{M+K} & \frac{1}{M+K} & \cdots & \frac{1}{M+K} & 0 & \frac{1}{M+K}\mathbf{1}_{\check{N}_U}^{\top} & \cdots & 0 & 0 & 0 & \cdots & 0 \\
\vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots \\
\frac{1}{M+K} & \frac{1}{M+K} & \cdots & \frac{1}{M+K} & 0 & 0 & \cdots & \frac{1}{M+K}\mathbf{1}_{\check{N}_U}^{\top} & 0 & 0 & \cdots & 0 \\
\frac{1}{2}\mathbf{1}_{\check{N}_U} & 0 & \cdots & 0 & \frac{1}{2}I_{\check{N}_U} & 0 & \cdots & 0 & 0 & 0 & \cdots & 0 \\
0 & \frac{1}{2}\mathbf{1}_{\check{N}_U} & \cdots & 0 & 0 & \frac{1}{2}I_{\check{N}_U} & \cdots & 0 & 0 & 0 & \cdots & 0 \\
\vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots \\
0 & 0 & \cdots & \frac{1}{2}\mathbf{1}_{\check{N}_U} & 0 & 0 & \cdots & \frac{1}{2}I_{\check{N}_U} & 0 & 0 & \cdots & 0 \\
0 & 0 & \cdots & 0 & 0 & 0 & \cdots & 0 & \frac{1}{2}I_{\check{N}_U} & 0 & \cdots & 0 \\
0 & 0 & \cdots & 0 & 0 & 0 & \cdots & 0 & 0 & \frac{1}{2}I_{\check{N}_U} & \cdots & 0 \\
\vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots \\
0 & 0 & \cdots & 0 & 0 & 0 & \cdots & 0 & 0 & 0 & \cdots & \frac{1}{2}I_{\check{N}_U}
\end{bmatrix},
$$

$$
A_H = \begin{bmatrix}
\frac{1}{M+K} & \frac{1}{M+K} & \cdots & \frac{1}{M+K} & \frac{1}{M+K}\mathbf{1}_{\check{N}_H}^{\top} & 0 & \cdots & 0 & 0 & 0 & \cdots & 0 \\
\frac{1}{M+K} & \frac{1}{M+K} & \cdots & \frac{1}{M+K} & 0 & \frac{1}{M+K}\mathbf{1}_{\check{N}_H}^{\top} & \cdots & 0 & 0 & 0 & \cdots & 0 \\
\vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots \\
\frac{1}{M+K} & \frac{1}{M+K} & \cdots & \frac{1}{M+K} & 0 & 0 & \cdots & \frac{1}{M+K}\mathbf{1}_{\check{N}_H}^{\top} & 0 & 0 & \cdots & 0 \\
0 & 0 & \cdots & 0 & 0 & 0 & \cdots & 0 & 0 & 0 & \cdots & 0 \\
0 & 0 & \cdots & 0 & 0 & 0 & \cdots & 0 & 0 & 0 & \cdots & 0 \\
\vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots \\
0 & 0 & \cdots & 0 & 0 & 0 & \cdots & 0 & 0 & 0 & \cdots & 0 \\
\frac{1}{2}\mathbf{1}_{\check{N}_U} & 0 & \cdots & 0 & 0 & 0 & \cdots & 0 & 0 & 0 & \cdots & 0 \\
0 & \frac{1}{2}\mathbf{1}_{\check{N}_U} & \cdots & 0 & 0 & 0 & \cdots & 0 & 0 & 0 & \cdots & 0 \\
\vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots \\
0 & 0 & \cdots & \frac{1}{2}\mathbf{1}_{\check{N}_U} & 0 & 0 & \cdots & 0 & 0 & 0 & \cdots & 0 \\
0 & 0 & \cdots & 0 & 0 & 0 & \cdots & 0 & 0 & 0 & \cdots & 0 \\
0 & 0 & \cdots & 0 & 0 & 0 & \cdots & 0 & 0 & 0 & \cdots & 0 \\
\vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots \\
0 & 0 & \cdots & 0 & 0 & 0 & \cdots & 0 & 0 & 0 & \cdots & 0
\end{bmatrix}.
$$

Both matrices consist of three row blocks. The first block has $\hat{N}_U$ rows corresponding to uninformed agents in the core. The second row block has $\hat{N}_U\check{N}_U$ rows corresponding to uninformed agents in clientele $1, 2, \ldots \hat{N}_U$. These are uninformed agents in the periphery who have a link with an uninformed agent in the core. The third row block has $\hat{N}_H \check{N}_U$ rows corresponding to uninformed agents in clienteles $\hat{N}_U + 1, \ldots, \hat{N}_U + \hat{N}_H$. These are uninformed agents in the periphery who have a link with an optimist in the core. The fourth row block has $\hat{N}_L \check{N}_U$ rows corresponding to uninformed agents in the remaining clienteles, each of which is linked to a pessimist in the core. The third and fourth row blocks look like a single row block in the case of A_U. The matrix A_L is analogous to A_H; the only difference is that the third and fourth row blocks are swapped.

Now we use Lemma A.1. The elements of the vector v corresponding to the four row blocks of A_U (also of A_H and A_L) are the same. We denote these four values by $v(1), v(2), v(3)$, and $v(4)$. We have

$$\begin{aligned} v(1) &= \left[\frac{\hat{N}_U + \check{N}_U}{M+K} - 1\right] \frac{V_H + V_L}{2} + \frac{\hat{N}_H + \check{N}_H}{M+K} V_H + \frac{\hat{N}_L + \check{N}_L}{M+K} V_L \\ &= \left[\frac{N_U}{N} - 1\right] \frac{V_H + V_L}{2} + \frac{N_H}{N} V_H + \frac{N_L}{N} V_L, \end{aligned}$$

which is strictly positive, from (22). Also $v(2) = 0$, and

$$\begin{aligned} v(3) &= \left[\frac{1}{2} - 1\right] \frac{V_H + V_L}{2} + \frac{1}{2} V_H = \frac{V_H - V_L}{4}, \\ v(4) &= \left[\frac{1}{2} - 1\right] \frac{V_H + V_L}{2} + \frac{1}{2} V_L = -\frac{V_H - V_L}{4}. \end{aligned}$$

We partition the vector $A_U^t v$ in the same way we did for v.

All elements of the first row block of $A_U v$ are the same, given by $[\hat{N}_U/(M+K)]v(1) > 0$. All elements of the first row block of $A_U^2 v = A_U(A_U v)$ are also the same, and strictly positive, and so on. Therefore, from (17), $\overset{\diamond}{V}_U(t)$ is strictly increasing in t, for $t \geq 1$. Directly computing $\tilde{V}(1)$, using (3), we see that $\overset{\diamond}{V}_U(1) > \overset{\diamond}{V}_U(0) = (V_H + V_L)/2$. Thus $\overset{\diamond}{V}_U(t)$ is strictly increasing in t, for $t \geq 0$.

All the elements of the second row block of $A_U v$ are the same, given by $(1/2)v(1) > 0$. All elements of the second row block of $A_U^2 v = A_U(A_U v)$ are also the same, and strictly positive, and so on. Using the same argument as in the preceding paragraph, $\overset{\diamond}{V}{}_U^U(t)$ is strictly increasing in t, for $t \geq 1$, and $\overset{\diamond}{V}{}_U^U(1) = \overset{\diamond}{V}{}_U^U(0) = (V_H + V_L)/2$.

