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Government Procurement and Access to Credit: Firm Dynamics and Aggregate Implications*

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Abstract

We provide a framework to study how different allocation systems of public procurement contracts affect firm dynamics and long-run macroeconomic outcomes. We start by using a newly created panel dataset of administrative data that merges Spanish credit register loan data, quasi-census firm-level data, and public procurement projects to study firm selection into procurement and the effects of procurement on credit growth and firm growth. We show evidence consistent with the hypotheses that there is selection of large firms into procurement, that procurement contracts provide useful collateral for firms —more so than sales to the private sector— and that procurement contracts facilitate firm growth beyond the contract duration. We next build a model of firm dynamics with both asset-based and earnings-based borrowing constraints and a government that buys goods and services from private sector firms. We use the calibrated model to quantify the long-run macroeconomic consequences of alternative procurement allocation systems. We find that granting procurement contracts to small firms, either by directly targeting them or by slicing large contracts into smaller ones, helps these firms grow and overcome financial constraints in the long run. However, we also find that reducing the average size of contracts —or making it less likely for large firms to access them— removes saving incentives for large firms, whose negative effects on capital accumulation can overcome the expansionary consequences for small firms and hence generate a drop in aggregate output.

JEL Classifications: E22; E23; E62; G32
Keywords: Government Procurement; Financial Frictions; Capital Accumulation; Aggregate Productivity

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1 Introduction

Governments play a key role in economic activity. They set taxes and transfers, they are large employers, and they purchase goods and services from the private sector. The purchases of goods and services are done by awarding *public procurement contracts* to private firms. The size of public procurement varies over time and across countries, but it consistently represents a large fraction of GDP —12.8% in the OECD countries, 14% in EU countries, and 9.3% in the United States.[1] Because of its large size and high level of discretion, governments can use the public procurement process to allocate resources to specific sectors or firms. In the U.S., for example, the Small Business Act aims to "ensure that a fair proportion of federal contracts is awarded to small business".[2] Similarly, in the EU, promoting the participation of small firms is at the core of the European Commission's agenda for public procurement regulation.[3] Yet, in contrast to the choice of different tax and transfer mixes, we have little understanding of how the procedure of awarding public procurement contracts to private firms may affect the macroeconomy.

In this paper we study the effects of public procurement on firm outcomes and the macroeconomy. We argue that the long-run macroeconomic impact of public procurement depends crucially on the severity of firm-level financial frictions as well as on their type. As a consequence, differences in the procedure to allocate contracts to firms can have first-order effects on macroeconomic outcomes. In particular, we show that granting procurement contracts to small firms —either by directly targeting smaller firms or by slicing large contracts into smaller ones— helps these firms grow and overcome financial constraints in the long run, but the aggregate effects can reduce output.

We carry out an analysis that integrates a novel firm-level dataset with a macroeconomic model of firm dynamics. Our dataset merges administrative data on public procurement, credit allocation at the bank-firm level, and firm outcomes for the Spanish economy over the 2000-2013 period. Our model builds on the canonical framework of firm dynamics with financial frictions (Midrigan and Xu, 2013; Buera and Moll, 2015) and incorporates two novel elements. First, there is a government that purchases goods and services from private sector firms. Firms that are willing to sell to the government must make a risky investment in advance, which reflects the costs of preparing the proposal or increasing the chances to win the auction. Second, we allow for both collateral- and earnings-based borrowing constraints. That is, firms not only borrow against their assets but also against their earnings.[4]

[1] See https://ec.europa.eu/growth/single-market/public-procurement_en and https://stats.oecd.org/Index.aspx?QueryId=94406# for details.

[2] See https://sgp.fas.org/crs/misc/R45576.pdf for details.

[3] See the Public Sector Directive 2014/24/EU for details. For a while, there has been strong support from the European Parliament for explicit regulation that discriminates in favor of small firms: "From this House, we must insist that...administrative bodies incorporate terms into their tender specifications that facilitate positive discrimination in favor of SMEs and remove contractual provisions that hinder their participation." See https://www.europarl.europa.eu/doceo/document/CRE-7-2011-05-11-ITM-012_EN.html

[4] Lian and Ma (2020) show that 80% of corporate debt in the US is based on cash flows from firms' operations.

Given these tools, we proceed in three steps. In the first step, we use our data to show a strong pattern of firm selection into procurement based on firm size. In particular, we document that firms involved in procurement are larger on average, and that firms that end up participating in procurement are 72% bigger in terms of value-added before they do so. We refer to this difference as the "procurement size premium."

In the second step, using the same data, we characterize the treatment effect of government procurement on several outcomes related to firm dynamics. We show evidence consistent with the hypotheses that procurement contracts (a) provide useful collateral for firms —and more so than sales to the private sector— and (b) facilitate firm growth beyond the duration of the granted procurement contract. Our empirical evidence contains a host of novel results. In particular, we show that the increase in credit after winning a procurement contract is larger for firms more likely to be financially constrained, comes exclusively from credit that is not backed by tangible collateral, and is associated with an increase in the acceptance of loan applications. Furthermore, we show that firms' credit increases with procurement revenues when controlling for total revenues, providing evidence of the extra pledgeability of procurement contracts. In terms of real variables, we show through local projections that sales to the private sector decline in the short run (a negative spillover of procurement on private sector sales) and increase afterwards as firms accumulate net worth and capital.

Finally, in the third step, we use our model to study the interplay between procurement and the macroeconomy. We calibrate the model to reproduce several of the micro-moments related to firm selection into procurement and to firm dynamics after procurement described above, as well as to macro-moments. In terms of selection, the model generates a procurement size premium through two state variables of firms: productivity (TFPQ) and net worth. As is standard in models with firm heterogeneity, the value of participating in a given market —the procurement market in this case— depends on the firms' ability to deliver large projects (e.g., Melitz (2003) in the context of international trade). In our model, this ability uniquely depends on firms' TFPQ in the case of financially unconstrained firms. However, for constrained firms, that ability also depends on their financing capacity, which itself depends on firms' net worth.[5] In our baseline calibration, where we match the 72% value-added procurement size premium mentioned above, our model implies a procurement premium of 36% in terms of TFPQ and 53% in terms of net worth.

Regarding the treatment effect of procurement on credit growth, the model is calibrated to reproduce a regression in which the change in firms' leverage, i.e., total credit divided by fixed assets, depends on two variables: the change in total earnings divided by fixed assets and the

Recent papers by Aguirre et al. (2021), Caglio et al. (2021), Drechsel (2021), Gupta et al. (2021) and Li (2022) also find empirical evidence of earnings-based borrowing constraints.

[5]This result is similar to the one in Chaney (2016), who shows how liquidity (and not only productivity) determines firms' ability to export.

change in total earnings from procurement divided by fixed assets. Through the lens of our model, the coefficient associated with the former pins down the parameter that governs the pledgeability of firms' earnings from selling to the private sector, whereas the latter pins down the difference between that and its counterpart from procurement.[6] We run this regression for firms that are likely to be financially constrained in our data, i.e., young firms, and find that firms can pledge 42% of their annual earnings from selling to the private sector and 110% of their annual earnings from procurement. In terms of the dynamics of real variables, the model is consistent with the empirical finding of firm growth beyond the duration of the contract. The high pledgeability of procurement contracts together with the extra profits generated by them reinforces the self-financing channel previously emphasized in the literature (Moll, 2014). In this respect, public procurement is a powerful policy tool to help small firms overcome financial frictions and achieve closer to optimal size in the long-run. Importantly, we find that this positive long-run effect takes place despite the fact that procurement temporarily crowds out constrained firms' sales to the private sector. In our model, this within-firm spillover occurs because financially constrained firms have to split their scarce collateral to serve both procurement and private sector operations. The fact that government sales can be collateralized partly alleviates but does not eliminate this problem.

To assess the interplay between procurement and the macroeconomy, we use our calibrated model to perform some *expenditure-neutral* counterfactual experiments that consist of reallocating procurement contracts across firms while keeping government expenditure unchanged. In particular, we compare our benchmark economy with counterfactual economies in which a higher share of procurement contracts is allocated to small firms. Our preferred counterfactual, which consists of encouraging small firms' participation by decreasing the size of the contracts, aims to mimic the European Commission's strategy to increase the presence of small firms in public procurement in Europe.[7] We find that increasing the fraction of firms to which the government allocates contracts (from 3.8% to 13.8%) has important macroeconomic implications. First, we find that aggregate GDP would fall by 2.68%, of which around 15% is explained by a fall in TFP and the rest is explained by a fall in aggregate capital. The fall in TFP is the result of a small increase in TFP in the private sector —which is explained by a reduction in misallocation across firms due to the reinforcement of the self-financing channel— and a big reduction of TFP in the procurement sector— which is explained by the fact that the selection pattern based on firms' TFPQ weakens.

To understand the mechanisms behind the evolution of capital accumulation and GDP, we conduct a decomposition of the policy experiment's effects that allows us to isolate three different channels. The first channel is a negative *short-run partial equilibrium effect* on directly affected firms, which is the result of aggregating the crowding-out effects at impact mentioned above. We

[6]This structural identification of the earnings-based constraints is similar to the one used by Li (2022) for the case of only private sector earnings,.

[7]See Trybus (2014) for details.

find that this channel would reduce GDP by 0.37%. The second channel is a positive *long-run partial equilibrium effect* on directly affected firms, which is the aggregate consequence of the strengthening of the self-financing channel. We find that this channel would increase GDP by 4.45%. The third channel is a negative *long-run general equilibrium effect* coming from the change in capital accumulation incentives of all firms (not only those that ex-post obtain procurement projects) and their responses to general equilibrium price changes. One of the main reasons why firms accumulate financial wealth in our model is the fact that they expect to obtain a public procurement contract at some point. That is, obtaining a procurement contract acts as a large demand shock in response to which firms want to expand their capital stock, causing even relatively big firms to accumulate precautionary savings. Intuitively, productive firms want to have enough net worth so that they minimize the probability of being constrained in case the procurement shock is realized. A procurement policy that targets smaller firms very aggressively will remove savings incentives for middle-size and large firms. This channel is the main driver of the 2.68% GDP decline.

We also conduct an alternative policy experiment that consists of promoting small firms' participation by directly targeting them in the procurement allocation system, as opposed to promoting their participation by reducing the average size of contracts. In particular, we solve for an economy in which we target a procurement premium of 50%, as opposed to the 72% in the baseline calibration. Our main finding is that this alternative policy counterfactual would reach out to a lower number of small firms than in our preferred counterfactual but would actually generate an increase in aggregate GDP of 2.07%. The reason is that, although large firms become less likely to get contracts under this alternative allocation system, the size of these contracts remains big, which implies that the reduction in large firms' incentives to accumulate assets is significantly smaller than in our main counterfactual.

Finally, we find that the aggregate effects of this type of policy counterfactuals would be significantly more detrimental for the macroeconomy in a world in which earnings from government contracts exhibit the same pledgeability as our calibrated pledgeability of earnings from selling to the private sector. For example, for the case of our preferred counterfactual, we find that GDP losses would be almost three times as large as those implied when running the same counterfactual under our baseline calibration (–7.04% vs. –2.68%). By reducing the extent to which borrowing capacity increases when participating in procurement, the above-mentioned positive *long-run partial equilibrium effect* weakens. This result points towards the importance of the extra collateral provided by government contracts when evaluating the aggregate effects of changes in the procurement allocation system.

1.1 Related literature

There is practically no literature that analyzes how the microeconomic aspects of public procurement can affect the macroeconomy. One recent exception is Cox et al. (2021), who document several new facts using micro-level data on public procurement contracts awarded by the U.S Federal Government, and investigate how accounting for these facts— in particular that government spending is concentrated in sectors where prices are more sticky— can affect the short run fiscal transmission mechanism in a New Keynesian model. Our interest instead is in quantifying the long-run macroeconomic effects of different procurement allocation systems.

Governments have been proposed as directly responsible for the long-run economic performance of countries through the implementation of policies that distort the allocation of resources across firms. Some examples are credit subsidies to state-owned-enterprises (Song et al., 2011), the reservation of goods for small firms (García-Santana and Pijoan-Mas, 2014), labor market regulations (Garicano et al., 2016), or tariffs (Berthou et al., 2019). However, one of the most important roles that governments play in modern economies, i.e., their role as buyers of goods and services from private sector firms, has been overlooked. We focus on this by analzying specific size-dependent procurement policies aimed at helping small firms. In this respect, our work is related to Guner et al. (2008), who show the importance of size-dependent policies in affecting misallocation across firms and aggregate productivity.

Our focus on firm-level financial frictions as a channel through which public procurement can affect the macroeconomy builds on the literature that quantifies the effects of financial constraints on aggregate output and productivity (Buera et al., 2011; Midrigan and Xu, 2013; David and Venkateswaran, 2019; Catherine et al., Forthcoming).[8] A few papers in this literature have studied the interplay of financial frictions with different forms of taxation (Erosa and González, 2019; Itskhoki and Moll, 2019; Guvenen et al., 2019; Blanco and Baley, 2022) but none has focused on the expenditure side of government policies. Our finding that the type of financial frictions matters in understanding the effects of procurement on the macroeconomy is also related to recent papers that show that the type of financial frictions, i.e., earnings- vs. asset-based, and not only their severity, plays a crucial role in explaining important economic outcomes: the gains from trade liberalization (Brooks and Dovis, 2020), aggregate productivity (Li, 2022), macroeconomic fluctuations (Drechsel, 2021), and the transmission of monetary policy (Caglio et al., 2021).

Our results on the treatment effects of winning procurement contracts on firms are related to the recent literature analyzing the relationship between public procurement and firm dynamics. Ferraz et al. (2016) and Lee (2021) use quasi-experimental designs for Brazil and South Korea, respectively to show that firms winning procurement contracts have a positive and permanent effect on firms' performance. Hebous and Zimmermann (2021) document for the US a positive relationship between

[8]See Buera et al. (2015) for a survey of this literature.

winning a procurement contract and firm investment, and show that the effect disappears when looking at firms that are less likely to be financially constrained. Our results are consistent with all this body of research. We provide novel evidence on loan acceptances and on the fact that only non-collateralized credit increases, which along with the other empirical facts that we document, can be taken as direct evidence of earnings-based financial constraints that are alleviated with procurement projects. Additionally, our results on the short-run crowding out of sales to the private sector by procurement sales are related to recent papers that investigate within-firm spillover effects across markets, like Almunia et al. (2021) with domestic versus foreign markets and Alfaro-Ureña et al. (Forthcoming) with multinational corporations versus other buyers. Finally, Cappelletti and Giuffrida (2021) use data for Italy to show that firms that receive public procurement contracts survive longer, a dimension of the data that we do not explore.

In the context of public procurement, our paper also relates to two different strands of literature. First, it relates to the literature that analyzes the factors that determine the outcomes in the allocation of procurement projects (Engel et al., 1997, 2001; Decarolis, 2018; Bosio et al., 2020). And second, it also relates to the recent empirical literature that investigates the capability of governments to generate desired economic outcomes (Bandiera et al., 2009).

1.2 Organization of the paper

The rest of the paper is structured as follows. Section 2 describes the construction of the dataset and provides summary statistics. Section 3 provides our empirical evidence organized in five stylized facts. Section 4 presents the model of firm dynamics with procurement, leaving the formal details to Appendix B. Section 5 discusses how we parameterize the model. Section 6 describes our benchmark economy. Section 7 provides the main quantitative results. Section 8 concludes.

2 Data and Summary Statistics

Our empirical work is based on merging three large datasets at the firm level. Specifically, we combine data from public procurement projects awarded to firms; firm-level data on balance sheets and income statements; and loan-level data between every bank and firm in the Spanish economy. In this Section we discuss each dataset, the merging process, and key summary statistics.

2.1 Public procurement data

Public procurement is defined in the *System of National Accounts (SNA)* as the sum of intermediate consumption (e.g., purchases of goods like medical consumables and services like accounting services), gross capital formation (e.g., building new roads), and social transfers in kind via market producers (e.g., medicines). Roughly speaking, one can think of public procurement as "government consumption expenditures and gross investment" (the G part of GDP) minus "compensation of employees" and "consumption of fixed capital."[9] The size of public procurement varies across countries and over time. For the case of OECD countries, public procurement represented approximately 12% of GDP and 30% of G averaged over the 2007-2017 period.

Main sample of projects published in BOE. According to Spanish law, all procurement contracts above a certain threshold awarded by public institutions must be published in official bulletins.[10] If the contract is awarded by the central government, the information on this contract must be published in the *Agencia Estatal Boletín Oficial del Estado* (BOE), which is the official bulletin of the central government of Spain. In contrast, if the entity that awards the contract is a regional government or a municipality, the information about this contract can alternatively be published at their respective regional or local bulletin.

We construct a novel dataset on Spanish public procurement contracts by scraping the BOE website over the 2000-2013 period. Each contract provides considerable information on each awarded project. In particular, we collect information on the type of contract (kind of good or service provided), the institution awarding the contract, the initial bidding and final price of the contract, the type of procedure used to allocate the contract, and the firm(s) that won the contract. In total, we scraped more than 150,000 projects over 2000-2013, which we assign to the month that the project was awarded. Of these, 130,633 projects have a value assigned to them that we were able to recover. The sum of all these projects totals around 220 billion euros. On average, our micro data account for around 13% of total public procurement as measured in Spanish National Accounts. Despite the level differences, our micro data are able to capture the overall evolution of public procurement over time, which increased from 9.9 to 13.8 percent between 2000 and 2009 and decreased from 13.8 to 10.0 percent between 2010 and 2013; see Figure A1.

Small sample of projects with information on bidders. Although the BOE website provides detailed information about the characteristics of the contracts, it does not provide the identity of the firms that competed for the project but did not win. This is a limitation of our dataset because it does not allow us to construct a well-defined control group. To overcome this limitation, we construct a sample of procurement projects for which we have detailed information about the awarding process. Although we did not find any government agency that provided information about the awarding process during our main sample period (2000-2013), we could identify around 50 agencies that started providing detailed information about their projects starting in 2013. Putting all these agencies together, we were able to uncover the identity of the firms competing for the same projects as well as their final rankings for around 1,000 contracts over the 2013-2016 period.