The analysis for the beliefs of uninformed agents in the periphery who are linked to an informed agent in the core is the same as for the star. From (23) it follows that $N_H \overset{\diamond}{V}{}_U^H(t) + N_L \overset{\diamond}{V}{}_U^L(t)$ is strictly increasing in t, and converges to $\bar{V}$.

Limiting beliefs for uninformed agents in the core, and uninformed agents in the periphery linked to an uninformed agent in the core, can be derived from (4). We have

$$\overset{\diamond}{V}_U^U(\infty) - \left[\frac{1}{2}\overset{\diamond}{V}_U(\infty) + \frac{1}{2}\overset{\diamond}{V}_U^U(\infty)\right] = 0,$$

implying that $\overset{\diamond}{V}_U(\infty) = \overset{\diamond}{V}_U^U(\infty)$. Using (4) once again,

$$\overset{\diamond}{V}_U(\infty) - \frac{\hat{N}_U + \check{N}_U}{M+K}\overset{\diamond}{V}_U(\infty) = \frac{(\hat{N}_H + \check{N}_H)V_H + (\hat{N}_L + \check{N}_L)V_L}{M+K},$$

or

$$\left[1 - \frac{N_U}{N}\right]\overset{\diamond}{V}_U(\infty) = \frac{N_H V_H + N_L V_L}{N},$$

so that $\overset{\diamond}{V}_U(\infty) = \overset{\diamond}{V}_U^U(\infty) = \bar{V}$. □

Proof of Proposition B3.2 As in the proof of Proposition 3.6, we introduce an additional fictitious round of trading at date 0, in which we assume that an uninformed agent in the core trades with L with probability 1. The price at any date depends on which agent is chosen to trade, whether the chosen agent is in the core or in the periphery, and in the latter case, what type of agent she is linked to. The notation for prices that arise in each case is just as for the star, except that the superscript is interpreted differently. We use a superscript only when an uninformed agent in the periphery is chosen to trade, and the superscript is the type of agent she is linked to. Thus $\overset{\diamond}{p}_{UX}^X$ is the price that arises when an uninformed agent in the periphery is chosen to trade and she trades with X (who must be the only agent she has a link to), while $\overset{\diamond}{p}_{UX}$ is the price that results when a core agent U is chosen to trade and she trades with X. The price $\overset{\diamond}{p}_{UU}^U$ arises when an uninformed agent in the periphery is chosen to trade and she trades with an uninformed agent in the core. Let $\mathbb{E}(\overset{\diamond}{p}_t|H)$ be the expected price at date t if H is chosen to trade, and similarly for L and U. We have

$$\begin{aligned}
\mathbb{E}(\overset{\diamond}{p}_t|H) &= \frac{M}{N}p_{HL} + \left(1 - \frac{M}{N}\right)\left[\frac{N_H}{N}p_{HH} + \frac{N_L}{N}p_{HL} + \frac{N_U}{N}\overset{\diamond}{p}_{HU}(t)\right],\\
\mathbb{E}(\overset{\diamond}{p}_t|L) &= \frac{M}{N}p_{LH} + \left(1 - \frac{M}{N}\right)\left[\frac{N_H}{N}p_{LH} + \frac{N_L}{N}p_{LL} + \frac{N_U}{N}\overset{\diamond}{p}_{LU}(t)\right],\\
\mathbb{E}(\overset{\diamond}{p}_t|U) &= \frac{M}{N}\overset{\diamond}{p}_{UL}(t) + \left(1 - \frac{M}{N}\right)\left[\frac{N_H}{N}\overset{\diamond}{p}_{UH}^H(t) + \frac{N_L}{N}\overset{\diamond}{p}_{UL}^L(t) + \frac{N_U}{N}\overset{\diamond}{p}_{UU}^U(t)\right].
\end{aligned}$$

We can now compute the expected price in the core-periphery network, using (5)–(8):

$$\mathbb{E}(\overset{\diamond}{p}_t) = \frac{N_H}{N}\mathbb{E}(\overset{\diamond}{p}_t|H) + \frac{N_L}{N}\mathbb{E}(\overset{\diamond}{p}_t|L) + \frac{N_U}{N}\mathbb{E}(\overset{\diamond}{p}_t|U)$$

$$
\begin{aligned}
&= \frac{M}{N^2}\Big[N_H p_{HL} + N_L p_{LH} + N_U \mathring{p}_{UL}(t)\Big] \\
&\quad + \frac{1}{N^2}\left(1-\frac{M}{N}\right)\Big[N_H^2 p_{HH} + N_H N_L p_{HL} + N_H N_U \mathring{p}_{HU}(t) + N_H N_L p_{LH} + N_L^2 p_{LL} \\
&\qquad\qquad + N_L N_U \mathring{p}_{LU}(t) + N_H N_U \mathring{p}^H_{UH}(t) + N_L N_U \mathring{p}^L_{UL}(t) + N_U^2 \mathring{p}^U_{UU}(t)\Big] \\
&= \frac{M}{N^2}\Big[N_H V_H + N_L V_L - \alpha(N_H - N_L)(V_H - V_L) + N_U[\alpha V_L + (1-\alpha)\mathring{V}_U(t)]\Big] \\
&\quad + \frac{1}{N^2}\left(1-\frac{M}{N}\right)\Big[N_H^2 V_H + N_L^2 V_L + N_H N_L (V_H + V_L) + N_H N_U[(1-\alpha)V_H + \alpha \mathring{V}_U(t)] \\
&\qquad + N_L N_U[(1-\alpha)V_L + \alpha \mathring{V}_U(t)] + N_H N_U[\alpha V_H + (1-\alpha)\mathring{V}^H_U(t)] \\
&\qquad + N_L N_U[\alpha V_L + (1-\alpha)\mathring{V}^L_U(t)] + N_U^2[\alpha \mathring{V}_U(t) + (1-\alpha)\mathring{V}^U_U(t)]\Big] \\
&= \frac{M}{N^2}\Big[N_H V_H + N_L V_L - \alpha(N_H - N_L)(V_H - V_L) + N_U[\alpha V_L + (1-\alpha)\mathring{V}_U(t)]\Big] \\
&\quad + \frac{1}{N^2}\left(1-\frac{M}{N}\right)\Big[N(N_H V_H + N_L V_L) + (1-\alpha)N_U\Big[N_H \mathring{V}^H_U(t) + N_L \mathring{V}^L_U(t)\Big] \\
&\qquad\qquad + N_U\Big[\alpha N \mathring{V}_U(t) + (1-\alpha)N_U \mathring{V}^U_U(t)\Big]\Big]. \qquad \text{(B43)}
\end{aligned}
$$

In the proof of Proposition B3.1 we showed that $\mathring{V}_U(t)$ is strictly increasing in t, $\mathring{V}^U_U(t)$ is increasing in t, and $N_H \mathring{V}^H_U(t) + N_L \mathring{V}^L_U(t)$ is strictly increasing in t, for $t \geq 0$. Therefore, $\mathbb{E}(\mathring{p}_t)$ is strictly increasing in t, for $t \geq 0$.

To calculate the limiting price, we note from Proposition B3.1 that $\mathring{V}^H_U(\infty) = V_H$, $\mathring{V}^L_U(\infty) = V_L$, and $\mathring{V}_U(\infty) = \mathring{V}^U_U(\infty) = \bar{V}$. Therefore, from (B43),

$$
\begin{aligned}
\mathbb{E}(\mathring{p}_\infty) &= \frac{M}{N^2}\Big[N\bar{V} - \alpha(N_H - N_L)(V_H - V_L) - \alpha N_U(\bar{V} - V_L)\Big] \\
&\quad + \frac{1}{N^2}\left(1-\frac{M}{N}\right)\Big[N(N_H + N_L)\bar{V} + (1-\alpha)N_U(N_H + N_L)\bar{V} + N_U[\alpha N + (1-\alpha)N_U]\bar{V}\Big] \\
&= \frac{M}{N^2}\Big[N\bar{V} - \alpha(N_H - N_L)(V_H - V_L) - \alpha N_U(\bar{V} - V_L)\Big] + \left(1-\frac{M}{N}\right)\bar{V} \\
&= \bar{V} - \alpha\frac{M}{N^2}\Big[(N_H - N_L)(V_H - V_L) + N_U(\bar{V} - V_L)\Big] \\
&= \bar{V} - \alpha\frac{M(V_H - V_L)}{N^2(N_H + N_L)}\Big[N(N_H - N_L) + N_L N_U\Big].
\end{aligned}
$$

This completes the proof. □

Proof of Proposition B3.3 *Proof of (i):* Since there is at least one pessimist in each clientele, i.e.

$$
\check{N}_L = \frac{N_L}{N}K = \frac{N_L}{N}\frac{N-M}{M} \geq 1,
$$

we have

$$M \leq \frac{NN_L}{N+N_L}. \tag{B44}$$

Therefore, from (B42) and (9),

$$\mathbb{E}(\overset{\diamond}{p}_\infty) \geq \bar{V} - \alpha \frac{N_L(V_H - V_L)}{N(N+N_L)(N_H+N_L)}\Big[N(N_H - N_L) + N_L N_U\Big] > \mathbb{E}(\overset{c}{p}_\infty).$$

Proof of (ii): We proceed along the same lines as in the proof of Proposition 3.7. We begin by calculating the expected price in the core-periphery network in the fictitious trading round at date 0. From (B43),

$$\begin{aligned}
\mathbb{E}(\overset{\diamond}{p}_0) = {} & \frac{M}{2N^2}\Big[2(N_H V_H + N_L V_L) - 2\alpha(N_H - N_H)(V_H - V_L) - \alpha N_U(V_H - V_L) + N_U(V_H + V_L)\Big] \\
& + \frac{1}{2N^2}\left(1 - \frac{M}{N}\right)\Big[2N(N_H V_H + N_L V_L) + (1-\alpha)N_U(N_H + N_L)(V_H + V_L) \\
& \qquad + N_U[\alpha N + (1-\alpha)N_U](V_H + V_L)\Big] \\
= {} & \frac{M}{2N^2}\Big[N(V_H + V_L) + [(1-2\alpha)(N_H - N_L) - \alpha N_U](V_H - V_L)\Big] \\
& + \frac{1}{2N}\left(1 - \frac{M}{N}\right)\Big[2(N_H V_H + N_L V_L) + N_U(V_H + V_L)\Big] \\
= {} & \frac{M}{2N^2}\Big[N(V_H + V_L) + [(1-2\alpha)(N_H - N_L) - \alpha N_U](V_H - V_L)\Big] \\
& + \frac{1}{2N}\left(1 - \frac{M}{N}\right)\Big[(N_H - N_L)(V_H - V_L) + N(V_H + V_L)\Big] \\
= {} & \frac{V_H + V_L}{2} + \frac{1}{2N}(N_H - N_L)(V_H - V_L) - \frac{\alpha M(V_H - V_L)}{2N^2}\Big[2(N_H - N_L) + N_U\Big].
\end{aligned}$$

Using (B44), we obtain

$$\mathbb{E}(\overset{\diamond}{p}_0 \geq \frac{V_H + V_L}{2} + \frac{1}{2N}(N_H - N_L)(V_H - V_L) - \frac{\alpha N_L(V_H - V_L)}{2N(N+N_L)}\Big[2(N_H - N_L) + N_U\Big].$$

Using this expression together with (9), and restricting α to the interval $[1/2, 1]$, we have

$$\begin{aligned}
\mathbb{E}(\overset{\diamond}{p}_0) - \mathbb{E}(\overset{c}{p}_\infty) \geq {} & \frac{V_H + V_L}{2} - \bar{V} + \frac{1}{2N}(N_H - N_L)(V_H - V_L) \\
& + \alpha \frac{V_H - V_L}{2N(N+N_L)(N_H+N_L)}\Big[2N(N+N_L)(N_H - N_L) + 2N_L N_U(N+N_L) \\
& \qquad - 2N_L(N_H + N_L)(N_H - N_L) - N_L N_U(N_H + N_L)\Big] \\
\geq {} & \frac{V_H + V_L}{2} - \bar{V} + \frac{1}{2N}(N_H - N_L)(V_H - V_L)
\end{aligned}$$

$$
\begin{aligned}
&+ \frac{V_H - V_L}{4N(N+N_L)(N_H+N_L)}\Big[2N(N+N_L)(N_H-N_L) + 2N_L N_U(N+N_L) \\
&\qquad\qquad - 2N_L(N_H+N_L)(N_H-N_L) - N_L N_U(N_H+N_L)\Big] \\
&= \frac{V_H - V_L}{4N(N+N_L)(N_H+N_L)}\Big[2(N+N_L)(N_H+N_L)(N_H-N_L) + 2N_L N_U(N+N_L) \\
&\qquad\qquad - 2N_L(N_H+N_L)(N_H-N_L) - N_L N_U(N_H+N_L)\Big] \\
&> 0.
\end{aligned}
$$

In the proof of Proposition B3.2 we showed that $\mathbb{E}(\overset{\diamond}{p}_t)$ is strictly increasing in t, for all $t \geq 0$. Also, from Proposition 3.4, $\mathbb{E}(\overset{c}{p}_t)$ is strictly increasing in t for $t \geq 1$. Therefore, we have the following chain of inequalities:

$$
\mathbb{E}(\overset{\diamond}{p}_t) > \mathbb{E}(\overset{\diamond}{p}_0) > \mathbb{E}(\overset{c}{p}_\infty) > \mathbb{E}(\overset{c}{p}_t), \qquad \forall t \geq 1.
$$