[9] See Appendix A.1 for details.

[10] The thresholds above which the contract must be advertised in official bulletins depend on the type of contract. In the case of supplies and services, for example, the threshold is 60,000 euros.

2.2 Balance sheet data

We use the balance sheets and income statements of the quasi-universe of Spanish companies between 2000 and 2016, a dataset that is maintained by the Banco de España and taken from the Spanish Commercial Registry. For each firm and year, this dataset includes information on the firm's name, fiscal identifier, sector of activity (4-digit NACE Rev. 2 code), age, net operating revenue, material expenditures, number of employees, labor expenditures, total fixed assets, total assets, and net worth. The final sample covers a total of 1,801,955 firms with an average of 993,876 firms per year. This represents around 85-90% of the firms in the non-financial market economy for all size categories in terms of both turnover and number of employees. This database is used by García-Santana et al. (2020) among others and is described in detail by Almunia et al. (2018).

2.3 Credit data

The *Central de Información de Riesgos* (CIR) is maintained by the Banco de España in its role as primary banking supervisory agency, and contains detailed monthly information on all outstanding loans over 6,000 euros to non-financial firms granted by all banks operating in Spain since 1984. Given the low reporting threshold, virtually all firms with outstanding bank debt appear in the CIR. In addition to the total amount of credit, CIR also contains information on whether or not a non-personal collateral ("Garantía real") was posted for a particular loan. These collaterals include assets like real estate, land, machinery, securities, deposits, and merchandise (i.e., hard collateral).[11] With this information, we can hence assess whether a particular loan for a bank-firm pair was granted on the basis of tangible collateral. We use data from 2000 to 2016.

Loan applications. Besides the information on outstanding loans, we also have information about loan applications at the firm-bank level. The construction of this dataset is as follows. Spanish banks can request information about a firm whenever this firm "seriously" approaches them to obtain credit.[12] Because banks already have information about the firms with which they have a credit relationship, banks only request information on firms that have never received a loan from them or that ended the credit relationship before the current request. By matching the loan applications with the information on outstanding loans from CIR, we can infer whether the loan was granted or not.[13]

[11] See Ivashina et al. (Forthcoming) for more details.

[12] The Law stipulates that a bank can not request information about the firm without its consent, which indicates the seriousness of the approach

[13] Both the CIR and loan application data provide the identity, i.e., fiscal identifier, of the firm involved in every loan, allowing us to easily match the loan data with the balance sheet and income statements of firms.

2.4 Summary statistics

We create a merged data set at the firm-quarter level.[14] In this Section we provide some summary statistics.

Types and size of procurement contracts in BOE. For many people, procurement is associated with large infrastructure projects. However, only 20% of the contracts in our BOE data are in the construction sector and the median size of procurement projects in construction (0.74 million euros) is not too different from the one in the other categories reported by BOE: services (0.42), consulting (0.37), supplies (0.37), and other sectors (0.35) respectively. The major differences in project size across sector appear in the right tail of the distribution, with the top 1% of projects in construction being much larger than in other sectors. We also note that there is a large number of relatively small projects in all sectors: 25% of projects have a value of less or equal to 230,000 euro in construction, 200,000 euro in services, and 170,000 in consulting and in supplies. See Table A1 for details.

Although we do not have direct information about the duration of the contracts in our sample, we were able to collect information about the duration of the contracts awarded in Spain in the year 2015. Around 71% of the contracts have a duration which is one year or less, and 91% have a duration which is two years or less.[15]

Presence of procurement firms. Looking at the firm-level data, we find that procurement firms are present in most industries of the economy: firms with at least one procurement contract in a given year operate in 71 out of the 91 industries based on NACE 2-digit classification. The share of procurement firms in our data is 0.5% percent, but it varies a lot across industries, with the highest fraction —around 15%— in industries like "Manufacture of coke and refined petroleum products" and "Manufacturing of Pharmaceutical Products." Because procurement firms tend to be larger, the share of employment, sales, assets, or credit of procurement firms tend to be larger than the share of firms, see Table A2 for details. We note that this 0.5% percent of procurement firms in our data is below the actual number in the population. First, as mentioned above, our procurement data only captures 13% of the total procurement value measured in national accounts. Second, because our data are biased towards contracts awarded by the central government, we capture contracts that are bigger than the average awarded contract by other governments (e.g., local governments) and hence are probably more concentrated in a few firms. We do a back of the

[14]The balance sheet and credit data can be merged by use of firm fiscal identifiers. Instead, the BOE data does not provide the fiscal identifier of firms, so we merge the procurement data using the name of the firm by fuzzy algorithms.

[15]As a reference for a different country, Cox et al. (2021) find that the median contract in the U.S. has a duration of 31 days and 90% of contracts last less than one year.

envelope calculation to infer the fraction of firms that are active in procurement in the population. The total value of the procurement contracts in our sample in a given year captures around 1.5% of GDP. By assuming that the fraction of firms active in procurement is proportional to the share of procurement in GDP, we can use the observed 12.1% share of procurement in GDP from national accounts to recover an implied fraction of procurement firms of 3.8% ($0.121/0.015 \times 0.005 = 0.038$). This number will be one of our targets in the calibration of the model in Section 5.[16]

Procurement vs. non-procurement firms. Table 1 shows the mean and selected percentiles of the distribution of some relevant variables for both procurement and non-procurement firms. We highlight three patterns. First, firms participating in procurement are significantly larger and older on average, but there is considerable overlap in the support of the size and age distribution for procurement and non-procurement firms. For example, the average number of employees of a procurement firm is around 6 times larger than for the rest of the firms (73.56 vs. 12.75), total sales are 7 times larger for procurement firms than for the rest (8.9 millions of euro vs. around 1.2 million), and procurement firms are 9 years older (20 vs 11 years). Yet, around 25% of procurement firms have less than 16 employees, have revenues which are lower than 1.14 million euro, and are 12 or less years old. Second, conditional on having at least one procurement project, there is a lot of variation on the importance of these projects as a fraction of firms' total revenue. The average ratio of all the procurement value to total revenue is 0.20, with 25th, 50th, and 75th percentiles of 0.01, 0.03 and 0.10 respectively. And third, we observe large differences between procurement vs. non-procurement firms in terms of their composition of credit. In particular, procurement firms seem to rely more on non-collateralized credit (86% vs 71% on average) despite holding higher levels of assets.

3 Procurement, Credit Growth, and Firm Dynamics

We begin by documenting several facts related to firms' participation in procurement. First, we show a positive relationship between obtaining a procurement contract and firms' credit growth. Second, we find evidence suggesting that this association arises, at least partly, due to an easing of financial constraints, in contrast to an increase in firms' demand for credit. Third, we show that loans growth is mostly explained by an increase in credit for which no tangible collateral is posted. Fourth, we provide evidence consistent with the fact that winning a procurement contract eases firms' financial constraints more than selling to the private sector. And fifth, we document the dynamic path followed by some key real variables —namely, fixed assets, net worth, total sales,

[16] To the best of our knowledge, there are no official statistics on the number of firms selling to governments in a given country. As a reference from a different country, Lee (2021) calculates that 5.3% participate in the procurement market in South Korea.

Table 1. Descriptive evidence from the final merged dataset, year 2006

	mean		**25th pctile**		**50th pctile**		**75th pctile**	
	Proc	NoProc	Proc	NoProc	Proc	NoProc	Proc	NoProc
Age	20.42	10.95	12.00	5.00	17.00	10.00	24.00	15.00
Employment	73.56	12.75	16.00	3.00	45.00	6.00	155.0	12.00
Sales	8.96	1.19	1.14	0.10	4.22	0.28	16.89	0.86
Procurement/Sales	0.20	0.00	0.01	0.00	0.03	0.00	0.10	0.00
Fixed Assets	3.80	0.85	0.21	0.03	0.82	0.14	3.58	0.50
Net worth	3.92	0.43	0.36	0.01	1.27	0.07	6.12	0.30
Credit	2.51	0.57	0.11	0.03	0.48	0.08	2.32	0.30
Coll. Credit (share)	0.14	0.29	0.00	0.00	0.00	0.00	0.14	0.74

Notes: This table presents summary statistics from our merged dataset for the year 2006, separately for firms with at least one procurement contract ($n = 2,411$) vs. the rest of the firms ($n = 406,261$). The variable *Employment* measures the number of full-time workers employed by the firm; the variable *Sales* is just firm's revenue measured in millions of euro; *Procurement/Sales* measures the value of all the procurement projects awarded to a firm in a given year divided by total revenue in that year; *Assets* measures the value of fixed assets; *net worth* measures total assets minus total debt; *Credit* measures the value of all firm's outstanding loans in millions of euro; *Coll. Credit (share)* is the share of *Credit* collateralized against firm's assets; *Def. Credit (share)* is the share of defaulted credit over total *Credit*; *age* measures the age of the firm. We winsorize the 1% tails of all variables to make numbers result to outliers.

and sales to the private sector— after a firm obtains a procurement project.

To document these facts we use two subsamples from our merged dataset: a sample of firms that obtain at least one procurement project between 2000 and 2013 (the main sample) and the sample of firms for which information and ranking of the other bidders is available (bidders sample).[17] For facts 1 to 3 we use data at quarterly frequency, but for facts 4 and 5 we will need to move to the annual frequency.

Fact 1. Procurement and credit growth

We start by regressing firms' credit growth on a dummy variable for procurement as follows:

$$\Delta \log l_{it} = \alpha_{iy} + \alpha_{st} + \beta_1 \text{PROC}_{it} + \beta_2 \log l_{it-1} + \varepsilon_{it}, \tag{1}$$

where the dependent variable $\Delta \log l_{it}$ is the quarterly growth of credit (loans) of firm i between quarter $t-1$ and quarter t defined as $\Delta \log l_{it} \equiv \log l_{it} - \log l_{it-1}$, winsorized between -1 (-100%) and $+2$ ($+200\%$). The regressor PROC_{it} is a dummy variable that takes value one if the firm obtained a procurement contract in quarter t. We control for the the firm's lagged credit at $t-1$, as well as for a stringent set of fixed effects. We use firm×year fixed effects, α_{iy}, in order to capture

[17] For the main sample, one could alternatively use a sample with all firms, but results would be very similar. This is because all our specifications use firm fixed effects, and hence the identification of the effects of procurement comes from the panel variation and not from comparing firms that participate in procurement on a regular basis with firms that never compete for procurement.

Table 2. Credit Growth and Procurement

	All firms			Bidders only	
				First	Second
	(1)	(2)	(3)	(4)	(5)
PROC_{it}	0.006^a	0.041^a	0.017^a	0.073^a	-0.061
	(0.002)	(0.015)	(0.006)	(0.028)	(0.049)
$\text{PROC}_{it}\times\text{Age}_{iy}$		-0.012^b			
		(0.005)			
$\text{PROC}_{it}\times\text{NW}_{iy}$			-0.027^c		
			(0.014)		
$\log(\text{Credit}_{it-1})$	-0.489^a	-0.492^a	-0.492^a	-0.175^a	-0.229^a
	(0.009)	(0.011)	(0.011)	(0.043)	(0.044)
Observations	130,295	73,572	73,572	8,310	3,683
R-squared	0.549	0.565	0.565	0.360	0.458
Sector×quarter FE	Yes	Yes	Yes	No	No
Firm×year FE	Yes	Yes	Yes	Yes	Yes
Quarter FE	No	No	No	Yes	Yes
Auction FE	No	No	No	Yes	Yes

Notes: This table presents results from estimating the relationship between total credit growth and procurement participation (PROC) by regression (1) (i) with firms obtaining at least one procurement project over 2000-13 in column (1), and (ii) with firms who participated in procurement contests over 2013–15 in columns (4) and (5) where the PROC dummy indicates the winning firm ('First') in column (4) and the runner-up firm ('Second') in column (5). Further, the table presents results from estimating the relationship between total credit growth and procurement participation and its interaction with firm age (Age) and net worth (NW) by regression (2) in columns (2) and (3), respectively, using the 2000-13 sample. All regressions use quarterly data. Standard errors are clustered at the firm level; [a] indicates significance at the 1% level, [b] at the 5% level, and [c] at the 10% level.

firm-level characteristics that vary over time at the yearly (y) level, such as total sales growth. We further include 4-digit sector×quarter effects, α_{st}, which control for both sector and macroeconomic conditions that vary over time. Therefore, identification of the key parameter of interest, β_1, comes from the variation of a firm's credit growth across quarters within a year conditional on obtaining a procurement contract.

Table 2, column (1), presents the results of this regression for the entire set of procurement firms (conditional on having obtained at least one procurement project between 2000 and 2013). The estimate of β_1 is positive and significant at the one-percent level.[18] The estimated coefficient implies that winning a procurement contract in a quarter translates into an increase of credit growth of 0.6 percent at the quarterly level, or roughly 2.4 percent in annual terms.

We next zoom in on the sample of procurement projects where we have information on all bidders as well as the final ranking. Doing so allows us to run regressions analogous to (1), except that we can identify the association between a firm's ranking in a given auction and its ensuing

[18]Note that we cluster standard errors at the firm-level in all regressions unless otherwise noted.

credit growth. To be more precise, we run two regressions similar to specification (1) at the auction level. In the first regression, we include all bidders and the PROC variable indicates which firm wins the auction ('First' place).[19] In the second regression, we drop the winner of the procurement contest and the PROC dummy now indicates which firm was runner-up ('Second' place). We run this second regression to make sure that winning the contract, as opposed to the relative ranking, is what is really associated with differences in credit growth across auction participants. Given that we run regressions at the project level, we are able to further include an auction fixed effect, which allows us to exploit variation across firms within the same project. We further include firm fixed effects that vary at the annual level as well as quarterly fixed effects.

Table 2, column (4), shows the results of estimating the regression where PROC indicates the firm that comes first place in the auction. We find that the winner of a procurement contract has higher credit growth relative to the firms it competes against in a given auction. Note that identification of the coefficient is exploiting the full time series of bidders, so the comparison is based on the within-auction group of firms but also with respect to each firm's annual credit growth given the inclusion of firm×year effects. The coefficient on the winner, 0.076, is considerably larger than that which we estimated on the PROC dummy in column (1). However, the regressions are not directly comparable given the difference in both composition and number of firms included in the two sets of regressions. Finally, column (4) of Table 2 shows the results of estimating the regression where we exclude the winner of the procurement auctions, and PROC captures the firm that finishes second in the auction. The estimated coefficient on PROC implies that there is no statistical difference in quarterly credit growth for the firm that placed second relative to other losers of the auction.

Fact 2. Procurement and firms' borrowing capacity

We next extend specification (1) to allow for the relationship between credit growth and procurement to vary depending on how likely a firm is financially constrained. To proxy for this likelihood, we use two variables that are usually considered as predictors of firms' ability to borrow: age and net worth (normalized by total assets). In particular, we run regressions that interact these two variables with the PROC_{it} variable:

$$\Delta \log l_{it} = \alpha_{iy} + \alpha_{st} + \beta_1 \text{PROC}_{it} + \beta_2 \text{PROC}_{it} \times \text{FC}_{iy-1} + \beta_3 \log l_{it-1} + \varepsilon_{it} \tag{2}$$

where FC indicates the proxy for a firm's ability to borrow: age or net worth (computed as assets minus debt divided by assets). As we only have information on these variables from firms' annual balance sheet data, we interact the firm's quarterly procurement dummies within a given year's

[19] In Appendix A.4, we show that the evolution of credit growth for winners and non-winners was similar before the time of the auction and diverge afterwards.

balance sheet value.[20]

Table 2 presents the main results of these regressions in columns (2) and (3), where we once again control for time-varying firm-level (annual) and sector (quarter) effects. We constrain the sample size such that we have observations both for age and net worth. The interaction of the procurement dummy and the age variable in column (2) is negative and significant, implying that younger firms experience higher credit growth upon receiving a procurement contract than older firms. Specifically, one year of age translates into 1.2 percent higher credit growth. The coefficient on the interaction with the net worth variable (NW) in column (3) is also negative, though only marginally significant. The negative coefficient also provides evidence that more credit constrained firms experience larger credit growth upon receiving a procurement as relatively low net worth firms will experience higher loan growth once receiving a contract.

We next ask whether firms are able to use their procurement contracts to access credit more easily at the extensive margin. A unique piece of information contained in the Banco de Espana's credit registry allows us answer this question: the information on the loan application process for firms and banks. In particular, we can see whether a firm has applied to a given bank and whether the loan application has been accepted or rejected throughout our sample period. We use this information to help identify a firm's access to credit based on the extensive margin. To do so, we now run regressions at the firm-bank level and relate the probability of firms obtaining a loan to whether they have received a procurement contract using the following linear probability specification:

$$\text{Loan granted}_{ibt} = \alpha_{ib} + \alpha_{bt} + \alpha_{st} + \beta \text{PROC}_{it} + \varepsilon_{ibt} \tag{3}$$

where the variable 'Loan granted' is a 0/1 dummy variable that is turned on when the firm receives a loan from bank b in quarter t conditional on the firm applying for it during that same quarter. We include firm×bank fixed effects, α_{ib}, which implies that we are identifying the coefficient β on the procurement variable via the variation within a firm-bank relationship over time. We further control for firm×year and sector×quarter effects as in the intensive margin credit growth regressions above, but we now also control for overall bank supply in a given period with the bank×quarter fixed effect α_{bt}.

Table 3 shows the results from running this regression. We include only firm×bank fixed effects in column (1), and augment these fixed effects with the time-varying firm and bank fixed effects in column (2). Overall, regardless of the specification, the probability of receiving a bank loan increases by approximately 2 percent in the quarter that a firm wins its first procurement project.

Fact 3. Procurement and the composition of credit

[20] Note that since we include firm×year fixed effects, α_{iy}, we do not need to include age or net worth on their own in the regressions.

Table 3. Probability of a New Loan and Procurement

	Accept	
	(1)	(2)
PROC_{it}	0.024[a]	0.023[b]
	(0.008)	(0.011)
Observations	36,857	26,924
R-squares	0.395	0.628
Firm×bank FE	Yes	Yes
Bank×quarter FE	No	Yes
Sector×quarter FE	No	Yes

Notes: This table presents results from estimating the relationship between loan participation and procurement participation (PROC) by regression (3) with firms obtaining at least one procurement project over 2000-13 using quarterly data. Standard errors are clustered at the firm level; [a] indicates significance at the 1% level, [b] at the 5% level, and [c] at the 10% level.

We next decompose the increase in credit associated with winning a procurement contract into that coming from collateralized vs. non-collateralized credit. To this end, we use the information on the composition of firms' loans which indicates whether these loans require collateral or not to be posted by a firm to receive financing from a bank. We therefore run the same regression as (1) but construct the dependent variable at the firm×credit type×quarter level. Similarly, we can run the same regressions on the bidders-only sample.

Table 4 presents the main results, where c denotes the additional collateral/non-collateral dimension that we exploit in the data. First, looking at the larger sample of firms, we see that a procurement contract is not significantly correlated with the growth rate of collateralized credit in column (1). However, when turning to column (2) we see a positive and significant association with a firm obtaining a procurement contract and non-collateralized credit growth.

We next run regressions on the bidders-only sample but split the estimation between collateralized and non-collateralized credit growth. Results in columns (3) and (4) mimic the findings for the larger sample of firms. That is, a firm winning a contract experiences significantly larger growth in non-collateralized loans relative to losing firms, but there is no differential for collateralized loan growth. Finally, regressions for the second vs. the rest samples in columns (5) and (6) do not yield any significant estimates. Overall, these findings point to the growth rate in overall credit associated with obtaining a procurement contract observed in Table 2 being driven by the growth in loans that do not require tangible-assets backing.

Fact 4. Differential impact of earnings from procurement on firms' credit

Our previous findings are consistent with the idea that higher earnings help ease financial con-

Table 4. Composition of Credit Growth and Procurement

	All firms		Bidders only			
			First		Second	
	Collat.	NoCollat.	Collat.	NoCollat.	Collat.	NoCollat.
	(1)	(2)	(3)	(4)	(5)	(6)
$PROC_{it}$	0.001	0.009^a	-0.011	0.080^b	-0.019	-0.058
	(0.002)	(0.003)	(0.029)	(0.031)	(0.044)	(0.057)
$\log(Credit_{ict-1})$	-0.214^a	-0.447^a	-0.449^a	-0.192^a	-0.461^a	-0.254^a
	(0.003)	(0.007)	(0.073)	(0.040)	(0.064)	(0.044)
Observations	130,295	130,295	2,690	8,110	1,423	3,606
R-squared	0.538	0.546	0.357	0.368	0.435	0.435
Sector×quarter FE	Yes	Yes	No	No	No	No
Firm×year FE	Yes	Yes	Yes	Yes	Yes	Yes
Quarter FE	No	No	Yes	Yes	Yes	Yes
Auction FE	No	No	Yes	Yes	Yes	Yes

Notes: This table presents results from estimating the relationship between collateralized (Collat.) and non-collateralized (NonCollat.) credit growth and procurement participation (PROC) by regression (1) with firms obtaining at least one procurement project over 2000-13 in columns (1) and (2), and with firms who participated in procurement contests over 2013-15 in columns (3), (4) and (5), (6) respectively, where the PROC dummy indicates the winning firm ('First') in columns (3)-(4) and the runner-up firm ('Second') in columns (5)-(6). All regressions use quarterly data. Standard errors are clustered at the firm level; [a] indicates significance at the 1% level, [b] at the 5% level, and [c] at the 10% level.

straints. The fourth fact that we document is that winning a procurement contract generates a differential impact on firm's credit conditions relative to selling to the private sector. Recent papers show evidence of earning based financial constraints (e.g., see Lian and Ma, 2020; Aguirre et al., 2021; Caglio et al., 2021; Drechsel, 2021; Li, 2022). These papers base their empirical specification on models where a firm's ability to borrow is not only a function of its net worth as in traditional macro-finance models (e.g., Kiyotaki and Moore, 1997), but also on its earnings. In particular, these papers consider financial constraint of the type $l_{it} \leq \varphi_a k_{it} + \varphi_y p_{it} y_{it}$, where l_{it}, k_{it}, and $p_{it}y_{it}$ are debt outstanding, capital stock, and revenues of firm i at time t, and φ_a and φ_y control the extent to which physical capital and firm revenues can be collateralized. Based on this financial constraint, and splitting total revenues into private sector sales $p_{ipt}y_{ipt}$ and public sector sales $p_{igt}y_{igt}$ we can run the following regression:

$$\Delta\left(\frac{l_{it}}{k_{it}}\right) = \alpha_{st} + \beta_1 \Delta\left(\frac{p_{it}y_{it}}{k_{it}}\right) + \beta_2 \Delta\left(\frac{p_{igt}y_{igt}}{k_{it}}\right) + \varepsilon_{it} \tag{4}$$

where we now define t at the annual level, so that Δ is the first-difference operator between year t and $t-1$, l_{it}/k_{it} is a firm's leverage, i.e., total credit divided by fixed assets; $p_{it}y_{it}/k_{it}$ is a firm's total average product of capital, measured as total earnings (value added minus wages) divided by total fixed assets; $p_{igt}y_{igt}/k_{it}$ is a firm's earnings from selling to the government divided by the firm's total stock of fixed assets.[21] We run these regressions at the annual level (because firm-level balance sheet data are only available at annual frequency) and we control for non-time varying firm fixed effects, α_i, as well as sector×year effects, α_{st}. The coefficient β_1 identifies the impact of a change in earnings on a firm's change in leverage, while β_2 identifies the differential effect between selling to the government and selling to the private sector, with $\beta_2 > 0$ indicating that the impact is larger when selling to the government.

Table 5 presents the main results for firms that receive procurement contracts in at least two consecutive years.[22] Column (1) presents results for all firms that meet this criterion and that have balance sheet information. We see that a positive increase in a firm's total revenues in relation to its capital stock is positively correlated with an increase in leverage, which is consistent with the existence of earnings-based financial constraints. However, we also find that the coefficient on the change in a firm's earnings coming from selling to the government is not significant. Therefore, when looking at all firms, we identify earnings-based constraints that are not different between private and public sector sales. However, it is important to note that equation (4) should hold with equality only for firms whose financial constraint is binding. Therefore, we next restrict our sample to young firms, which are firms more likely to be constrained in their finance. Column (2) restricts the sample to firms that are ten years or younger (the median of the age distribution), while columns (3) and (4) further cut the sample to nine or eight years and less, respectively. The coefficients on total earnings remain significant, but now the coefficients on government earnings are also positive and significant, indicating that government earnings ease financial constraints to a larger extent.

Fact 5. Dynamic effects

Finally, we investigate whether obtaining a procurement contract generates dynamic effects at the firm level. In particular, we look at two types of variables. First, we are interested in understanding whether obtaining a procurement contract boosts firms' capital accumulation. Second, we want to examine the reaction of firms' sales to the private sector and its evolution over time upon winning a procurement contract.

To conduct the analysis, we extend the structure of equation (1) and estimate local projection panel regressions (Jordà, 2005). In particular, we regress the cumulative difference of a variable x, $\Delta_h \log(x_{i,t+h}) \equiv \log(x_{i,t+h}) - \log(x_{i,t-1})$ on the regressor PROC_{it}, the firm's lagged credit at $t-1$,

[21] We do not observe the use of factors separately for what is used to deliver sales to the private vs. the government sector. To compute value added generated by selling to the government, we assume that the intermediate goods and the labor share in total expenditure is constant within the firm; i.e., it does not change depending on whether the firm sells to the private sector or the government.

[22] This is what we need in order to exploit intensive margin variation on $\Delta p_{igt} y_{igt}$.

Table 5. Change in Leverage and Procurement

	(1)	(2)	(3)	(4)
$\Delta p_{it}y_{it}/k_{it}$	0.576^a	0.425^c	0.543^b	0.419^c
	(0.065)	(0.227)	(0.257)	(0.229)
$\Delta p_{igt}y_{igt}/k_{it}$	0.280	0.682^c	0.797^c	1.047^c
	(0.199)	(0.391)	(0.478)	(0.588)
Observations	11,376	579	403	282
R-squared	0.211	0.391	0.437	0.421
Sector×year FE	Yes	Yes	Yes	Yes
Sample by age	All	≤ 10 yrs	≤ 9 yrs	≤ 8 yrs

Notes: This table presents results from estimating the relationship between the change in firm's leverage and the change in its average product of capital and change in its earnings coming from selling to the government divided by the firm's total stock of capital. Regression (4) is estimated with firms obtaining at least one procurement project over 2000-13 using annual data. Standard errors are clustered at the firm level; [a] indicates significance at the 1% level, [b] at the 5% level, and [c] at the 10% level.

firm fixed effects, and sector×year fixed effects:

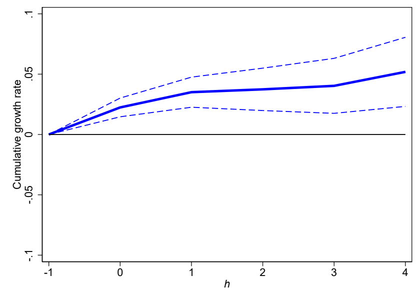
$$\Delta_h \log(x_{i,t+h}) = \alpha_{ih} + \alpha_{sth} + \beta_1^h \text{PROC}_{it} + \beta_2^h \log\ l_{it-1} + \varepsilon_{ith+h} \tag{5}$$

where $x_{i,t+h}$ is the firm-level variable of interest, such as sales or net-worth measured for firm i at time $t+h$. Therefore, $h = 0, 1, ..., H$ denotes the horizon at which the impact of procurement is estimated. Notice that when $h = 0$, equation (5) collapses to equation (1). We show the results from running these regressions at annual frequency in Figure 1, where we plot the estimated h coefficients and their associated confidence intervals.

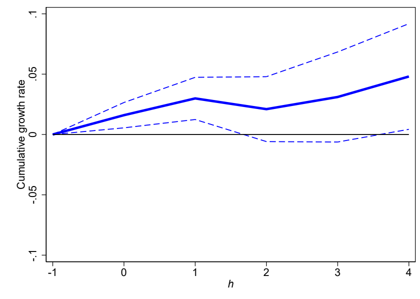
We start by looking at firms' capital accumulation. In particular, we run regression (5) for the case of x = firms' fixed assets and x = firms' net worth. The main result from these regressions is that winning a procurement contract is associated with a permanent effect on firms' capital accumulation. On impact ($h = 0$), winning a procurement contract is associated with an increase in firms' fixed assets of around 2 percentage points. We also find that this effect reinforces over time. For example, the 5 years ($h = 4$) cumulative effect is around 5 percentage points.

We next look at the evolution of firms' total sales and sales to the private sector. In particular, we run regression (5) for the case of x = firms' total sales and x = firms' sales to the private sector. Several important results emerge from running these regressions. First, as expected, we find that firms' total sales increase after winning a procurement contract. In particular, we find that firms' total sales increase by around 5 percentage points. Second, we find that this effect is persistent, which is consistent with the previous result on capital accumulation. Obtaining a procurement contract allows firms to accumulate capital faster which translates into higher output in the subsequent years. Third, we find that sales to the private sector fall right after a firm wins a procurement contract. This result points towards a crowding out effect of selling to the government to the amount

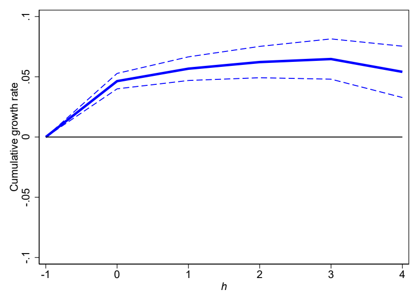
Figure 1. Dynamic effects: firms' capital accumulation



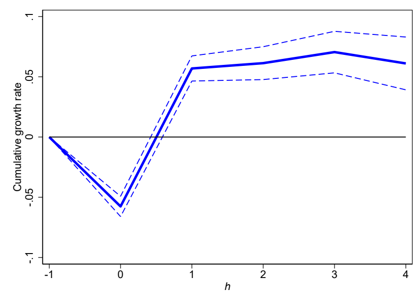
(a) Fixed assets



(b) Net worth



(c) Total sales



(d) Sales to the private sector

Notes: This figure shows the cumulative impact of the estimate of β_2^h from regression (5) for different time horizons $h = 0, 1, 2, 3, 4$. Panel (a) shows the results for the case of x being firms' fixed assets. Panel (b) shows the results for the case of x being firms' net worth. Panel (c) shows the results for the case of x being firms' total sales. Panel (d) shows the results for the case of x being firms' sales to the private sector.

of output that the firms sell to the private sector. In the model we will present below, this result will be interpreted through the fact that financial constraints will limit firms' production capacity and hence firms will be forced to decrease the amount of output sold to the private sector in order to deliver their procurement contracts. Finally, we find that this crowding out effect disappears and actually gets reversed over time. Through the lens of our model, this will be interpreted as firms becoming less financially constrained over time as the result of the extra earnings generated by the procurement contracts and the associated increases in assets.

4 The Model

We set up a model of privately held heterogeneous firms. We build on standard models of firm dynamics with collateral constraints —as Midrigan and Xu (2013), Moll (2014), or Buera and Moll (2015)— and extend this setting to allow for (a) earnings-based borrowing constraints, (b) a public sector demanding goods from private firms, (c) downward-sloping demands in both the private and public sectors, and (d) a choice to compete for procurement projects.

4.1 Technology

Time is discrete and we omit the subscript t unless it is strictly needed. The economy is populated by a continuum of size 1 of heterogeneous infinitely-lived households indexed by i. Each household is also an entrepreneur running a firm that produces a differentiated intermediate good y_i. There are two final goods in the economy: the "private sector" good, Y_p, used by households to consume, invest in productive capital, or prepare applications for procurement projects, and the "public sector" good Y_g, purchased by the government to produce (useless) public consumption.

The two final goods are assembled by two final good producers combining the differentiated intermediate goods y_i through the following CES aggregators:

$$Y_p = \left(\int_{[0,1]} y_{ip}^{\frac{\sigma_p - 1}{\sigma_p}} \, di \right)^{\frac{\sigma_p}{\sigma_p - 1}} \quad \text{and} \quad Y_g = m_g^{\frac{1}{1-\sigma_g}} \left(\int_{I_g} y_{ig}^{\frac{\sigma_g - 1}{\sigma_g}} \, di \right)^{\frac{\sigma_g}{\sigma_g - 1}} \quad \text{with } \sigma_p, \sigma_g > 1 \tag{6}$$

where I_g is the subset of goods purchased by the public sector and m_g is the measure of this set. Note that Y_g is corrected by m_g to prevent love for variety.[23] We also note that I_g (and the implied m_g) is a policy variable and the identity of firms in this set is discussed below. The final goods producers are perfectly competitive and choose the optimal demand of intermediate goods y_{ip} and y_{ig}, respectively, to maximize profits taking intermediate good prices p_{ip} and p_{ig}, final good prices P_p and P_g, and the set I_g as given. We assume that firms compete independently in each sector and face the following downward-sloping demands,

$$p_{ip} = B_p y_{ip}^{-1/\sigma_p} \tag{7}$$

$$p_{ig} = B_g y_{ig}^{-1/\sigma_g} \tag{8}$$

where for convenience we define $B_p \equiv P_p Y_p^{1/\sigma_p}$ and $B_g \equiv m_g^{-1/\sigma_g} P_g Y_g^{1/\sigma_g}$. The prices p_{ip} and p_{ig} faced by the private and public sector producers in the purchase of the same intermediate good i may differ because intermediate good i producer has monopoly power over its variety and may be selling different quantities to each market. Y_g is the demand of the public good from the government and is a policy variable in the model, while Y_p is the demand of the private good from the households and it is determined in equilibrium. The aggregate prices P_p and P_g of the private and public goods are given by the usual aggregators:

$$P_p = \left(\int_{[0,1]} p_{ip}^{1-\sigma_p} di \right)^{\frac{1}{1-\sigma_p}} \quad \text{and} \quad P_g = \left(\int_{I_g} \frac{1}{m_g} p_{ig}^{1-\sigma_g} di \right)^{\frac{1}{1-\sigma_g}} \tag{9}$$

We will be using the final private good as the numeraire, and therefore we set $P_p = 1$ in what is to follow.

[23]Governments purchase only a fraction of goods and services provided by the private economy mainly because their needs are different than the needs of private households and firms. By removing 'love-for-variety' we want to eliminate this trivial effect from the analysis of the effects of the number of contracts offered.

The intermediate inputs are produced by heterogeneous firms. At any period in time, these firms are characterized by their idiosyncratic stochastic productivity s_i, their capital stock k_i (which depreciates at rate δ), their debt level l_i (when $l_i > 0$ the firm is a net borrower), and whether they currently hold a procurement project $d_i = 1$ or not $d_i = 0$. Output is given by a simple CRS production function, $f(s_i, k_i)$ that depends on capital k_i and managerial productivity s_i:

$$f(s_i, k_i) = s_i k_i \tag{10}$$

The firm-specific s_i follows a first order Markov process, specified in more detail below. If a procurement project is active ($d_i = 1$) a fraction of output u_i, chosen by the firm, is sold to the private sector and a fraction $1 - u_i$ is sold to the public sector, otherwise all output is sold to the private sector. Our simple production function implies that u_i is also the fraction of capital used for the production of the private sector variety, that is, $k_{ip} = u_i k_i$.