This establishes the result. $\square$

Proof of Proposition B3.4 Calculations for the variance of the ex ante payoff in the core-periphery network are analogous to those for the star, with the analogous notation ($\diamond$ replaces $\star$). We use a superscript only for payoffs that arise when an uninformed agent in the periphery is chosen to trade; the superscript identifies the type of agent she is linked to. Limiting beliefs of uninformed agents mirror those in the star (Proposition B3.1). Hence payoffs that involve a trade with an uninformed agent are given by (30)–(35), where we simply replace $\star$ with $\diamond$, and drop the U superscript in (31)–(34), since these payoffs do not involve a trade with an uninformed agent in the periphery. Payoffs from trades between two informed agents are given by (25)–(28). Using this information, and (13), we can calculate the variance of the ex ante payoff $\tilde{\Pi}^{\diamond}$:

$$
\begin{aligned}
\mathrm{Var}(\tilde{\Pi}^{\diamond}) = \mathbb{E}(\overset{\diamond}{\Pi}{}^2) &= \left[\frac{M}{N}\frac{N_H}{N} + \left(1 - \frac{M}{N}\right)\left(\frac{N_H}{N}\frac{N_L}{N}\right)\right]\Pi_{HL}^2 + \left[\frac{M}{N}\frac{N_L}{N} + \left(1 - \frac{M}{N}\right)\left(\frac{N_L}{N}\frac{N_H}{N}\right)\right]\Pi_{LH}^2 \\
&+ \left(1 - \frac{M}{N}\right)\frac{N_H}{N}\frac{N_H}{N}\Pi_{HH}^2 + \left(1 - \frac{M}{N}\right)\frac{N_L}{N}\frac{N_L}{N}\Pi_{LL}^2 \\
&+ \left(1 - \frac{M}{N}\right)\frac{N_U}{N}\frac{N_H}{N}(\overset{\diamond}{\Pi}{}_{UH}^{H})^2 + \left(1 - \frac{M}{N}\right)\frac{N_U}{N}\frac{N_L}{N}(\overset{\diamond}{\Pi}{}_{UL}^{L})^2 \\
&+ \frac{M}{N}\frac{N_U}{N}\overset{\diamond}{\Pi}{}_{UL}^2 + \left(1 - \frac{M}{N}\right)\frac{N_H}{N}\frac{N_U}{N}\overset{\diamond}{\Pi}{}_{HU}^2 \\
&+ \left(1 - \frac{M}{N}\right)\frac{N_L}{N}\frac{N_U}{N}\overset{\diamond}{\Pi}{}_{LU}^2 + \left(1 - \frac{M}{N}\right)\frac{N_U}{N}\frac{N_U}{N}(\overset{\diamond}{\Pi}{}_{UU}^{U})^2 \\
&= \frac{(V_H - V_L)^2}{N^3}\Bigg[MN\left(\alpha^2 N_H + (1-\alpha)^2 N_L + N_U\left(\frac{\alpha N_H + N_L}{N_H + N_L}\right)^2\right)
\end{aligned}
$$

$$+ (N-M)\Bigg([\alpha^2+(1-\alpha)^2]N_H N_L + N_L(N_L+N_U)$$

$$+ N_H N_U\left(\frac{\alpha N_L}{N_H+N_L}\right)^2 + N_L N_U\left(\frac{(1-\alpha)N_H+N_L}{N_H+N_L}\right)^2 + N_U^2\left(\frac{N_L}{N_H+N_L}\right)^2\Bigg)\Bigg].$$

Comparing this expression with (29), we get

$$\begin{aligned}\mathrm{Var}(\tilde{\Pi}^c) - \mathrm{Var}(\tilde{\Pi}^\diamond) = \frac{(N-M)(V_H-V_L)^2}{N^3}\Bigg[&N\left(\alpha^2 N_H + (1-\alpha)^2 N_L + N_U\left(\frac{\alpha N_H+N_L}{N_H+N_L}\right)^2\right)\\ &-\Bigg([\alpha^2+(1-\alpha)^2]N_H N_L + N_L(N_L+N_U) + N_H N_U\left(\frac{\alpha N_L}{N_H+N_L}\right)^2\\ &+ N_L N_U\left(\frac{(1-\alpha)N_H+N_L}{N_H+N_L}\right)^2 + N_U^2\left(\frac{N_L}{N_H+N_L}\right)^2\Bigg)\Bigg].\end{aligned}$$

It is easy to check that this expression is increasing in α for $\alpha \geq 1/2$. Therefore, in order to prove that $\mathrm{Var}(\tilde{\Pi}^c) - \mathrm{Var}(\tilde{\Pi}^\diamond) > 0$ for $\alpha \geq 1/2$, it suffices to show that $\mathrm{Var}(\tilde{\Pi}^c) - \mathrm{Var}(\tilde{\Pi}^\diamond) > 0$ for $\alpha = 1/2$. We have

$$\begin{aligned}\left[\mathrm{Var}(\tilde{\Pi}^c) - \mathrm{Var}(\tilde{\Pi}^\diamond)\right]\Big|_{\alpha=1/2} &\propto N(N_H+N_L) + NN_U\left(1+\frac{N_L}{N_H+N_L}\right)^2\\ &\quad - 2N_H N_L - 4N_L(N_L+N_U) - N_H N_U\left(\frac{N_L}{N_H+N_L}\right)^2\\ &\quad - N_L N_U\left(1+\frac{N_L}{N_H+N_L}\right)^2 - 4N_U^2\left(\frac{N_L}{N_H+N_L}\right)^2\\ &= N_H(N_H+N_U) - 3N_L(N_L+N_U)\\ &\quad + N_U(N_H+N_U)\left(1+\frac{N_L}{N_H+N_L}\right)^2 - N_U(N_H+4N_U)\left(\frac{N_L}{N_H+N_L}\right)^2\\ &= (N_H+N_U)^2 - 3N_L(N_L+N_U) + \frac{2N_U N_L(N_H+N_U)}{N_H+N_L} - 3N_U^2\left(\frac{N_L}{N_H+N_L}\right)^2\\ &= (N_H+N_U)(N_H+N_U-3N_L) + 3N_L(N_H-N_L)\\ &\quad + \frac{2N_U N_L(N_H+N_U)(N_H-N_L+2N_L)}{(N_H+N_L)^2} - 3N_U^2\left(\frac{N_L}{N_H+N_L}\right)^2,\end{aligned}$$

which is positive if $N_H + N_U \geq 3N_L$. $\square$

B4 Instructions for the Experiments

Instructions for experiments for fully connected markets (FC) with no public information

This is an economic experiment on decision making in financial markets. The instructions are simple and if you follow them carefully, you can earn a considerable amount of money. Your earnings will be communicated to you privately, and paid in cash at the end of the experiment. During the experiment your earnings will be measured in points that will become € at the end of the experiment using a rate of €1 for every 50 accumulated points, plus a fixed amount of €3, as show-up fee for participating. The corresponding amount in € will be paid in cash at the end of the experiment. At the beginning of the experiment each of you has been assigned a number. From now on, you and the rest of the participants will be identified by that number. No communication is allowed among the participants during the experiment. Any participant who does not comply will be invited to leave the experiment without payment.

<u>The market</u>

The experiment consists of 15 periods of 3 minutes each. The market is composed of 15 participants. At the beginning of each period you will be randomly paired with another 14 participants. At the beginning of each period your initial portfolio consists of 10 units of the asset and 1000 ECU cash. Each participant has the same initial portfolio. In each period, you and the other participants will have the opportunity to buy and/or sell the asset. You can buy and sell as many units of the asset as you want, although each bid, ask and transaction involves the exchange of a single unit. Therefore, the asset is exchanged one unit at a time.

<u>Information and dividends</u>

At the end of each period, you will receive a given dividend for each unit of the asset you hold in your portfolio. The value of the dividend can be 0 or 10 with the same probability. Thus, without additional information, the value of the asset can be 0 or 10 with a probability of 50%. Likewise, you will receive private information about the value of the dividend at the end of the period in the form of signals.

- A private signal equal to 0 means that with a probability of 80% the value of the dividend will be 0 at the end of the period.
- A private signal equal to 10 means that with a probability of 80% the value of the dividend will be 10 at the end of the period.