4.2 Participation in public procurement

The government has control over the subset I_g of goods purchased by the public sector, and a choice of the subset I_g naturally implies its measure m_g. In order to introduce structure in this choice, we consider that the government follows a simple stochastic rule for the allocation of procurement contracts based on the quality of the proposals. In particular, we assume that firms who wish to sell to the government next period, i.e., obtain $d_{it+1} = 1$, must invest an amount of private sector good $b_{it} > 0$ today. This quantity may reflect the costs of learning how the process works, the actual costs of preparing a proposal, or the costs of establishing connections with government officials. There is always uncertainty in the outcome of the application, which reflects the fact that "equally capable" firms usually compete in the same auction with only one winner.[24] The probability $Pr(d_{it+1} = 1 \mid b_{it})$ of being able to sell to the government next period depends on the amount invested,

$$Pr(d_{it+1} = 1 \mid b_{it}) = g(b_{it}) = 1 - e^{-\eta_0 b_{it}^{\eta_1}}$$

with $\eta_0 > 0$ and $1 > \eta_1 > 0$ to ensure positive and diminishing returns. Also notice that $\lim_{b \to 0} \frac{\partial g(b)}{\partial b} \to \infty$, so there will always be an interior solution in the optimal choice of b_{it}. This probability function captures in reduced form the competition for procurement projects. As such, we think of η_0 as an equilibrium object that ensures that the fraction of firms obtaining a procurement project equals the measure m_g of goods purchased by the public sector. Hence, the probability of procurement depends on firms' own actions through b_{it} as well as on the actions of all other firms through the

[24] In practice, the final ranking of firms is decided based on a number of attributes as the price, quality, and technical requirements. Therefore, firms always face uncertainty about how the public entity awarding the contract will perceive them and their competitors fulfilling these attributes.

equilibrium object η_0.[25] The winners of the competition for procurement form the set I_g in that period.

4.3 Entry and exit

A fraction $1-\theta$ of households die every period and are replaced by the same number of new households running new firms. To avoid changing the composition of the goods produced in the economy, the entrant households produce the varieties left vacant by the exiting households. Dying households leave accidental bequests that for simplicity are taken by the government. Entrant households start with a joint distribution of financial wealth and productivity Γ_0 and with no procurement project. The wealth of the entrants is provided by the government. Alternatively, we could have assumed that all accidental bequests go the newborns, but we want to break this link in order to have the flexibility to choose the amount of financial wealth for entrants.

4.4 Preferences and constraints

Firms are owned by entrepreneurs that have preferences over consumption. Their objective is to maximize the discounted sum of utilities:

$$\mathbb{E}\left[\sum_{t=0}^{\infty}(\theta\beta)^t\,\frac{c_t^{1-\mu}-1}{1-\mu}\right] \tag{11}$$

They obtain income only from their firm so their budget constraint is given by:

$$c_{it}+b_{it}+k_{it+1}-l_{it+1}\leq p_{ipt}y_{ipt}+p_{igt}y_{igt}+(1-\delta)k_{it}-(1+r_t)l_{it}-\text{tax}_{it} \tag{12}$$

where

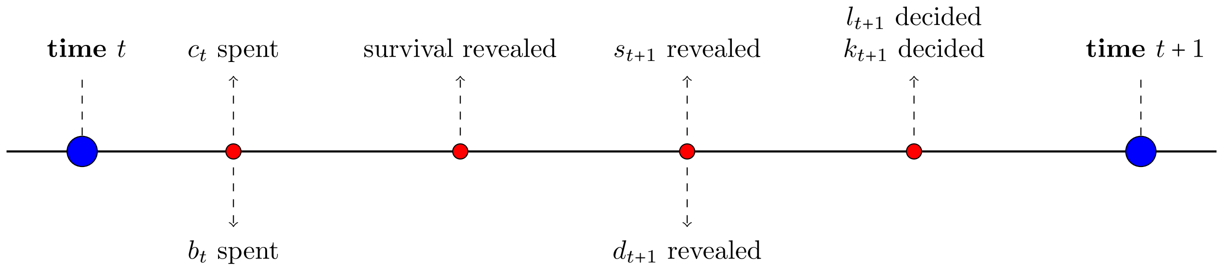
$$\text{tax}_{it}=\tau\left[p_{ipt}y_{ipt}+p_{igt}y_{igt}-(r_t+\delta)\,k_{it}\right]$$

denotes the proportional taxes on profits paid by entrepreneur i at time t. The tax function is purposely simple because we focus on revenue neutral counterfactuals. As it is standard in the literature, we only allow for one-period debt contracts l_t that pay a risk-free interest rate r_t. The amount of debt is limited by the repayment capacity of the firm through a combination of earnings-based and asset-based collateral constraint. In particular, the amount of debt of a firm coming into $t+1$ is limited by,

$$l_{it+1}\leq\varphi_a\,k_{it+1}+\varphi_p\,p_{ipt+1}y_{ipt+1}+\varphi_g\,p_{igt+1}y_{igt+1} \tag{13}$$

[25] Alternatively, we could have followed a more structural approach in modelling the competition for public contracts. For instance, in a different setting, Michelacci and Pijoan-Mas (2012) model competition for jobs with a job finding probability depending on individual human capital relative to the average human capital of the economy. Yet, our formulation is flexible and does not require taking a stand on the complex procurement competition process.

Figure 2. Timing in the Model



If $\varphi_a = 0$, $\varphi_p = 0$, and $\varphi_g = 0$ no external finance is available and all production needs to be self-financed. With $\varphi_a > 0$ the firm can lever up. With $\varphi_p > 0$ and $\varphi_g > 0$ firms can borrow against the revenues generated in the private and the public sector respectively.[26]

4.5 Timing and state space

Regarding the non-procurement part of the model, we follow the timing convention commonly used in the firm dynamics literature. First, we assume that resources devoted to consumption are spent at the beginning of each period t. Second, we assume that production in $t+1$ is carried out using capital installed at the end of period t. Third, we assume that the household survival shock and the firms' productivity in $t+1$ are revealed (in this order) before firms decide how much capital to install for next period, k_{it+1}, and how much debt to issue for next period, l_{it+1}. Regarding the variables related to procurement, we follow a similar logic. The amount of resources devoted to increase the probability of being active in procurement in $t+1$, i.e., b_{it}, is spent at the beginning of each period t. Whether or not the firm is successful and becomes active in procurement in $t+1$, i.e, $d_{it+1} = 1$, is revealed at the same time as productivity in $t+1$ and right after the survival shock. This means that procurement applications of dying households are ignored by the government and hence dying households are not awarded a procurement project that cannot be delivered. See Figure 2 for a summary of the timing assumptions.

These assumptions on timing simplify the state-space dimensionality of the problem. In particular, let $a_{it+1} \equiv k_{it+1} - l_{it+1}$ be the firm's net worth to be carried to next period in units of private good today. Then we can redefine the budget constraint as

$$c_{it} + b_{it} + a_{it+1} \leq (1-\tau)\left[p_{ipt}y_{ipt} + p_{igt}y_{igt} - (r_t + \delta)k_{it}\right] + (1+r_t)a_{it} \tag{14}$$

[26] An alternative and more structural borrowing constraint would limit repayment $(1+r_{t+1})l_{it+1}$ explicitly by a fraction of undepreciated capital $(1-\delta)k_{it+1}$ plus revenues,

$$(1+r_{t+1})l_{it+1} \leq \tilde{\varphi}_a\,(1-\delta)k_{it+1} + \tilde{\varphi}_p\,p_{ipt+1}y_{ipt+1} + \tilde{\varphi}_g\,p_{igt+1}y_{igt+1}$$

In steady state with constant r this specification would be equal to (13) with the redefinitions: $\varphi_a \equiv \frac{(1-\delta)\tilde{\varphi}_a}{1+r}$, $\varphi_p \equiv \frac{\tilde{\varphi}_p}{1+r}$, and $\varphi_g \equiv \frac{\tilde{\varphi}_g}{1+r}$. In counterfactual exercises, increases (decreases) in the equilibrium r would tighten (loosen) the borrowing constraints. Our formulation ignores this effect, but this is quantitatively second order, as seen by the results from our counterfactuals below.

The collateral constraint becomes

$$k_{it} \le \phi_a a_{it} + \phi_p p_{ipt} y_{ipt} + \phi_g p_{igt} y_{igt} \tag{15}$$

where the parameters in the borrowing constraint are re-defined as:

$$\phi_a \equiv \frac{1}{1-\varphi_a} \in [1, \infty), \quad \phi_p \equiv \frac{\varphi_p}{1-\varphi_a} \in [0, \phi_a), \quad \phi_g \equiv \frac{\varphi_g}{1-\varphi_a} \in [0, \phi_a) \tag{16}$$

Hence, the production decisions (capital and sales composition) are intratemporal, while the accumulation of net worth and the investment in procurement are intertemporal. This allows to split the firm's problem in two: a *static production* problem and a *dynamic consumption-saving* problem. Next, we describe them in turn.

4.6 The static production problem

The intratemporal production problem is characterized by firm productivity s, firm net worth a, and the availability of a procurement project d. For simplicity we drop the firm subindex i. Firms with $d = 0$ only have to choose their optimal size k subject to the borrowing constraint, while firms with $d = 1$ also decide on the fraction of output $u \in [0, 1]$ sold to the private sector. We can write the formal maximization problem for the firm of type $(s, a, d = 1)$ as,

$$\pi(s, a, 1) = \max_{k,u} \{p_p y_p + p_g y_g - (r + \delta) k\}$$

subject to:

$$p_p y_p = B_p [\quad u \quad sk]^{\frac{\sigma_p - 1}{\sigma_p}}$$

$$p_g y_g = B_g [(1-u) sk]^{\frac{\sigma_g - 1}{\sigma_g}}$$

$$k \in [0, \phi_a a + \phi_p p_p y_p + \phi_g p_g y_g], \ u \in [0, 1]$$

while for the firm of type $(s, a, d = 0)$ all the terms $p_g y_g$ trivially disappear and u becomes equal to 1. Let λ be the multiplier of the intratemporal borrowing constraint and let's consider the general case with $d = 1$. The optimal choices are described by the following FOC:

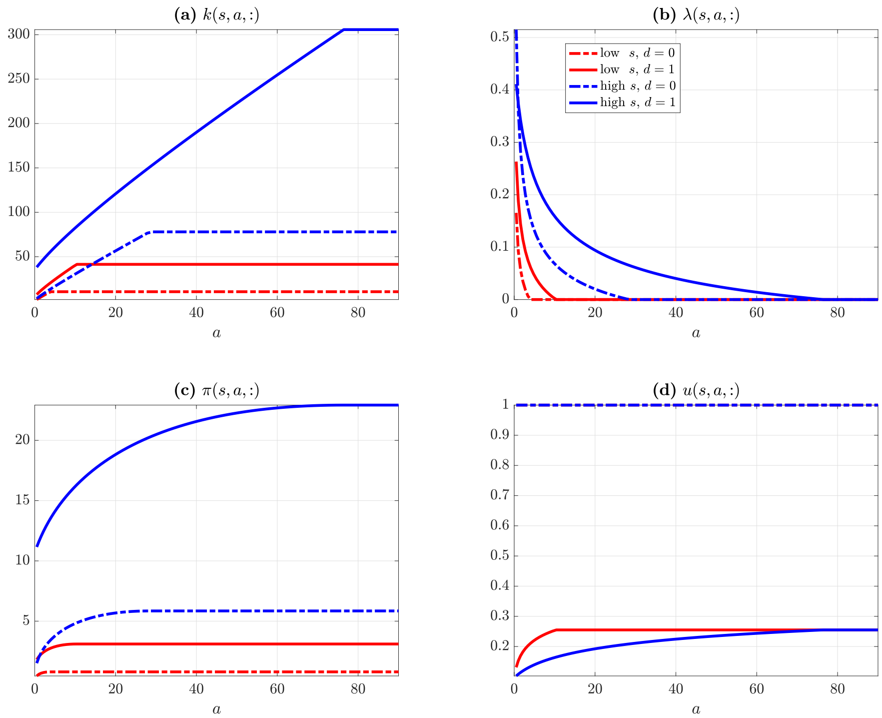
$$(1 + \lambda\phi_p) \frac{\partial p_p y_p}{\partial u} + (1 + \lambda\phi_g) \frac{\partial p_g y_g}{\partial u} = 0 \tag{17}$$

$$(1 + \lambda\phi_p) \frac{\partial p_p y_p}{\partial k} + (1 + \lambda\phi_g) \frac{\partial p_g y_g}{\partial k} = r + \delta + \lambda \tag{18}$$

$$\lambda \ge 0, \ \phi_a a + \phi_p p_p y_p + \phi_g p_g y_g - k \ge 0, \ \lambda [\phi_a a + \phi_p p_p y_p + \phi_g p_g y_g - k] = 0 \tag{19}$$

These optimality conditions show how financial frictions distort the two decisions faced by firms: production composition and firm size. Equation (17) characterizes the composition of sales. With $\lambda = 0$, the optimal choice requires to equalize the marginal revenues obtained from each sector. Because of the concave revenue functions in both sectors, there is always an interior solution to this

Figure 3. Solution of the static profit maximization problem



Notes: This figure shows the solution to the firm's problem. Panel (a) shows the size of the firm represented by the amount of capital $k(s,a,:)$; Panel (b) shows the multiplier of the financial constraint $\lambda(s,a,:)$; Panel (c) shows the profits $\pi(s,a,:)$. All of them are plotted against firm's productivity s, for two different levels of net worth, and for the cases $d=0$ and $d=1$.

problem. With binding financial constraints ($\lambda > 0$), production is shifted towards the sector whose output can be better collateralized. For instance, if procurement contracts offer better collateral value than sales to the private sector ($\phi_g > \phi_p$) the optimal choice requires lower marginal revenues from public procurement relative to the marginal revenues from the private sector, which happens when production is shifted towards the public sector and away from the private sector.

Equation (18) determines optimal firm size. With $\lambda = 0$ the optimal choice requires to equalize the marginal revenue product of capital to its cost, which is just $r + \delta$. With binding financial constraints ($\lambda > 0$), the effective cost of capital is $\frac{r+\delta+\lambda}{1+\lambda\phi_p}$ for sales to the private sector and $\frac{r+\delta+\lambda}{1+\lambda\phi_g}$ for sales to the public sector. The multiplier of the financial constraint λ has two opposite effects on the cost of capital: on the one hand it increases the cost of capital as in standard asset-based financial constraints, but on the other hand it decreases the cost of capital because a fraction of the generated output can also be collateralized. We will restrict ϕ_p and ϕ_g as indicated in Assumption 1 below to ensure that the earnings-based constraints cannot self-finance production, that is, to ensure that the financial constraints are binding for at least the entrepreneurs with zero net worth. Otherwise all firms would be unconstrained, see Lemma 2 and Proposition 2 in Appendix B. An implication of

Assumption 1 is also that the values of ϕ_p and ϕ_g are below $(r+\delta)^{-1}$. This implies that the effective costs of capital for the private and public sector, $\frac{r+\delta+\lambda}{1+\lambda\phi_p}$ and $\frac{r+\delta+\lambda}{1+\lambda\phi_p}$, are monotonically increasing in λ, which in turn means that financially constrained firms operate with less capital, see Lemma 1 and Proposition 1 in Appendix B.

Assumption 1 *The model parameters satisfy the following boundary constraints:* $\phi_p < \frac{\sigma_p-1}{\sigma_p}(r+\delta)^{-1}$ *and* $\phi_g < \frac{\sigma_g-1}{\sigma_g}(r+\delta)^{-1}$.

Here $\frac{\sigma_p-1}{\sigma_p}(r+\delta)^{-1}$ and $\frac{\sigma_g-1}{\sigma_g}(r+\delta)^{-1}$ are the capital to revenues ratios for the unconstrained problem in the private and public sector respectively.

Static policy functions. The solution of this problem yields optimal choices $k(s,a,d)$ and $u(s,a,d)$, an associated shadow value of the financial constraint $\lambda(s,a,d)$, and a profit function $\pi(s,a,d)$. In Appendix B we characterize analytically these objects for both non-procurement ($d=0$) and procurement firms ($d=1$) whenever $\sigma_g=\sigma_p$. In Figure 3, we illustrate the numerical solution for both cases with the parameterization discussed in Section 5. First, as it is common in standard models of firm dynamics with collateral constraints, constrained firms with no procurement see their capital and profits increase with net worth (while the shadow value of the borrowing constraints declines) until the point in which the financial constraints stop binding and net worth plays no role. Second, different from models with only asset-based collateral constraints, financially constrained firms without procurement increase capital and profits when productivity increases. This happens through the earnings-based constraint, which allows more productive firms to generate more revenues at the same level of net worth and hence expand production. Note also that more productive firms are more financially constrained at any level of net worth (their shadow value of the borrowing constraint is larger) because the expansion of borrowing possibilities with s is lower than the increase in the optimal size. Third, looking at firms with procurement, the fraction of output sold by constrained firms to the private sector is decreasing in productivity s and increasing in net worth a, which simply says that more financially constrained firms, conditional on participating, have a higher fraction of their capital allocated to the production of goods sold to the government. This last result is true under $\phi_g > \phi_p$ and it would be the opposite if $\phi_g < \phi_p$. Finally, note also that capital, profits, and the shadow value of the borrowing constraint for firms with procurement evolve with s and a as in the case without procurement.

A procurement shock. We can also analyze the static effect of a procurement shock by comparing the solutions of the $d=1$ and $d=0$ cases at any value of the state variables s and a. For unconstrained firms, a procurement shock leaves operations in the private sector unchanged and increases firm size (and profits) to serve the public demand. This is due to the constant returns to scale production assumption and the absence of adjustment costs. For constrained firms, a procurement shock tightens the financial constraint whenever $\phi_g \leq \phi_p$. With $\phi_g = \phi_p$ this is because the firm with $d = 1$ has two demands to serve, which are equally pledgeable, and has the same net worth to finance capital in the two different markets. As a result the firm scales down the operations in the private sector to free up collateral for the production in the public sector, which generates a negative within-firm private sector spillover of the procurement contract, that is, $k_p(s, a, 1) \equiv u(s, a, 1) k(s, a, 1) < k(s, a, 0)$. When $\phi_g < \phi_p$ the financial situation is aggravated because the public sector demand can be self-financed to a lesser extent than the private sector one and the negative private sector spillover is larger. When $\phi_g > \phi_p$, instead, public procurement may alleviate the firm financial situation because the public sector demand can be self-financed to a larger extent. This will only be relevant for firms with little or no wealth, which will be less constrained when obtaining a procurement project and will use the extra financing capacity coming from the public sector to scale up operations in the private sector. This is precisely stated in Proposition 13 in Appendix B. In our numerical exercises with $\phi_g > \phi_p$, with a realistic calibration, and endogenously accumulated net worth distributions, however, a procurement shock always increases firm size, makes the firm more constrained, and almost always generates a *negative* spillover on the private sector sales for constrained firms that obtain procurement. Finally, a procurement shock always increases profits. Among unconstrained firms, this is more so for the more productive ones because more productive firms can deliver larger projects. Among constrained firms, and for the empirically relevant case $\phi_g > \phi_p$, this is more so for the more productive and the richer firms, because these two variable determine the capacity to deliver large projects. The only exception is for the firms with little or no wealth discussed above, in which case the value of procurement actually falls with net worth, see Proposition 15 in Appendix B.