This will be your private information and therefore you will be the only one able to see it. At the end of each period, your profit will be the cash you have at the end of the period plus the dividend for each unit of the asset you own minus the 1000 ECU that is given to you at the beginning of the period. Your payment at the end of the experiment corresponds to the accumulated profit in all periods. If at any time you have any question or problem, do not hesitate to contact the experimentalist. Remember that it is important that you understand correctly the operation of the market, since your earnings depend both on your decisions and on the decisions of the other participants in your market.

Instructions for experiments for medium/low connected markets (MC, LC) with no public information

Welcome. This is an economic experiment on decision making in financial markets. The instructions are simple and if you follow them carefully, you can earn a considerable amount of money. Your earnings will be communicated to you privately, and paid in cash at the end of the experiment. During the experiment your earnings will be measured in points that will become € at the end of the experiment using a rate of €1 for every 50 accumulated points, plus a fixed amount of €3, as show-up fee for participating. The corresponding amount in € will be paid in cash at the end of the experiment. At the beginning of the experiment each of you has been assigned a number. From now on, you and the rest of the participants will be identified by that number. No communication is allowed among the participants during the experiment. Any participant who does not comply will be invited to leave the experiment without payment.

<u>The market</u>

The experiment consists of 15 periods of 3 minutes each. The market is composed of 15 participants connected to each other through a network. At the beginning of each period you will be paired randomly with a subset of participants. At the beginning of each period your initial portfolio consists of 10 units of the asset and 1000 ECU cash. Each participant has the same initial portfolio. In each period, you and the other participants will have the opportunity to buy and/or sell the asset only with those participants with whom you are connected. You can buy and sell as many units of the asset as you want, although each bid, ask and transaction involves the exchange of a single unit. Therefore, the asset is exchanged one unit at a time.

<u>Information and dividends</u>

At the end of each period, you will receive a given dividend for each unit of the asset you hold in your portfolio. The value of the dividend can be 0 or 10 with the same probability. Thus, without additional information, the value of the asset can be 0 or 10 with a probability of 50%. Likewise, you will receive private information about the value of the dividend at the end of the period in the form of signals.

- A private signal equal to 0 means that with a probability of 80% the value of the dividend will be 0 at the end of the period.
- A private signal equal to 10 means that with a probability of 80% the value of the dividend will be 10 at the end of the period.

This will be your private information and therefore you will be the only one able to see it. At the end of each period, your profit will be the cash you have at the end of the period plus the dividend for each unit of the asset you own minus the 1000 ECU that were given to you at the beginning of the period. Your payment at the end of the experiment corresponds to the accumulated profit in all periods. If at any time you have any question or problem, do not hesitate to contact the experimentalist. Remember that it is important that you understand correctly the operation of the market, since your earnings depend both on your decisions and on the decisions of the other participants in your market.

Instructions for experiments for medium/low connected markets (MC, LC) with public information

Welcome. This is an economic experiment on decision making in financial markets. The instructions are simple and if you follow them carefully, you can earn a considerable amount of money. Your earnings will be communicated to you privately, and paid in cash at the end of the experiment. During the experiment your earnings will be measured in points that will become € at the end of the experiment using a rate of €1 for every 50 accumulated points, plus a fixed amount of €3, as show-up fee for participating The corresponding amount in € will be paid in cash at the end of the experiment. At the beginning of the experiment each of you has been assigned a number. From now on, you and the rest of the participants will be identified by that number. No communication is allowed among the participants during the experiment. Any participant who does not comply will be invited to leave the experiment without payment.

<u>The market</u>

The experiment consists of 15 periods of 3 minutes each. The market is composed of 15 participants connected to each other through a network. At the beginning of each period you will be paired randomly with a subset of participants. At the beginning of each period your initial portfolio consists of 10 units of the asset and 1000 ECU cash. Each participant has the same initial portfolio. In each period, you and the other participants will have the opportunity to buy and/or sell the asset only with those participants with whom you are connected. You can buy and sell as many units of the asset as you want, although each bid, ask and transaction involves the exchange of a single unit. Therefore, the asset is exchanged one unit at a time.

Information and dividends

At the end of each period, you will receive a given dividend for each unit of the asset you hold in your portfolio. The value of the dividend can be 0 or 10 with the same probability. Thus, without additional information, the value of the asset can be 0 or 10 with a probability of 50%. You will receive additional information about the value of the dividend in the form of signals. All market participants will receive at the beginning of each period public information in the form of a public signal that will be correct with a probability of 80%, that is:

- A public signal equal to 0 means that with a probability of 80% the final value of the asset will be 0 at the end of the period.
- A public signal equal to 10 means that with a probability of 80% the final value of the asset will be 10 at the end of the period.

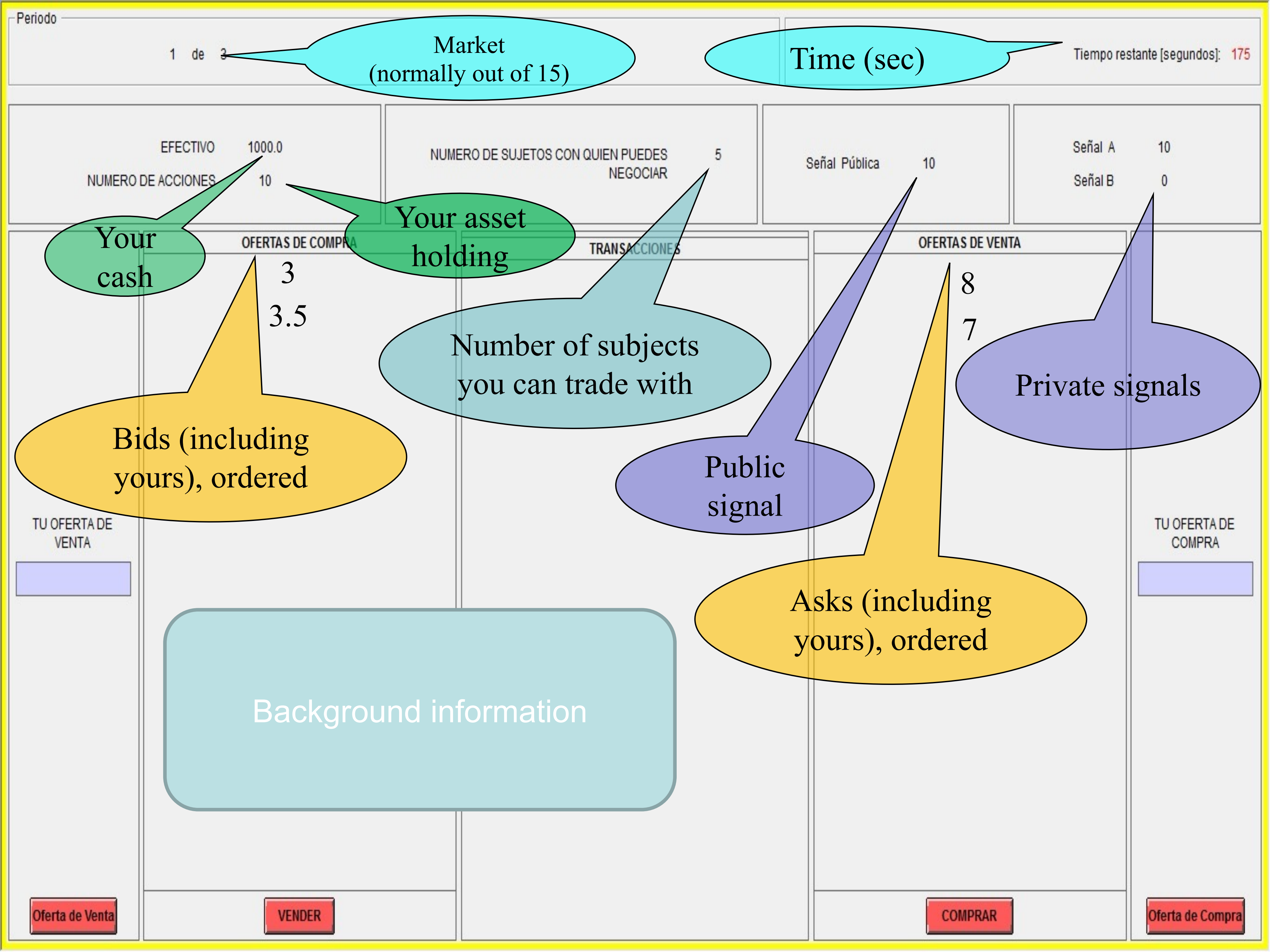
Recall that the public signal will be the same for all market participants. Likewise, you will receive private information about the value of the dividend at the end of the period in the form of two independent signals.

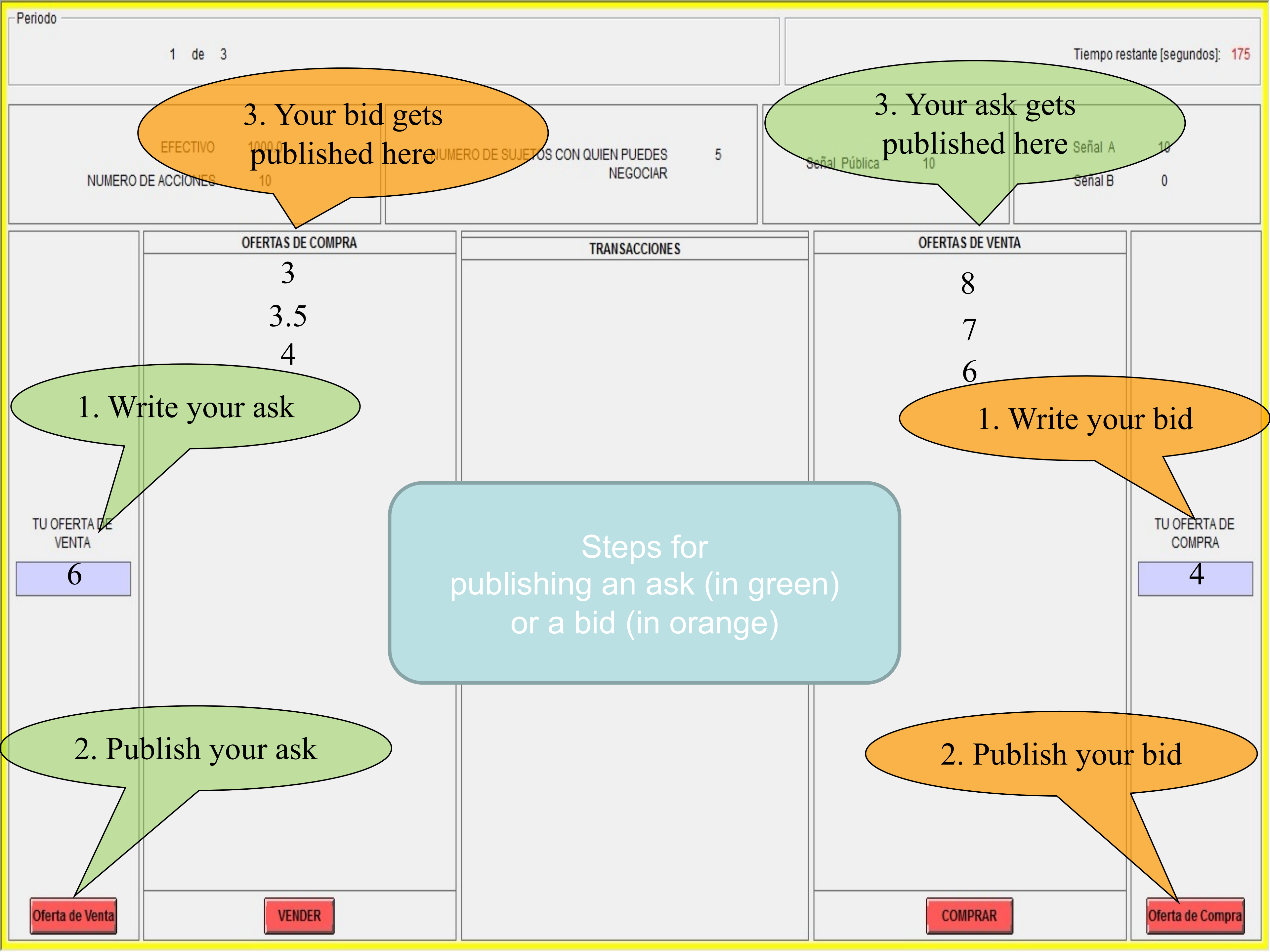
- A private signal equal to 0 means that with a probability of 80% the value of the dividend will be 0 at the end of the period.
- A private signal equal to 10 means that with a probability of 80% the value of the dividend will be 10 at the end of the period.