4.7 The dynamic problem

The *dynamic consumption-saving* problem can be written in recursive form,

$$V(s, a, d) = \max_{c,b,a'} \left\{ u(c) + \beta\theta \, \mathbb{E}_{s',d'|s,b} \left[V\left(s', a', d'\right) \right] \right\} \tag{20}$$

subject to

$$\mathbb{E}_{s',d'|s,b} \left[V\left(s', a', d'\right) \right] = g(b) \, \mathbb{E}_{s'|s} V\left(s', a', 1\right) + (1 - g(b)) \, \mathbb{E}_{s'|s} V\left(s', a', 0\right)$$
$$c + b + a' = (1 + r) a + (1 - \tau) \pi(s, a, d)$$
$$a' \geq 0$$

The first constraint says that the expected firm's value for tomorrow is an average of the firm's value under procurement, i.e., $d' = 1$, and no procurement, i.e., $d' = 0$, weighted by the endogenous probability of procurement $g(b)$. This is why the expectations operator $\mathbb{E}$ depends on b in addition to s.

The FOC for the choices of a' and b are:

$$u_c(c) \geq \beta\theta\,\mathbb{E}_{s',d'|s,b}\left[\left(1 + r + (1-\tau)\frac{\partial\pi(s',a',d')}{\partial a'}\right)u_c(c')\right] \tag{21}$$

$$u_c(c) = \beta\theta\frac{\partial g(b)}{\partial b}\mathbb{E}_{s'|s}\left[V(s',a',1) - V(s',a',0)\right] \tag{22}$$

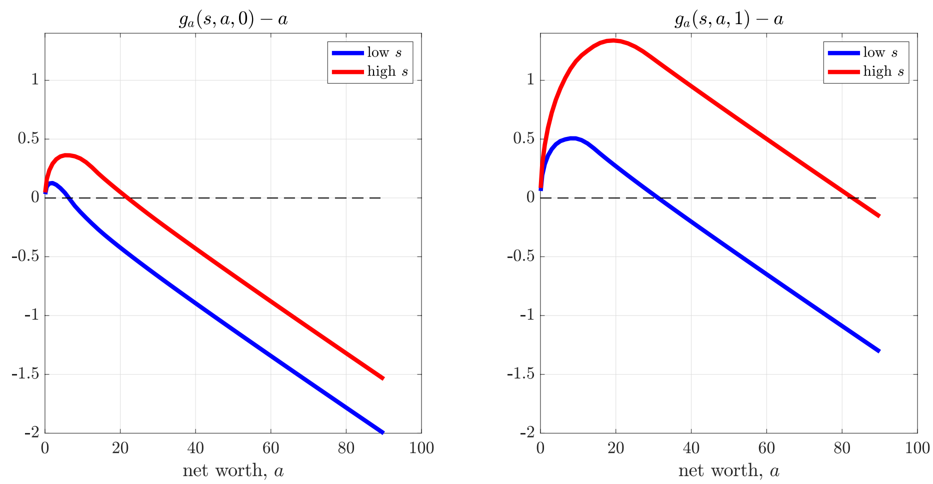
The first equation is the standard Euler equation that emerges in models of heterogeneous firms with financial constraints. If a firm is expected to be financially constrained next period in the *static profit maximization* problem, that is $\partial\pi(s,a,d)/\partial a = \phi_a\lambda(s,a,d) > 0$, then there is an extra return above r to accumulating net worth that is given by the increase in (after tax) profits due to relaxing the firm's collateral constraint, see Appendix B. The second equation determines the optimal spending in b: the entrepreneur will equalize its marginal utility of consumption to the marginal return of b, which is given by the expected increase of the firm's value coming from the possibility of selling to the government. Because of the properties of $g(b)$ and because $E_{s'|s}[V(s',a',1) - V(s',a',0)] > 0)$ the right hand side declines with b.[27]

Panel (a) of Figure 4 illustrates the net saving decision $a' - a$ of firms with and without procurement ($d = 1$ right panel, $d = 0$ left panel). At low levels of net worth there is a hump-shaped relationship between net savings and net worth that is driven by the tradeoff between smoothing consumption vs. relaxing future borrowing constraints, a feature present in similar models like Midrigan and Xu (2013). At larger levels of wealth, the saving behavior follows the logic in Aiyagari (1994): net savings decrease monotonically with net worth and there is a target level of wealth that is larger for larger productivity s. This figure also shows big differences between procurement and non-procurement firms in terms of saving decisions. In particular, procurement firms save more conditional on their current net worth a and productivity shock s. This difference is driven by the fact that profits are higher for firms that are active in procurement, which relaxes their budget constraint and hence allow them to save more without sacrificing too much consumption.

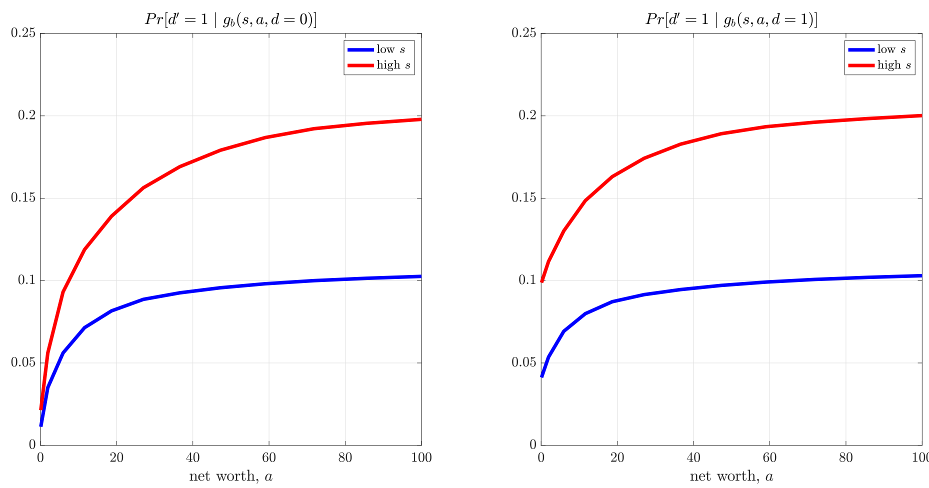
Panel (b) of Figure 4 shows the function $g(b)$ evaluated at the actual choice of b for firms with different levels of net worth and productivity, both for non-procurement (left panel) and procurement firms (right panel). The first thing to notice is that high-net worth firms invest more resources in increasing their probability of being able to sell to the government. This emerges as a result of an interesting trade off. In the dynamic problem there are two competing mechanisms to lessen borrowing constraints. On the one hand, households can accumulate wealth to relax the asset-based constraint and increase profits next period (right hand side of equation (21)). On the other hand, they can alternatively invest in applications for procurement projects that will relax

[27] Proposition 15 in Appendix B shows that $\pi(s,a,1) - \pi(s,a,0) > 0$, and $V(s,a,1) - V(s,a,0)$ inherits this property as d plays no other role than increasing profits π in the value function.

Figure 4. Decision rules



(a) net worth accumulation



(b) probability of participating in procurement

Notes: Panel (a) shows the net saving rules, i.e., $g_a(s, a, d)$), for firms of different levels of productivity and net worth, both for non-procurement (left subpanel) and procurement firms (right subpanel). Panel (b) shows the endogenous probability of obtaining procurement contracts evaluated at the optimal rules b, i.e., $g_b(s, a, d)$, for firms with different levels of productivity and net worth, both for non-procurement (left subpanel) and procurement firms (right subpanel).

the earnings-based constraint if $\phi_g > \phi_b$ and allow to increase revenues and accumulate net worth in any case (right hand side of (22)). Appendix B shows that $\frac{\partial^2 \pi(s,a,d)}{\partial a^2} < 0$, which means that the return of accumulating net worth is lower for firms with more net worth. The profit premium of a procurement project $\pi(s, a, 1) - \pi(s, a, 0)$ increases with net worth for constrained firms, see Proposition 15 in Appendix B, and so does $V(s, a, 1) - V(s, a, 0)$, which means that the return of investment in procurement is larger for firms with more net worth. This happens because selling to the government does not relax the borrowing constraints completely, which means that firms still rely on their own assets for determining the size of their procurement contracts. This reflects a "size effect": the bigger the procurement projects the firm expects to be able to deliver, the higher the expected profits that participating in procurement generates. Therefore, we obtain the result that the investment in procurement projects increases with firm net worth.

The second thing to notice is that there are almost no differences between procurement and non-procurement firms. The reason is that, conditional on b, the probability of obtaining contracts tomorrow is independent from whether the firm is active in procurement today. Procurement firms spend a bit more on b though for low levels of a, which simply reflects the fact that these firms have more available resources at hand.

4.8 Steady state equilibrium

Let $\mathbf{X} \equiv S \times A \times \{0,1\}$ be the state space of the household problem, $\mathbf{X}_1 \equiv S \times A \times \{1\}$ the subset of the state space for firms with a procurement project, $\mathcal{X}$ a σ-algebra generated by $\mathbf{X}$, and Γ a probability measure over $\mathcal{X}$. Then, given government policy parameters Y_g and m_g and a distribution of entrants Γ_0, the steady state equilibrium requires:

a) Entrepreneurs solve their optimization problem

b) The probability measure Γ is stationary

c) The market for the private good clears:

$$\int_{\mathbf{X}} p_p(s,a,d)\, u(s,a,d)\, y(s,a,d)\, d\Gamma = Y_p = \int_{\mathbf{X}} \left[b(s,a,d) + c(s,a,d) + \delta k(s,a,d)\right] d\Gamma$$

d) The market for the public good clears:

$$\int_{\mathbf{X}_1} p_g(s,a,1)\left[1 - u(s,a,1)\right] y(s,a,1)\, d\Gamma = P_g Y_g$$

e) The probability of obtaining procurement projects is consistent with the measure of goods bought by the public sector,

$$\int_{\mathbf{X}} Pr\left(d' = 1 \mid b(s,a,d)\right) d\Gamma = \int_{\mathbf{X}_1} d\Gamma = m_g$$

f) The budget constraint of the government holds

$$P_g Y_g = rD + \tau \int_{\mathbf{X}} \pi(s,a,d)\, d\Gamma + (1-\theta)\left[\int_{\mathbf{X}} a'(s,a,d) d\Gamma - \int_{\mathbf{X}} a d\Gamma_0\right]$$

g) By Walras law, the credit market clears.

$$D = \int_{\mathbf{X}} \left[k(s,a,d) - a(s,a,d)\right] d\Gamma$$

Several comments are in order. First, the parameter η_0 driving the average probability of a procurement project is an equilibrium object that ensures meeting equilibrium condition (e). It summarizes in reduced form the competition for projects. Second, the government can accumulate financial wealth D (a negative value of D would be public debt), which serves as an aggregate counterpart for the loans of entrepreneurs such that loans do not need to be in zero net supply in condition (g). Indeed, D will be a calibrated parameter to match the total amount of debt relative to capital held by firms in the data, at a targeted interest rate r. Third, condition (f) establishes that the government budget constraint in steady state is such that procurement is financed by taxes and revenues from the stationary amount of government wealth D. Plus, we assume that the real resources left after consumption by dying entrepreneurs in the form of net worth a' are collected by the government. The government then provides initial net worth to newly born entrepreneurs as dictated by the exogenously fixed distribution of entrants Γ_0. Fourth, the aggregate objects which are determined in general equilibrium and are relevant for private agents to solve their optimization problems are Y_p, r, τ, and P_g.

4.9 Two types of misallocation

Our model generates two types of misallocation. First, the presence of financial frictions generates misallocation of capital *across firms*. This is a type of misallocation that is well understood by the literature that studies the effects of financial frictions on aggregate productivity, see for instance Midrigan and Xu (2013) or Moll (2014). After some manipulations of the firm's FOC (equations (18) and (17)), defining $k_p = uk$ and $k_g = (1-u)k$, we obtain the following expressions:

$$\text{MRPK}_{ip} \equiv \frac{\partial p_p y_p}{\partial k_p} = \frac{r+\delta+\lambda}{1+\lambda\phi_p}$$
$$\text{MRPK}_{ig} \equiv \frac{\partial p_g y_g}{\partial k_g} = \frac{r+\delta+\lambda}{1+\lambda\phi_g}$$

Unconstrained firms ($\lambda = 0$) equalize their marginal products of capital to $r+\delta$ and hence operate at their optimal size. In contrast, constrained firms ($\lambda > 0$) face an effectively higher cost of capital in the private and public sectors $(r+\delta+\lambda)/(1+\lambda\phi_p)$ and $(r+\delta+\lambda)/(1+\lambda\phi_g)$ respectively and hence operate at a suboptimal scale, that is, they operate at inefficiently high MRPK_{ip} and MRPK_{ig} compared to unconstrained firms.

In an economy with binding financial frictions this has two consequences: first, the average marginal revenue products in the private and public sectors, $\overline{\text{MRPK}}_p$ and $\overline{\text{MRPK}}_g$ defined in Appendix C, will be inefficiently high and the average capital in each sector, K_p and K_g, inefficiently low. Second, because λ depends on the firm state variables, s and a, there will be heterogeneity in the MRPK_{ip} and MRPK_{ig} across constrained firms, which lowers TFP_p and TFP_g (defined in Appendix C). This type of misallocation across firms within a sector is similar to the one emphasized by Hsieh and Klenow (2009), with the key difference that in our model the same firm may produce in the two sectors at the same time, and hence the marginal products of capital are firm-sector specific.

Second, the model also generates misallocation of capital *within firms*. As it is apparent from the two equations above, unconstrained firms ($\lambda = 0$) equalize their marginal products across the two sectors. Constrained firms ($\lambda > 0$), instead, shift their production towards the procurement sector given its higher collateral value. In particular, whenever $\phi_g > \phi_p$ the marginal product of capital from selling to the private sector, MRPK_{ip}, will be inefficiently large relative to the one from selling to the government, MRPK_{ig}. This has again two consequences. First, the average marginal revenue product in the private sector, $\overline{\text{MRPK}}_p$, will be inefficiently higher than the one in the public sector, $\overline{\text{MRPK}}_g$. Second, the dispersion in λ across firms generates a larger dispersion in MRPK_{ig} than in MRPK_{ip}, which lowers TFP_g more than TFP_p. Therefore, and for both reasons, there would be efficiency gains from reallocating capital from the public to the private sector within the firm.

5 Calibration

We classify model parameters into four different blocks. The first block contains parameters related to preferences, technology, and productivity that we set to predetermined values. The second block contains parameters related to the financial constraints. The third block includes parameters related to public procurement participation and the size of public procurement in the economy. The fourth block includes the parameters governing the size of the government's debt/lending, the parameter governing firms' average productivity, and the survival probability. The model period is one year. We calibrate the parameters in blocks 2-4 such that the model matches several moments measured in the data for the year 2006.

Block #1: preferences and technology. The first subset of parameters in this block are the relative risk aversion coefficient μ, which we set equal to 2, and the CES elasticities σ_p and σ_g, which we both set to 3. We also include the discount factor parameter β, which we set to 0.94. We set the annual depreciation rate δ to 0.10. All these values are within the range of standard values used in the literature. In this block, we also include the parameters governing firms' idiosyncratic productivity. We assume that the log of a firm's productivity process s evolves over time according to an AR(1) process with Gaussian shocks and unconditional mean $\bar{s} \equiv \mathbb{E}[\log(s)]$. We set the correlation coefficient ρ to 0.80 and the standard deviation of the AR(1) innovations σ_s to 0.30, as estimated by Ruiz-García (2020) using the same dataset of firms. We discretize the process following the Rouwenhorst method, allowing for $N_s = 5$ different states.

Block #2: financial constraints. Our model contains three parameters governing firms' financial constraints: ϕ_a, ϕ_p, and ϕ_g. We choose a value of ϕ_a so that the model matches the credit-to-capital ratio observed in our micro-level data, 0.55. The identification of ϕ_p and ϕ_g requires more explanation. Given the credit constraint in equation (13), and after dividing by k and taking first differences, changes in firms' leverage for constrained firms are given by:

$$\Delta\left(\frac{l_{it}}{k_{it}}\right) = \varphi_p\,\Delta\left(\frac{p_{it}y_{it}}{k_{it}}\right) + (\varphi_g - \varphi_p)\Delta\left(\frac{p_{igt}y_{igt}}{k_{it}}\right) \tag{23}$$

where l_{it}/k_{it} is the firms' leverage, i.e., total credit divided by fixed assets; $p_{it}y_{it}/k_{it}$ is the firms' total average product of capital, measured as total value added (minus wages) divided by total fixed assets; $p_{igt}y_{igt}/k_{it}$ is the firms' value added (minus wages) coming from selling to the government divided by the firm's total stock of capital.[28] Therefore, for constrained firms, the coefficients from an OLS regression directly pin down φ_p and $(\varphi_g - \varphi_p)$, which together with ϕ_a allow to recover ϕ_p and ϕ_g (see equation (16)). Notice that equation (23) maps directly into the regression equation given by (4). Because our model implies that this equation holds with equality only for financially constrained firms, we run its empirical counterpart for a restricted sample of firms that are likely to be financially constrained. Our preferred specification corresponds to column (2) of Table 5, which shows the results of running the regression using only firms that are not older than the median age in our sample, i.e., 10 years.

Block #3: participation and size of procurement. There are four parameters driving the size and participation into procurement. The parameters Y_g and m_g are policy parameters governing the relative size of procurement in the economy and the fraction of goods bought by the government. We set Y_g to match the share of procurement in GDP equal to 12.1%, which is the value we measure in the Spanish national accounts in the year 2006. We set m_g equal to the share of firms that participate in procurement. Since our sample does not cover all the procurement contracts in Spain, we do a back of the envelope calculation and recover an implied fraction of procurement firms of 3.8% (see Section 2.4). Regarding the probability function of winning a contract we proceed as follows. We calibrate the level parameter η_0 to ensure that the the fraction of firms doing procurement equals the fraction of goods bought by the government m_g, which is the equilibrium condition e). Regarding the curvature parameter η_1, we identify it by making sure that the selection pattern of firms into procurement is as in the data. We proceed as follows. In the data, we select firms with no procurement contracts between 1999 and 2005. Then, we classify as procurement firms those firms that obtain at least one contract in 2006. We define the "procurement premium"

[28] We do not observe the use of factors separately for what is used to deliver sales to the private vs. the government sector. To compute value added generated by selling to the government, we assume that the intermediate goods and the labor share in total expenditure is constant within the firm, i.e., it does not change depending on whether the firm sells to the private sector or the government.

as the difference in size (measured by value added) between procurement and no procurement firms in 2005. That is, we want the model to match the ex-ante difference in size between procurement and non-procurement firms. We measure this procurement premium to be around 72%. The intuition why the parameter η_1 affects the selection of firms into procurement is as follows. When η_1 approaches zero, the probability function $g(b)$ exhibits strong diminishing marginal returns in b: the marginal increase in probability falls quickly as firms invest more. This makes differences in b across firms inconsequential for their probability of selling to the government, and hence generates very little selection, with complete randomness in allocation when $\eta_1 = 0$. Conversely, when η_1 approaches 1, the diminishing marginal returns are small: the marginal increase in probability falls slowly with b as firms invest more. This implies that differences in b translate into big differences in the probability of participating in procurement, which generates a strong selection pattern.

Block #4: rest of the parameters. We use firms' average productivity level $\bar{s}$ to match the capital-to-output ratio observed in our firm level data. The reason why this moment is informative of the average productivity in the economy has to do with our AK assumption on firms' technology. The construction of "output" in the data requires some explanation. Following the recent literature on earnings-based constraints (Lian and Ma, 2020; Drechsel, 2021), we assume that the flow variable that firms can collateralize is EBITDA: sales net of overhead and labor costs, without subtracting investment, interest payments or taxes. Because we do not have labor in our model, that variable is equal to the firm's value added $p_{ipt}y_{ipt} + p_{igt}y_{igt}$. However, in the data, we compute the counterpart of that variable as:

$$p_{ipt}y_{ipt} + p_{igt}y_{igt} = \text{VA}_i - \text{wage bill}_i \tag{24}$$

Using this measure of output and firms' fixed-capital stock, we compute an aggregate capital-output ratio of 3.88. To discipline government's wealth D, we target an equilibrium interest rate equal to 5%. Finally, we calibrate the survival probability $\theta = 0.95$ to the firms' exit rate in Spain of 0.05.

5.1 Calibration results

Our model matches all the targeted moments. Panel A in Table 6 shows the definition of the parameters as well as their inferred values. Panel B shows the description of moments and their value in the data and in the model. With respect to financial frictions, we find $\phi_a = 2.17$, which implies a $\varphi_a = 1 - 1/\phi_a = 0.54$. Therefore, our calibration implies that firms can collateralize 54% of their capital stock. Regarding earnings-based constraints, the coefficients estimated in the data, in column (2) of Table 5 for financially constrained firms, identify the pledgeability of the earnings from the private and public sectors. These two coefficients imply values for φ_p and φ_g of 0.42 and

Table 6. Calibration

Panel A: parameters		(1) Baseline	(2) $\phi_p = \phi_g$	Panel B: Moments	
Block 1					
μ	CRRA coefficient	2.00	2.00		
σ_p	CES private sector	3.00	3.00		
σ_p	CES government	3.00	3.00	PREDETERMINED	
β	Discount factor	0.94	0.94		
δ	Depreciation rate	0.10	0.10		
ρ	AR(1) correlation	0.80	0.80		
σ_s	AR(1) variance	0.30	0.30		
Block 2					Data = Model
ϕ_a	borrowing const. (a)	2.17	2.34	Credit/K	0.55
ϕ_p	borrowing const. $(p_p y_p)$	0.92	0.99	reg. coefficient (φ_p)	0.42
ϕ_g	borrowing const. $(p_g y_g)$	2.40	0.99	reg. coefficient $(\varphi_g - \varphi_p)$	0.68
Block 3					
η_0	probability function (level)	0.21	0.21	Consistency of $g(b)$ with m_g	–
η_1	probability function (slope)	0.53	0.55	Procurement premium	0.72
Y_g	demand shifter	0.83	0.63	Share of procurement in GDP	0.12
m_g	measure of procurement goods	0.038	0.038	Percentage of procurement firms	3.8%
Block 4					
D	Government lending	0.86	0.84	Interest rate	5%
$\bar{s}$	Productivity shifter	-6.51	-6.53	K/Y (aggregate)	3.88
θ	Survival probability	0.95	0.95	Exit rate	5%

Notes: This table summarizes our baseline calibration. All moments, with the exception of the regression coefficients, have been computed for the year 2006. Government lending D is expressed as a fraction of total credit in the model economy. In column (1), we show the parameter values in our baseline calibration. In column (2), we show the parameter values in our alternative calibration where we set $\phi_p = \phi_g$ (see Section 7.4) for details. Notice that we do not report data and model's moments separately because the model matches the data moments perfectly in the two calibrations.

1.10 respectively. Together with $\varphi_a = 0.54$, these numbers translate into $\phi_p = 0.92$ and $\phi_g = 2.40$.[29] Hence, we find that firms can pledge 42% of the annual earnings from selling to the private sector and 110% of their annual earnings from selling to the government. In terms of how that translates into firm size, we find that firms can increase their capital by 92% of their annual earnings in the private sector and by 240% of their annual earnings from selling to the government. These two last numbers are the result of a multiplier effect: firms can borrow against their revenues, allowing them to buy more capital, which can be partly collateralized to obtain further credit. This is an important interaction: how earnings-based constraints affect a firm ability to grow also depends on

[29] We note that both ϕ_p and ϕ_g satisfy Assumption 1, which means that capital cannot be self-financed through the earnings-based constraints, see Lemma 2 in Appendix B. This is true despite $\varphi_g > 1$ because the optimal unconstrained capital to output ratio in procurement is $\frac{\sigma_g - 1}{\sigma_g}(r+\delta)^{-1} = 4.44$, which means that ϕ_g should equal 4.44 and φ_g should equal 2.04 for procurement to be self-financed.

the value of φ_a.

Regarding the probability function of winning procurement contracts, we find the level η_0 to be equal to 0.21 and the slope η_1 to be equal to 0.53. To match the aggregate capital-output ratio of 3.88 the model needs an average log productivity $\bar{s} = -6.51$. Finally, the model needs a high level of government lending to match an interest rate r of 5%. In particular, the amount of government lending represents around 86% of the total amount of credit in the economy.

6 The Benchmark Economy

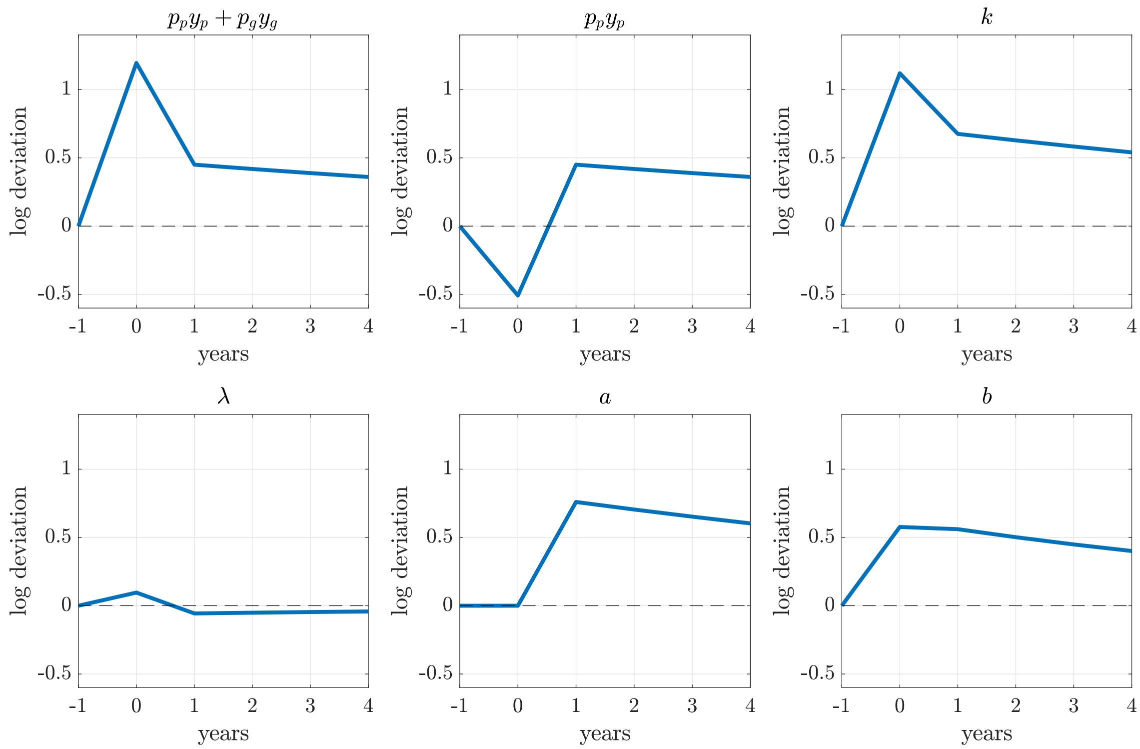
In this Section we describe three dimensions of our benchmark economy: the selection pattern of firms into procurement, the treatment effect of a procurement shock on firm dynamics; and the macroeconomic consequences of procurement.

Selection. Our benchmark economy generates a strong pattern of selection of firms into procurement based on both productivity s and net worth a. As discussed in Section 4.6, the value of procurement, $V(s, a, d = 1) - V(s, a, d = 0)$, is increasing in firms' ability to deliver large projects. For unconstrained firms, this ability is determined only by productivity s, but for constrained firms, it also depends on net worth a. To quantify the strength of selection on s and a, we calculate the implied "procurement premium" for each of these variables. In particular, in Section 5 we have calibrated our model economy to match a "procurement premium" of 72% in value added by measuring the relative size of procurement vs. non-procurement firms before they obtain a procurement project. When we compute the "procurement premium" for s and a we find that procurement firms are ex-ante 36% more productive and hold ex ante 53% more net wealth. That is, richer and more productive firms self-select into procurement.

Treatment. We next describe the treatment effects of procurement on firm dynamics by looking at the firm-level impulse-response functions to a procurement shock. In particular, in Figure 5, we report the evolution of some key variables for a firm that is financially constrained at time -1 (a firm with median net worth and median productivity), has no procurement at time -1, and obtains a procurement contract in year 0. All variables are reported relative to their counterparts (measured in log deviations) under the scenario in which the firm does not get any procurement contract.

These impulse-response functions are in line with the local projections estimated in Section 3. First, we find that the firm's total sales and capital increase on impact. Instead, net worth increases one year after the shock because it is predetermined at time 0. As in the data, the procurement shock generates a permanent positive effect on these three variables. Second, on impact, procurement generates a crowding out of a firm's sales to the private sector, and a crowding in during the subsequent years. As discussed in Section 4.6, a constrained firm has to split resources between the

Figure 5. Impulse responses from the model



Notes: The left panel of this figure shows the impulse responses of some key variables to a procurement shock in period 1. We show the results for a firm with productivity s and net worth a equal to the median values in the baseline calibration.

two sectors, despite the extra credit generated by selling to the government, which explains the fall in sales to the private sector. In fact, the firm becomes more constrained, i.e., higher λ, on impact as a result of the increase in demand. However, the new profits generated from procurement allow the firm to accumulate more net worth over time. This higher level of net worth will ease the firm's financial constraint and hence allow it to increase output in the private sector in the subsequent periods. Finally, we also find that the procurement contract has a permanent positive effect on b, which is the result of an increase in the firm's cash on hand. This is a channel through which the model endogenously generates persistence in firms' participation in procurement. To have a sense of how strong this channel is as compared to the data, we compute the probability of obtaining a contract over the next three years, i.e., $t+1$, $t+2$, or $t+3$, conditional on a firm having a contract in t. In the data, this number is quite high, 75%, which compares to its model counterpart of 16%. That is, our model generates around 1/5 of the persistence observed in the data.

The macroeconomy. We report the most relevant aggregate numbers of the benchmark economy in column (1) of Table 7. The main aggregate results from this economy are as follows.

We find significant differences in TFP across the two sectors. In particular, our model implies that TFP in the procurement sector is 21% higher than its counterpart in the private sector (0.308 vs. 0.255), see Appendix C for derivations. This difference is mainly due to selection on s. To see why, note that absent financial frictions but keeping firm selection into procurement, the *first-best* level of TFP in the private sector would be 19% higher than its equivalent in the public sector. Although our model predicts modest levels of misallocation in both sectors, it does predict a significantly higher level in the private one: 4.7% vs. 3.3%. There are two reasons for the difference of misallocation between the two sectors. The first reason is a consequence of ϕ_g being larger than ϕ_p, which makes financial frictions less severe for firms producing in the procurement sector. The second reason has to do with the fact that, as explained above, firms with higher net worth self select into procurement. These two reasons together imply a variance in the log MRPK across firms in the procurement sector which is around 18% lower than that in the private sector: 0.023 vs. 0.026.

We next analyze the price of public goods relative to private goods, which will be important to understand some of the results from the policy counterfactuals. The price of public goods relative to private goods can be written as:

$$\frac{P_g}{P_p} = \frac{\overline{\text{MRPK}}_g}{\overline{\text{MRPK}}_p} \frac{\text{TFP}_p}{\text{TFP}_g} \tag{25}$$

where $\overline{\text{MRPK}}_g$, $\overline{\text{MRPK}}_p$, TFP_g, and TFP_p are the weighted average marginal revenue products and sectoral TFP's (see Appendix C for derivations). As in standard multi-sector models, the ratio of relative prices is inversely related to sectorial TFPs. This expression also implies that the relative price is positively related to the ratio of average marginal revenue products in each sector. That is, a relatively high sectorial "wedge" will be associated with a higher relative price. Because firms active in procurement are on average more financially constrained, i.e., have a higher λ, the average wedge in the procurement sector is higher. In particular, in our benchmark economy, $\overline{\text{MRPK}}_g$ is around 8% higher than $\overline{\text{MRPK}}_p$. However, as mentioned above, TFP_g is 21% higher than TFP_p. This higher TFP in the procurement sector more than compensates its higher wedge, implying a relative price of public goods, $\frac{P_g}{P_p}$, of 0.899.

7 Policy Experiments

Our empirical evidence in Section 3 and model results in Section 6 show that procurement contracts help firms grow out of their financial constraints. At the same time, in Section 5 we have seen that smaller firms, typically the most constrained, do not participate in procurement. This suggest that making procurement contracts available to smaller firms may lead to aggregate output gains. For this reason, in this Section we quantify the aggregate effects of reforming the public procurement allocation system through *expenditure-neutral* changes that favor small firms.

7.1 Counterfactual 1: Decreasing contract size

We first run an experiment that consists of reducing the size of contracts to reach out to more firms, while keeping the same level of expenditure P_gY_g. This experiment is motivated by the fact that decreasing the size of procurement contracts as a tool to promote the participation of small firms is at the core of the European Commission's agenda for public procurement regulation.

In practice, we solve for a counterfactual economy in which the fraction of firms from which the government buys, m_g, increases by 10 percentage points, i.e., from 3.8% to 13.8%.[30] We do so by increasing η_0 and adjusting Y_g so that P_gY_g remains unchanged.

We present the main results from this exercise in column (2) of Table 7, which shows the relative change of some relevant variables compared to their counterparts in the benchmark economy.[31] We use aggregate GDP in private good units as the main measure to assess the macroeconomic impact of the policy. We report two different measures of GDP: "nominal" GDP, which uses the relative price of procurement P_g in the reformed economy, and "real" GDP, which keeps the price P_g of the benchmark economy. We also report changes in the levels of capital in the two sectors as well as in the aggregate, together with changes in variables related to misallocation and TFP.

Our model predicts a significant fall in GDP as a result of the policy reform, no matter how we measure it. In the case of "nominal" GDP, we find a reduction of 2.68%. Since we are keeping P_gY_g constant in our experiment, this reduction comes entirely from a 3.05% reduction in Y_p. We find a reduction in "real" GDP, 2.19%, which is around 20% lower than that in "nominal" GDP. The reason is that the relative price of procurement P_g declines as a result of the policy experiment. This is an important result: despite the fall in GDP, the government is able to provide a higher amount of public goods without increasing its expenditures.

As explained above, this relative price depends on the ratio of sectoral wedges times the inverse of relative TFPs. As a consequence of the reform, TFP in the private sector slightly increases and its counterpart in the procurement sector decreases (see below for details). This increase of around 2.7% in the relative TFP of the private sector pushes P_g up. However, the ratio $\overline{\text{MRPK}}_g/\overline{\text{MRPK}}_p$ decreases by around 6.3%, which is mostly explained by a reduction in $\overline{\text{MRPK}}_g$. The intuition for this result is related to the reduction of the average size of procurement contracts, which makes procurement firms less financially constrained on average (compared to the benchmark). In particular, the ratio of λ's for procurement firms vs. non-procurement firms decreases by around

[30] This represents a large change in the average size of contracts. In the counterfactual economy, the average size of the contract is 27% of that in the benchmark economy. The European Commission is not explicit about by how much governments should decrease the size of the contracts: "[...] Such division could be done on a quantitative basis, making the size of the individual contracts better correspond to the capacity of SMEs [...]." (see the Public Sector Directive 2014/24/EU for details.)