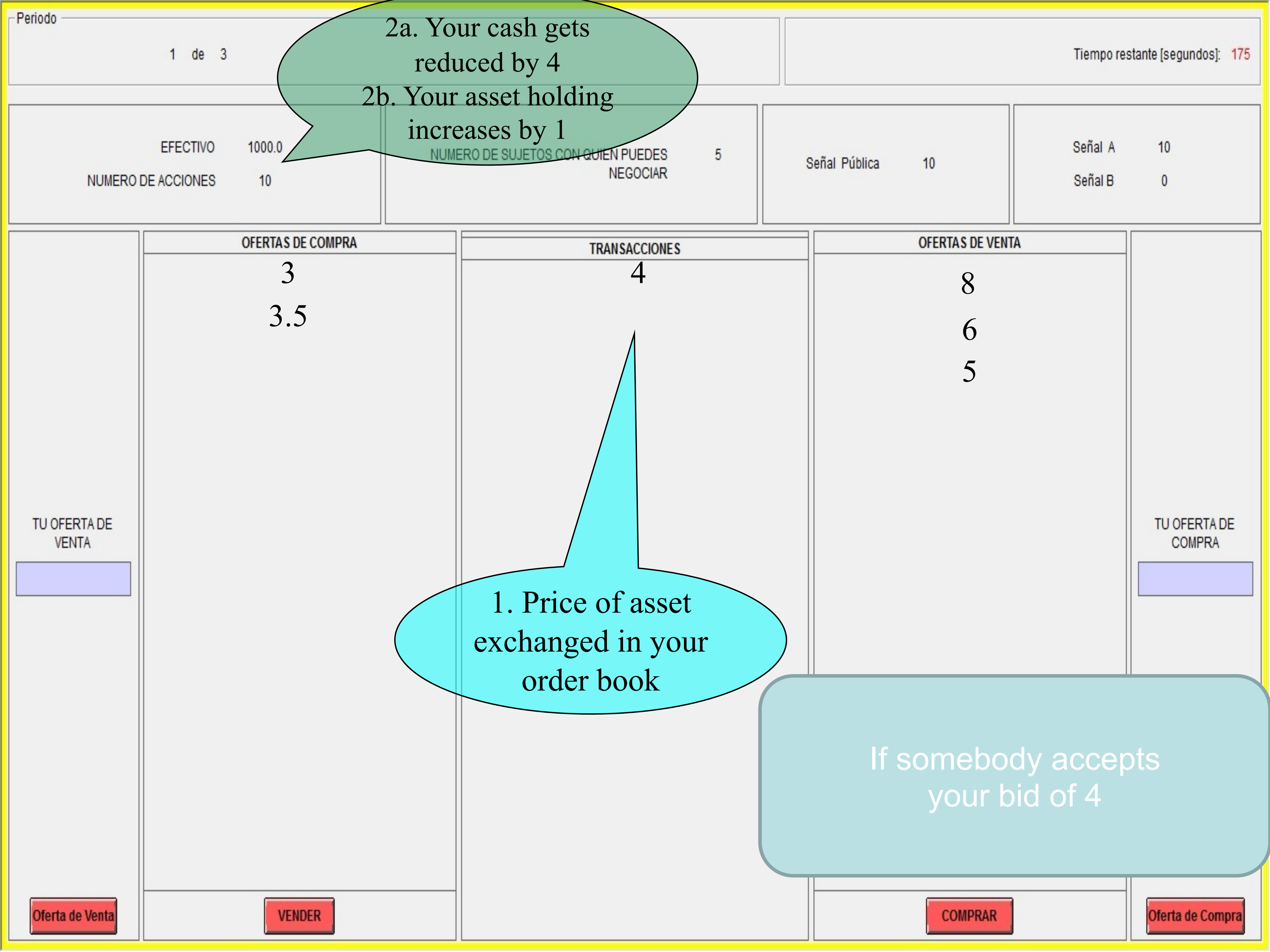
This will be your private information and therefore you will be the only one able to see it. At the end of each period, your profit will be the cash you have at the end of the period plus the dividend for each unit of the asset you own minus the 1000 ECU that were given to you at the beginning of the period. Your payment at the end of the experiment corresponds to the accumulated profit in all periods. If at any time you have any question or problem, do not hesitate to contact the experimentalist. Remember that it is important that you understand correctly the operation of the market, since your earnings depend both on your decisions and on the decisions of the other participants in your market.