[31] We report the difference not the relative change for variables that are already in shares, i.e., the percentage of procurement firms and the share of procurement in GDP, as well as for the interest rate r, the tax τ, and the parameters η_0 and η_1.

36% after the policy change.

We now decompose the reduction in "nominal" GDP into that coming from capital accumulation vs. TFP. We find that most of the reduction in GDP (around 85%) is accounted for by a fall in the aggregate stock of capital K, which decreases by 2.26%. In Section 7.3 below, we will provide more details that will help understand this reduction in the stock of capital. The rest is explained by a reduction of 0.43% in aggregate "nominal" TFP, which is the result of a slight increase in TFP_p (0.34%), a reduction in TFP_g (2.31%), and the above-mentioned reduction in P_g. When keeping constant P_g to its value in the benchmark economy, our model predicts a slight increase in TFP (0.07%).

We next explain in more detail the changes in TFP_p and TFP_g. The increase in TFP_p is the result of the long-run crowding in of procurement to the private sector: in the new steady state, firms that had a relatively high MRPK_p in the benchmark economy are more likely to be active in procurement, which allows them to accumulate more assets and hence operate with a higher level of capital in the private sector (see Section 6). This reallocation of procurement contracts towards relatively high MRPK_p firms implies a reduction in the dispersion of MRPKs in the private sector and hence in misallocation. The decrease in TFP_g is explained by a change in the composition of procurement firms that directly decreases its "first-best" level. In particular, firms' average productivity decreases by around 3%. This fall in the "first-best" level of productivity more than compensates the reduction in misallocation in the procurement sector.

7.2 Counterfactual 2: Targeting the selection pattern

We perform a second counterfactual in which we aim to change the selection pattern of firms into procurement without changing the size of contracts and the share of procurement firms. To do so, we reduce the parameter η_1 and hence solve for a new economy in which the procurement system gives relatively lower weight to firms' investment in b, making it easier for small firms to participate. We also change η_0 and Y_g so that the fraction of procurement firms m_g and total government expenditure $P_g Y_g$ remain unchanged. Policies that map into the reduction of η_1 would be those that facilitate access to the competition for procurement contracts —like better publicity or direct assistance to prepare the process— or provide more transparency of the whole process— which should diminish the importance of political connections.

We show the results from running this policy experiment in column 3 of Table 7. In contrast to the previous counterfactual, we find that the reform increases nominal GDP by around 2.07%. Out of this increase, around 28% is explained by an increase in TFP and the rest is explained by an increase in capital accumulation. As in the previous counterfactual, we will explain the behavior of capital accumulation below (see Section 7.3).

In contrast to the first counterfactual, the model predicts an increase in P_g, which explains why

Table 7. Counterfactuals

	Panel A: $\phi_g > \phi_p$			Panel B: $\phi_g = \phi_p$		
	(1) Benchmark	(2) Count 1	(3) Count 2	(4) Benchmark	(5) Count 1	(6) Count 2
Output						
Y_p	5.462	-3.05%	2.36%	5.365	-8.00%	-0.04%
Y_g	0.835	4.03%	-7.42%	0.636	18.78%	-12.19%
GDP	6.214	-2.68%	2.07%	6.092	-7.04%	0.38%
real GDP	6.214	-2.19%	1.18%	6.092	-4.80%	-1.07%
Capital						
K_p	21.385	-3.37%	2.34%	21.422	-8.16%	0.61%
K_g	2.710	6.50%	-1.81%	2.230	19.63%	-5.70%
$K_p + K_g$	24.094	-2.26%	1.88%	23.653	-5.54%	0.01%
Productivity						
TFP_p	0.255	0.34%	0.02%	0.250	0.17%	-0.18%
TFP_g	0.308	-2.31%	-5.72%	0.285	-0.71%	-7.70%
TFP	0.258	-0.43%	0.59%	0.254	0.17%	1.00%
real TFP	0.258	0.07%	-0.69%	0.260	0.78%	-1.08%
$\overline{\text{MRPK}}_p$	0.256	0.32%	-0.01%	0.251	0.17%	-0.21%
$\overline{\text{MRPK}}_g$	0.277	-6.09%	1.84%	0.327	-16.28%	6.27%
TFP_p gain	0.047	-7.39%	-0.37%	0.042	-4.71%	4.14%
TFP_g gain	0.033	-21.45%	2.10%	0.090	-36.60%	9.49%
Prices/tax						
P_g/P_p	0.899	-3.87%	8.01%	1.144	-15.78%	13.80%
r	0.050	0.001	0.000	0.049	-0.001	0.000
τ	-0.070	0.005	-0.008	-0.071	0.026	-0.002
Procurement						
% firms	0.038	0.100	0.000	0.038	0.100	0.000
Share GDP	0.121	0.003	-0.002	0.121	0.008	-0.002
η_0	0.209	0.555	-0.047	0.213	0.602	-0.057
η_1	0.527	0.000	-0.110	0.550	0.000	-0.128
ratio mean s	1.246	-0.96%	-6.47%	1.259	-2.60%	-7.97%
ratio mean a	1.727	-17.47%	-13.90%	1.902	-20.75%	-16.14%
ratio mean lamb	2.973	-36.72%	8.74%	6.221	-56.68%	14.79%

Notes: Panel A of shows the results from running the two policy experiments under our baseline calibration, i.e., $\phi_g > \phi_p$. Panel B shows the results from running the experiments for the alternative calibration in which we impose that $\phi_g = \phi_p$. Columns (1) and (4) show the variables from the respective benchmark economies. Columns (2) and (5) show the results from running counterfactual 1, which consists in increasing η_0 so that the model generates a % of procurement firms of 13.8% and decreasing the average size of contracts accordingly so that $P_g Y_g$ remains constant, while keeping η_1 constant. Columns (3) and (6) show the results from running counterfactual 2, which consists in changing η_0 and η_1 so that the procurement premium decreases by 50% (while keeping the % of procurement firms equal to 3.8%) and changing the average size of contracts accordingly so that $P_g Y_g$ remains constant.

Table 8. Channels

	Panel A: Count. 1			
	(0)	(1)	(2)	(3)
	Benchmark	Step 1	Step 2	Full
Y_p	5.462	-0.93%	5.06%	-3.05%
K	24.094	0.21%	5.22%	-2.26%
TFP	0.258	-1.01%	-0.73%	-0.43%
GDP	6.214	-0.81%	4.45%	-2.68%
r	0.050	0.000	0.000	0.001
	Panel B: Count. 2			
	Benchmark	Step 1	Step 2	Full
Y_p	5.462	-0.12%	2.50%	2.36%
K	24.094	-0.14%	2.09%	1.88%
TFP	0.258	0.05%	0.10%	0.59%
GDP	6.214	-0.10%	2.19%	2.07%
r	0.050	0.000	0.000	0.000

Notes: This table shows the results from running different versions of our model. Columns (0) and (4) show the valuex of the variables both in our benchmark economy and in the new steady state. Column (1) refers to the "Short-run partial equilibrium effect." Column (2) refers to the "Long-run partial equilibrium effect." Importantly, in columns (1) and (2), we solve the model by using the η_0 and η_1 that we use to compute the new steady states, and adjust Y_g so that $P_g Y_g$ remains unchanged.

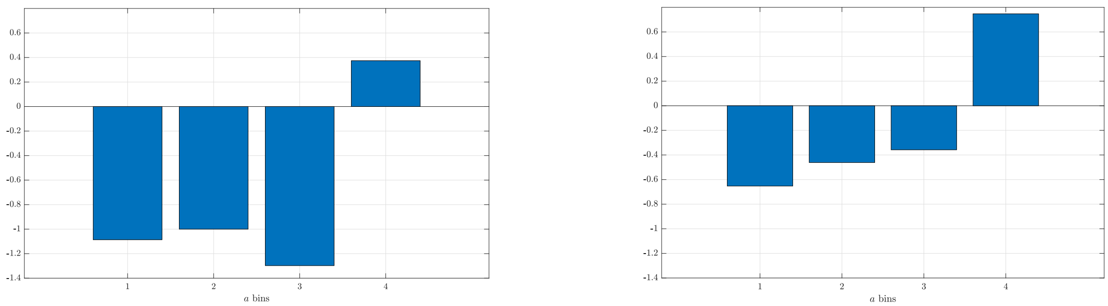
the difference in GDP growth between the two counterfactuals is particularly high when computing it using nominal GDP. The reason why P_g increases in this counterfactual is two-fold. First, the average wedge in the procurement sector $\overline{\text{MRPK}}_g$ increases by 1.84%. The reason is that the policy allocates procurement contracts to relatively smaller firms without reducing the average contract size, which was the main factor driving a fall in $\overline{\text{MRPK}}_g$ in the previous counterfactual. As a result, procurement firms become more constrained on average in the new steady state— with an increase in the ratio of λs for procurement vs. non-procurement firms of 8.74%.

This different behavior in the relative price explains why nominal TFP decreases in the previous counterfactual and increases in the current one. In fact, sectoral TFPs evolve qualitatively similarly in the two counterfactuals: TFP_p increases and TFP_g decreases. As in the previous counterfactual, the reason for the increase in TFP_p is a reduction of misallocation due to the dynamic crowding in. Also similarly to the previous counterfactual, the weaker selection pattern in s is behind the fall in TFP_g.

7.3 Channels

The main factor driving changes in aggregate GDP between the two steady states is capital accumulation. In the first counterfactual, a lower K explains around 85% of the fall in GDP. In the

Figure 6. Static spillover effects



(a) % in $p_p y_p$ by bins of a (counterfactual 1) **(b)** % in $p_p y_p$ by bins of a (counterfactual 2)

Notes: This figure shows the relative change (in %) in total $p_p y_p$ across equally-sized (in terms of total $p_p y_p$) groups of firms split according to the distribution of a as a result of a change in the procurement allocation system (step 1). Panel A and B refer to the first and second counterfactual exercises respectively.

second counterfactual, a higher K explains around 72% of the rise in GDP. In this section, we provide more details about the specific mechanisms driving changes in K, as well as some other interesting effects that our model generates.

To that end, we solve different versions of the economy that aim to isolate the different channels at play. In Table 8, we report the main aggregate variables across these different economies for the counterfactual 1 (Panel A) and 2 (Panel B). In columns (0) and (4), for completeness, we report again the value of the variables both in our benchmark economy and in the new steady state. We refer to this last column as "Full," capturing the idea that it contains all the different mechanisms we want to isolate in the previous columns. Importantly, in steps 1 and 2, we solve the model by using the η_0 and η_1 that we use to compute the new steady states, and adjust Y_g so that $P_g Y_g$ remains unchanged.

Short-run partial equilibrium effect. As we discussed in previous sections, a procurement shock ($d = 0 \rightarrow d = 1$) makes constrained firms decrease their output sold in the private sector on impact. This within-firm spillover manifests with the opposite sign when a firm becomes inactive in procurement ($d = 1 \rightarrow d = 0$). Our policy reforms reallocate procurement contracts across firms, and hence generate crowding out effects for some firms (constrained firms that start selling to the government), crowding in effects for others (constrained firms that stop selling or sell smaller contracts to the government), and no change for the rest (unconstrained firms that either obtain or lose a procurement contract).

In column (1) of Table 8, we isolate the aggregate effects of this crowding out/in effects in the short run. To do that, we solve the static firm's problem using the parameters that characterize the new procurement allocation system, i.e., the new η_0 and η_1, without taking any general equilibrium or dynamic effects into account. Not taking any general equilibrium effects into account means that we do not iterate over Y_p, r, or τ to make sure that markets clear or the government's budget constraint is satisfied. Not taking any dynamic effects into account means that we keep the distribution of a and b unchanged.

In Figure 6, we provide some evidence on how these crowding out/in effects operate for different types of firms. In particular, we plot the relative change in total $p_p y_p$ across groups of firms with different levels of net worth a. We do so by ordering firms based on the benchmark distribution of a, splitting them in four groups so that each group accounts for 1/4 of the production of aggregate Y_p, and calculating the change in total $p_p y_p$ produced by each group, as caused by the procurement reform at impact. We find that the crowding out effect dominates within the first, second, and third bins of the distribution of a, and that the opposite is true for the fourth bin. This result reflects the fact that our policy reforms consists of reallocating procurement contracts from relatively big to relatively small firms. That is, firms with relatively lower a are more likely to be "new procurement firms" as a result of the policy change, whereas for firms with relatively higher a, procurement contracts are now a lot smaller. Because firms with lower a are also more constrained on average, we also find that the crowding out effects tend to be stronger than the crowding in ones. As predicted by our theory, no crowding out or in operates for unconstrained firms. Because the fraction of unconstrained firms is the highest in the fourth bin, the extent of the crowding effect within that bin tends to be lower. Overall, we find that the policy reform generates a fall in the private sector aggregate output Y_p and hence GDP in step 1 of the two counterfactuals. In particular if prices and the distributions of a and b were fixed, the policy reform would generate a fall in GDP of 0.81% in the first counterfactual of 0.10% in the second one.

Long-run partial equilibrium effect. As shown in the previous sections, financially constrained firms that get procurement projects increase their revenues and can accumulate net worth at a faster pace and hence increase their private sector activity in the long run. In step 2, we quantify the aggregate effects of this strengthening of the self-financing mechanism. To do that, we solve for a new steady state distribution of a and s of our model under the new η_0 and η_1, but imposing that the policy functions of the dynamic problem $c(s,a,d)$, $a'(s,a,d)$ and $b(s,a,d)$, as a ratio of the entrepreneurs' cash on hand $(1+r)a + (1-\tau)\pi(s,a,d)$, remain unchanged. Our goal is to isolate the mechanical dynamic effect that procurement generates on directly affected firms, without taking into account adjustments in dynamic decisions. We also abstract from general equilibrium effects by using the same r and τ as in our benchmark economy. Column 2 of Table 8 shows a significant positive effect of this channel on the macroeconomy. In counterfactual 1, the implied capital stock and GDP are 5.22% and 4.45% higher than in the benchmark. In counterfactual 2, their counterparts are 2.09% and 2.19% higher. Hence, if we keep the policy functions of the dynamic problem and the interest rate unchanged, reforming the procurement allocation system in a way that favors small firms would generate a positive aggregate effect because it allows more constrained firms to accumulate more net worth, grow out of their financial constraints, and hence produce more in the long run.

Full effect. Finally, we study the aggregate effect of the changes in the policy functions of the dynamic problem and prices in general equilibrium. We find that the reforms reduce the incentives for big firms to accumulate assets over time, which shrinks (in counterfactual 2) or even turns negative (in counterfactual 1) the output gains associated to the reforms.

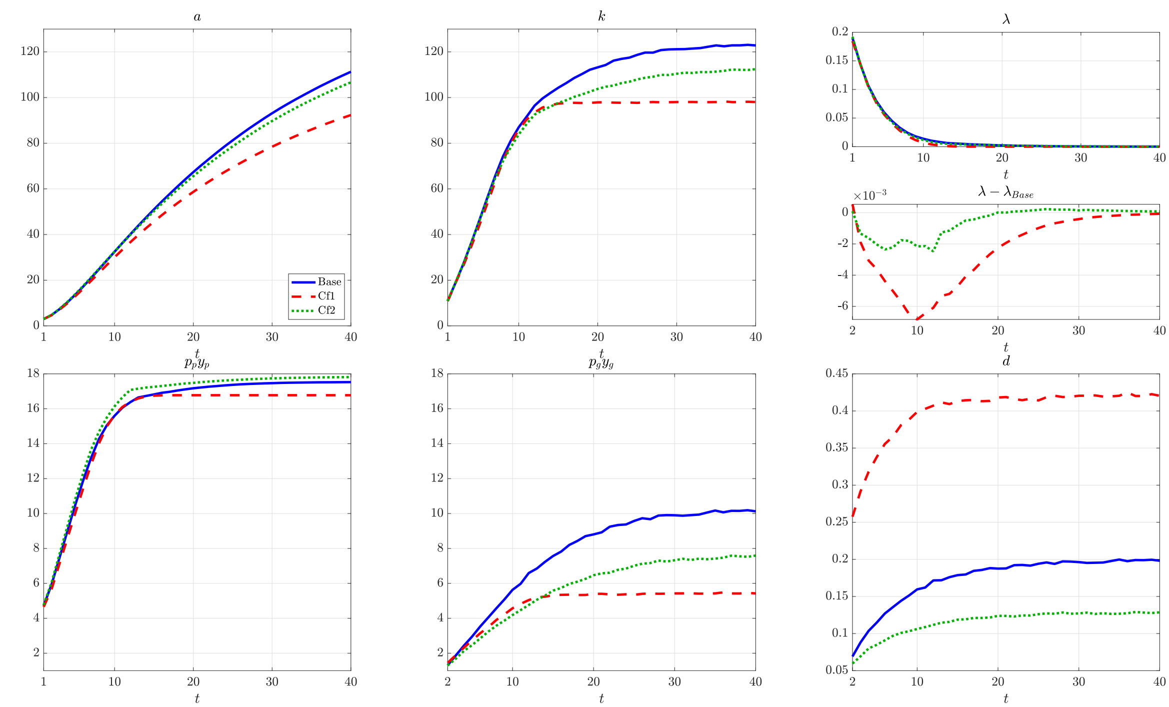
One of the main reasons why firms accumulate assets in our model is the fact that they expect to obtain a public procurement contract at some point. That is, obtaining a procurement contract is a big demand shock in response to which firms want to expand their invested capital stock, causing even relatively big firms to accumulate precautionary savings. Intuitively, productive firms want to have enough net worth so that they minimize the probability of being constrained in case the procurement shock is realized. In a context in which the average contract size is lower or in which obtaining a contract is less likely, this precautionary savings motive becomes weaker.