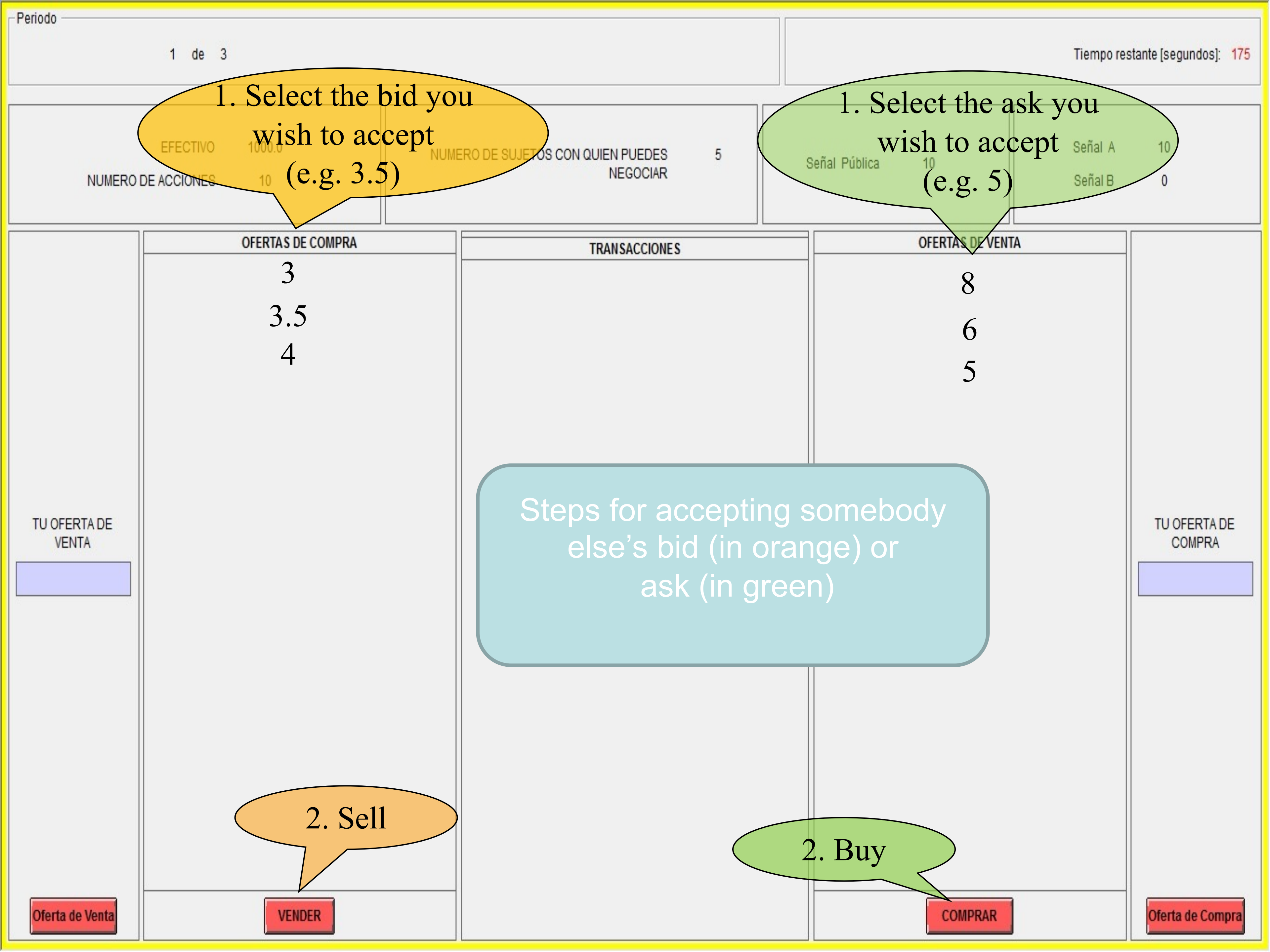
B5 Presentation for Subjects

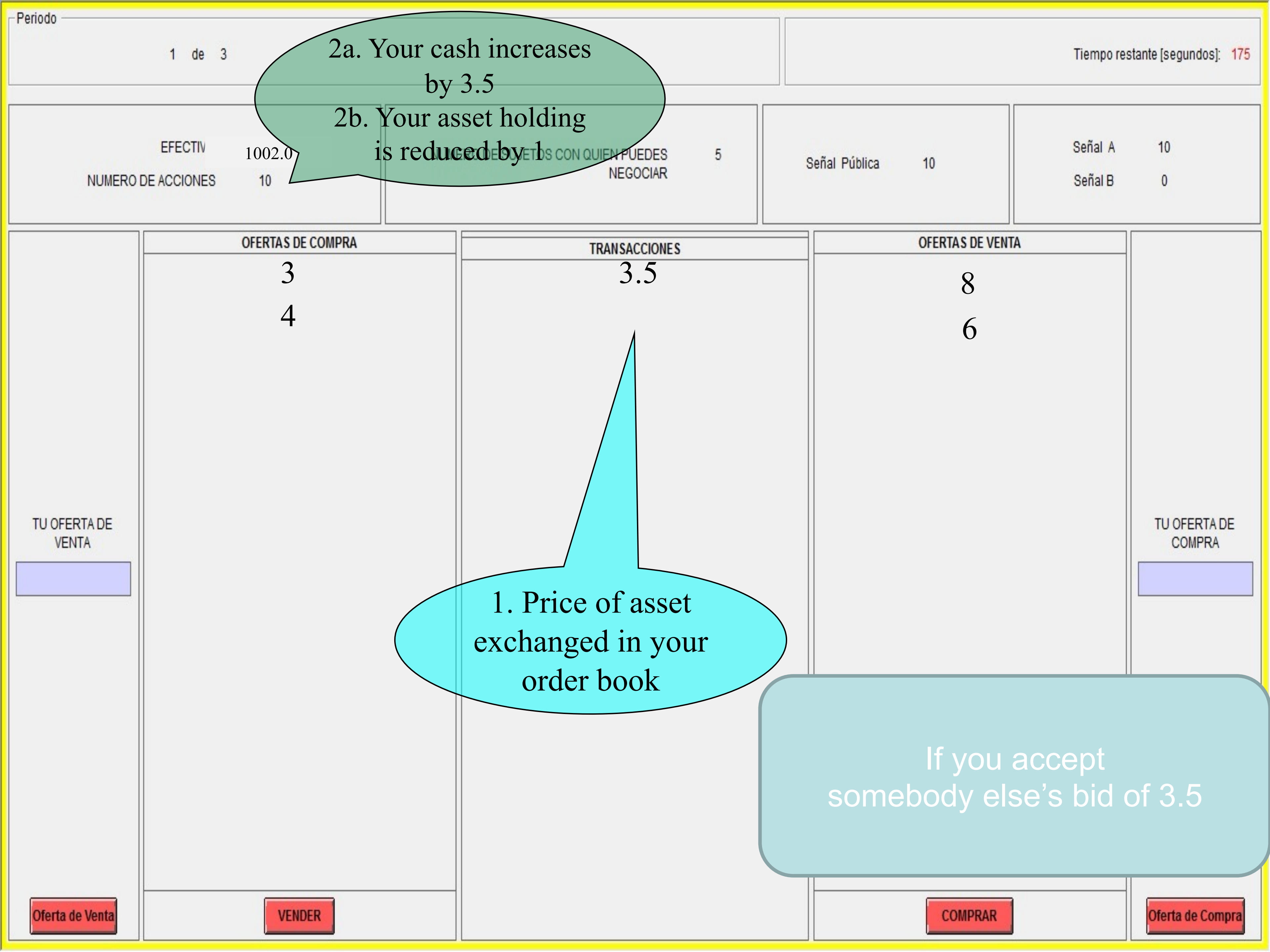
The following are screenshots from the presentation for subjects in the experiments on how to submit and/or accept bids or asks. The screenshots are for the MC and LC treatments with public information. In the FC treatment, the number of traders that subjects could trade with was not included, since the network was fully connected. The slides show, in turn, (i) background information, (ii) how to submit a bid or ask, (iii) what happens if one of the subject's own bids gets accepted, (iv) how to accept a bid or ask, and (v) what happens if the subject accepts a bid. If a subject tries to accept one of her own bids or asks, she receives a message saying that it is not possible.


Periodo
1 de 3
Market
(normally out of 15)
Time (sec)
Tiempo restante [segundos]: 175
EFECTIVO 1000.0
NUMERO DE ACCIONES 10
NUMERO DE SUJETOS CON QUIEN PUEDES NEGOCIAR 5
Señal Pública 10
Señal A 10
Señal B 0
Your cash
Your asset holding
OFERTAS DE COMPRA
TRANSACCIONES
OFERTAS DE VENTA
3
3.5
8
7
Number of subjects you can trade with
Private signals
Public signal
Bids (including yours), ordered
Asks (including yours), ordered
TU OFERTA DE VENTA
TU OFERTA DE COMPRA
Background information
Oferta de Venta
VENDER
COMPRAR
Oferta de Compra


Periodo
1 de 3
Tiempo restante [segundos]: 175
3. Your bid gets published here
3. Your ask gets published here
EFECTIVO
NUMERO DE ACCIONES 10
NUMERO DE SUJETOS CON QUIEN PUEDES NEGOCIAR 5
Señal Pública 10
Señal A 10
Señal B 0
OFERTAS DE COMPRA
3
3.5
4
TRANSACCIONES
OFERTAS DE VENTA
8
7
6
1. Write your ask
1. Write your bid
TU OFERTA DE VENTA
6
TU OFERTA DE COMPRA
4
Steps for publishing an ask (in green) or a bid (in orange)
2. Publish your ask
2. Publish your bid
Oferta de Venta
VENDER
COMPRAR
Oferta de Compra


Periodo
1 de 3
Tiempo restante [segundos]: 175
2a. Your cash gets reduced by 4
2b. Your asset holding increases by 1
EFECTIVO 1000.0
NUMERO DE ACCIONES 10
NUMERO DE SUJETOS CON QUIEN PUEDES NEGOCIAR 5
Señal Pública 10
Señal A 10
Señal B 0
OFERTAS DE COMPRA
3
3.5
TRANSACCIONES
4
OFERTAS DE VENTA
8
6
5
TU OFERTA DE VENTA
TU OFERTA DE COMPRA
1. Price of asset exchanged in your order book
If somebody accepts your bid of 4
Oferta de Venta
VENDER
COMPRAR
Oferta de Compra


Periodo
1 de 3
Tiempo restante [segundos]: 175
EFECTIVO 1000.0
NUMERO DE ACCIONES 10
NUMERO DE SUJETOS CON QUIEN PUEDES NEGOCIAR 5
Señal Pública 10
Señal A 10
Señal B 0
1. Select the bid you wish to accept (e.g. 3.5)
1. Select the ask you wish to accept (e.g. 5)
OFERTAS DE COMPRA
3
3.5
4
TRANSACCIONES
OFERTAS DE VENTA
8
6
5
Steps for accepting somebody else's bid (in orange) or ask (in green)
TU OFERTA DE VENTA
TU OFERTA DE COMPRA
2. Sell
2. Buy
Oferta de Venta
VENDER
COMPRAR
Oferta de Compra



Periodo
1 de 3
Tiempo restante [segundos]: 175
2a. Your cash increases by 3.5
2b. Your asset holding is reduced by 1
EFECTIV 1002.0
NUMERO DE ACCIONES 10
SUJETOS CON QUIEN PUEDES NEGOCIAR 5
Señal Pública 10
Señal A 10
Señal B 0
OFERTAS DE COMPRA
3
4
TRANSACCIONES
3.5
OFERTAS DE VENTA
8
6
TU OFERTA DE VENTA
TU OFERTA DE COMPRA
1. Price of asset exchanged in your order book
If you accept somebody else's bid of 3.5
Oferta de Venta
VENDER
COMPRAR
Oferta de Compra