In Figure 7, we show how this reduction in firms' incentives manifests in our model. This figure simulates the average life cycle profile of a cohort of firms in the benchmark economy (solid blue line), in the counterfactual 1 (dashed red line), and in the counterfactual 2 (dashed green line). In particular, we take a large number of newborn firms that draw the highest productivity state in every period and stochastically obtain procurement projects according to the probability $g(b)$ and their choices of b. We focus on firms that draw the highest productivity state in every period, to capture the fact that changes in saving incentives will be specially apparent in firms that expect to operate at large scales.

We find that the three economies exhibit common patterns. As they age, firms become larger —accumulate more net worth, operate with more capital, and sell more both in the private and procurement sector— and less financially constrained. The panels for $p_g y_g$ and d show the differences in the procurement allocation systems across the three economies. The probability of participating in procurement, given by d, is the highest in counterfactual 1 (there is a higher number of contracts) and the lowest in counterfactual 2 (the number of contracts is the same but high s and high a firms are less likely to get them). In terms of the revenues from procurement, $p_g y_g$, the highest ones are under the benchmark economy, despite the fact that the probability of getting a contract is significantly higher in the counterfactual 1. This is driven by the fact that contracts are considerably bigger in the benchmark.

The most important finding from this figure has to do with the evolution of a and k over the firms' life cycle. We find that high-TFPQ firms' net worth accumulation is the highest in the

Figure 7. Firms' life cycle profiles



Notes: This figure shows the "average" life cycle profile of a large number of firms, all drawing the highest productivity level s in every period, simulated in our model under three different scenarios: the benchmark economy (blue line), the counterfactual 1 (red line), and the counterfactual 2 (blue line). This particular figure uses firms with a productivity shock which is the highest among the five productivity shocks that we use to solve our model.

benchmark economy. In counterfactual 2, these firms prefer to accumulate slightly less because, although the size of contracts is still big, it is less likely for high s firms to obtain them.[32] The big difference becomes visible in counterfactual 1, where firms' net worth accumulation is significantly lower. This also becomes apparent when looking at the evolution of k. In the counterfactual 1 economy, firms reach their optimal size at an age that is considerably earlier than in the other two economies. To provide some intuition on this result, let's go back to the Euler equation given by equation (21). The strength of the precautionary savings motive (or self financing channel) is given by the term $\frac{\partial \pi(s',a',d')}{\partial a'}$, which is equal to $\phi_a \lambda'$ (see Appendix B.2 and B.3). That is, the expected value of the financial constraint multiplier represents an extra return to asset accumulation. In other words, firms that expect to be financially constrained next period will accumulate more assets today.

The panel $\lambda - \lambda_{Base}$ in Figure 7 compares the λ's in the two counterfactual economies with the one in the benchmark economy. We find that the λ's tend to be smaller in the two counterfactuals

[32]Instead, in counterfactual 2, all firms with s lower than the highest state, accumulate *more* net worth compared to the baseline (this is not shown in the paper, but it is available upon request).

for high s firms, and particularly so in counterfactual 1. Importantly, these differences become bigger as firms approach their optimal size in the counterfactual from below, which points towards the fall in incentives to accumulate assets being particularly high for relatively bigger firms. This is driven by the fact that, in counterfactual 1 the procurement contracts are smaller so firms do not need much financial capacity to expand in order to service them, and in counterfactual 2 the old high productivity firms become less likely to win procurement contracts as these are being reallocated to firms with lower a and s.

7.4 The importance of $\phi_g > \phi_p$

In our final exercise we want to show the quantitative importance of the fact that revenues from public procurement help obtain credit to a larger extent than revenues from the private sector ($\phi_g > \phi_p$). To do so, in this section we compare the macroeconomic effects of the policy reforms in a world where $\phi_g = \phi_p$. We apply the same calibration strategy presented in Section 5, but imposing that $\varphi_p = \varphi_g$ both equal the value of $\varphi_p = 0.42$ in the baseline, and ignore the targets associated with φ_p and φ_g in the calibration. In column 2 of Table 6, we show that most of the parameters are similar to those found in our baseline calibration. Because the model has to generate the same credit-to-capital ratio as before and ϕ_g is lower by construction, ϕ_a must be higher, mechanically increasing ϕ_p.

In columns 4, 5, and 6 of Table 7, we show the benchmark economy and its associated counterfactual exercises for this new calibration. Comparing the benchmark economy in both cases we can understand the role of ϕ_g in the aggregate economy. There are three main results. First, as discussed in Section 4.9, there is no within firm misallocation when $\phi_g = \phi_p$, that is, the fraction of output sold to the private and public sectors does not depend on the financial situation of the firm. Second, whenever $\phi_g = \phi_p$ the pattern of selection into procurement in terms of net worth a is more acute. This is because it is harder to finance procurement with lower ϕ_g, and hence procurement becomes relatively more attractive for firm with more financing capacity. And third, with $\phi_g = \phi_p$ the public good become more expensive relatively to private good. This is the result of an increase in the two components of P_g/P_p. The ratio TFP_p/TFP_g increase slightly because there is more misallocation and hence lower TFP in the government sector due to firms operating in procurement being more financially constrained. The ratio $\overline{\text{MRPK}}_g/\overline{\text{MRPK}}_p$ increases mainly due to the loss of within-firm misallocation, which decreases capital in the procurement sector for all firms.

Regarding the procurement reforms, our main finding is that, if government contracts were equally pledgeable as revenues from selling to the private sector, changes in the procurement system that facilitate the presence of small firms would be associated with worse macroeconomic outcomes. In the case of reducing the average size of contracts, i.e., counterfactual 1, we find that the fall in nominal GDP would be more than twice as big as in the baseline calibration. In the case of keeping the average size of contracts but increasing the strength of diminishing returns to b, i.e., counterfactual 2, the increase in nominal GDP would be around 1.69 percentage points smaller. In fact, we find that GDP would also fall as a result of counterfactual 2 when we measure the change in GDP in real terms. The reason for these results is as follows. When $\phi_g = \phi_p$, the private sector negative spillover of procurement in the short run is larger because there is no extra financing through public revenues to alleviate the problem of scarce collateral, see Proposition 13 in Appendix B. In addition, by reducing the extent to which borrowing capacity increases when participating in procurement, the long run positive effects also weaken. Overall, procurement is less effective in helping constrained firms increase their production.

8 Conclusion

In this paper, we quantify the macroeconomic impact of changes in the public procurement allocation system. To do so, we use a comprehensive framework that builds on three steps: selection, treatment, and the interplay between procurement and the macroeconomy. We use our framework to evaluate some of the policy reforms that are at the core of the European Commission's agenda for industrial policy. In particular, we quantify the long-run macroeconomic effects of a size-dependent expenditure-neutral policy reform that consists of breaking down big projects into smaller ones, and hence reaching out to small firms in the economy.

Our results point towards the presence of long-run positive effects for directly affected firms, but also suggest the existence of important changes in big firms' dynamic behaviors that could shrink the expansionary effects or even make them negative. Our findings show that both the sign and size of these effects and hence the overall macroeconomic impact of this type of policies crucially depends on the severity and type of financial frictions in the economy. But they also depend on the type of reform, which determines how the change in procurement harms larger firms. These findings suggest that the optimal procurement allocation system in a country would depend on the specific institutional characteristics of the economy.

We view our contribution as part of a broader research agenda on the macroeconomic effects of government procurement, a policy that is surprisingly understudied. In our work, we only investigate the long run consequences of expenditure-neutral changes in the procurement allocation system. Issues like the short-term consequences of reforms, or the potential implications for the effectiveness of fiscal policy are still unexplored. We emphasize that pushing this research agenda will deliver important policy implications.

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Appendix A Details on the data

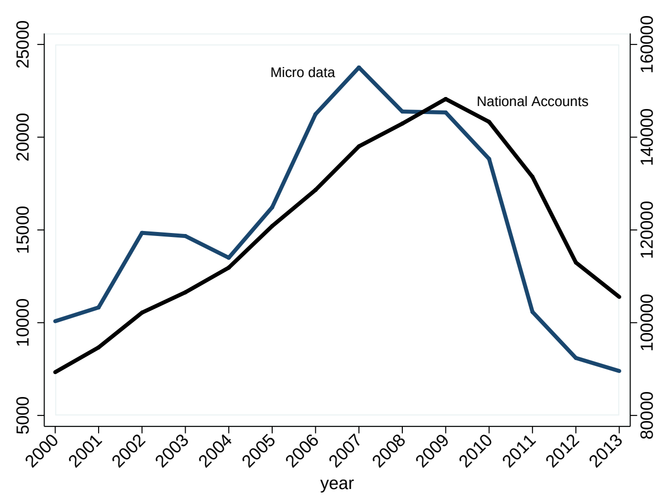
A.1 Public procurement in National Accounts.

According to the System of National Accounts (SNA), "Government consumption expenditures and gross investment", i.e., G, measures the fraction of GDP, or final expenditures, that is accounted for by the government sector. In that respect, the government is treated as a consumer/investor. In addition, the SNA treats the government as a producer that uses labor, capital, and intermediate goods to provide its own consumption and investment. The total value of this output, which equals G, is measured by the total cost incurred:

$$\begin{aligned} G &= \text{Gross Output of General Government} + \text{Gross Investment} \\ &= \text{Value Added} + \underbrace{\text{Intermediate goods and services} + \text{Structures} + \text{Equipment}}_{\text{public procurement}} + \text{IPP} \end{aligned}$$

Figure A1 shows the evolution of procurement value as measured with our micro data and compare it to the counterpart from national accounts. On average, our micro data accounts for around 13% of total government procurement as measured in Spanish national accounts. As apparent in the figure, our micro data reproduces well the cyclical aspect of pubic procurement expenditure, increasing during the boom and decreasing during the recession.

Figure A1. Evolution of Public Procurement in Spain, 2000-13



Notes: This figure shows the evolution of public procurement in Spain over 2000-13. The blue line ("Micro data", left y-axis) is computed by aggregating the individual projects scraped from the BOE, https://www.boe.es/. The black line ("National accounts", right y-axis) is measured from Spanish national accounts.

Table A1. Value of Procurement projects (budget value in millions of euro), pool of years 2000-13

Sector	mean	10th	25th	50th	75th	99th	obs.
Construction	5.28	0.13	0.23	0.74	4.00	70.84	22,549
Consulting	0.66	0.10	0.17	0.37	0.84	3.91	12,427
Services	1.22	0.11	0.20	0.42	1.05	13.47	44,581
Supplies	0.95	0.10	0.17	0.37	0.86	10.20	45,552
Others	1.99	0.09	0.15	0.35	0.99	38.18	5,524

Notes: This table presents summary statistics on the size of procurement projects in our sample as measured by the budget value. All the numbers have been computed after trimming the top 1% of the projects in terms of value, which mostly correspond to typos in the numbers, e.g., displaced comma, reported in BOE.

A.2 Types and size of procurement contracts

Table A1 provides descriptive evidence for the pool of projects and years between 2000 and 2013. In particular, we report statistics on the number of projects and distribution of projects' values, separately for the five broad sector categories reported by the BOE: construction, consulting, services, supplies, and others.

A.3 Procurement firms across different industries

In Table A2, we present summary statistics for the top 20 NACE 2-digit industries in terms of the fraction of firms in that industry that sell to the government. In column (1), we show the share of firms active in procurement in that industry. In columns (2-5), we show the share of employment, sales, fixed assets and credit accounted for by procurement firms in that industry. We provide the numbers for the year 2006, but the ranking and shares are similar for the rest of the years.

A.4 Pre-trends for winners vs. the rest

Graphically, the right panel in Figure A2 shows the average growth of credit without collateral of firms that win a procurement project in quarter 0 before and after winning the project, and compares it to the rest of firms. Again, there is a similar evolution of credit growth before procurement (parallel trends) and a clear (and persistent) divergence after that.

Appendix B Details on the static production problem

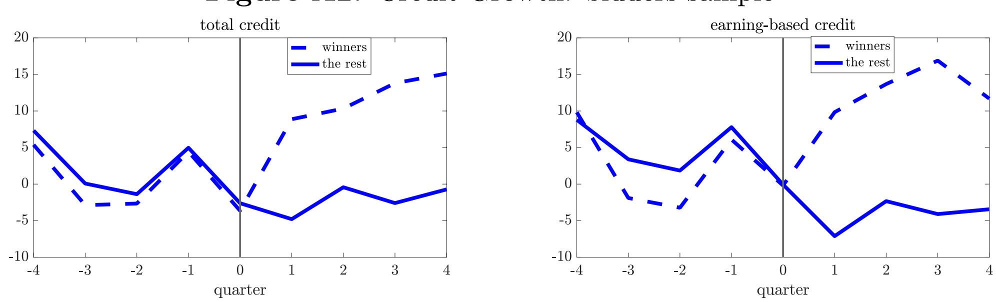
In this Appendix we characterize the solution of the static production problem. First, in Section B.1 we derive the results that serve to restrict the parameters ϕ_p and ϕ_g such that the problem is well-behaved. Then, in Section B.2 we characterize analytically the solution to the production

Table A2. Importance of procurement firms, 2006

Sector	Description	Firms (1)	Emp. (2)	Sales (3)	Assets (4)	Credit (5)
19	Manufacture of coke & refined petroleum prod.	0.150	0.332	0.315	0.310	0.243
21	Manufacturing of Pharmaceutical Products	0.149	0.240	0.225	0.231	0.288
42	Civil Engineering	0.093	0.260	0.324	0.366	0.386
80	Security and investigation activities	0.064	0.198	0.299	0.269	0.312
30	Manufacturing of Transport Equipment	0.052	0.176	0.177	0.205	0.180
94	Activities of membership organisations	0.051	0.069	0.127	0.037	0.018
36	Collection, purification and distribution of water	0.040	0.116	0.117	0.088	0.121
61	Telecommunications	0.038	0.217	0.192	0.189	0.207
51	Air transportation	0.033	0.054	0.049	0.078	0.142
81	Services of Buildings Maintenance	0.031	0.137	0.232	0.151	0.211
63	Information services	0.026	0.127	0.100	0.080	0.087
62	Programming, consultancy, other IT activities	0.025	0.151	0.193	0.157	0.214
26	Manufacturing of IT, electronic, & optical prod.	0.025	0.087	0.095	0.125	0.165
71	Technical services of architecture & engineering	0.024	0.152	0.159	0.084	0.103
2	Forestry and logging	0.019	0.069	0.068	0.033	0.080
6	Extraction of crude petroleum and natural gas	0.017	0.021	0.036	0.016	0.026
91	Libraries, archives, museums and cultural activities	0.016	0.061	0.051	0.021	0.017
29	Manufacture of motor vehicles and trailers	0.015	0.030	0.036	0.030	0.086
72	R&D activities	0.014	0.017	0.014	0.003	0.003
17	Paper industry	0.014	0.033	0.032	0.038	0.067

Notes: This table presents summary statistics on the number of firms, employment sales, and credit for the year 2006 at the 2-digit sectors. 'Firms' refers to the share of procurement firms, 'Emp.', 'Sales', 'Assets' , and 'Credit' are the share of employment, sales, assets and credit accounted for by procurement firms.

Figure A2. Credit Growth: bidders sample



Notes: These graphs plot the evolution of the average change in credit for winning vs. non-winning firms, before and after the quarter in which the auction takes place (Quarter=0). The left panel is for all credit. The right panel is for non-collateral credit only.

problem for firms without procurement ($d = 0$), which is useful to understand the interaction of asset based and earnings based financial constraints. Next, in Section B.3 we characterize analytically some of the solutions to the production problem for firms with procurement ($d = 1$) for the case $\sigma_p = \sigma_g$. Finally, in Section B.4 we show analytically the effect of a procurement shock for the case $\sigma_p = \sigma_g$, that is, the differences in allocations and profits between a firm with $(s, a, d = 1)$ and a firm with $(s, a, d = 0)$.

Before going to all these results, we start the Appendix by rewriting the FOC of the static production problem as follows. First, note that because the FOC for u, equation (17), states that the marginal revenue per unit of output sold —including its value as collateral— has to be equalized across the two sectors, and using the fact that $\frac{\partial p_p y_p}{\partial k} / \frac{\partial p_p y_p}{\partial u} = u/k$ and $\frac{\partial p_g y_g}{\partial k} / \frac{\partial p_g y_g}{\partial (1-u)} = (1-u)/k$ we can write the FOC for k, equation (18), as,

$$\frac{\partial p_p y_p}{\partial k} \frac{1}{u} = \frac{\partial p_p y_p}{\partial k_p} = \frac{r+\delta+\lambda}{1+\lambda\phi_p} \tag{B.1}$$

or as

$$\frac{\partial p_g y_g}{\partial k} \frac{1}{1-u} = \frac{\partial p_g y_g}{\partial k_g} = \frac{r+\delta+\lambda}{1+\lambda\phi_g} \tag{B.2}$$

or combining them both,

$$\frac{\partial\left[p_p y_p + p_g y_g\right]}{\partial k} = u\left(\frac{r+\delta+\lambda}{1+\lambda\phi_p}\right) + (1-u)\left(\frac{r+\delta+\lambda}{1+\lambda\phi_g}\right)$$

That is, the revenue marginal product of capital in each sector is equal to the capital cost of each sector and the revenue marginal product of capital for the whole firm is a weighted average of the capital costs in the two sectors, with the weights given but the cost shares of each sector.

It will be useful later on to use the actual revenue functions and substitute in equations (B.1) and (B.2) to obtain,

$$\left(\frac{\sigma_p - 1}{\sigma_p}\right) \frac{p_p y_p}{k} \frac{1}{u} = \frac{r+\delta+\lambda}{1+\phi_p\lambda} \tag{B.3}$$

$$\left(\frac{\sigma_g - 1}{\sigma_g}\right) \frac{p_g y_g}{k} \frac{1}{1-u} = \frac{r+\delta+\lambda}{1+\phi_g\lambda} \tag{B.4}$$

and using the production function one can write them as

$$\left(\frac{\sigma_p - 1}{\sigma_p}\right) p_p\, s = \frac{r+\delta+\lambda}{1+\lambda\phi_p} \tag{B.5}$$

$$\left(\frac{\sigma_g - 1}{\sigma_g}\right) p_g\, s = \frac{r+\delta+\lambda}{1+\lambda\phi_g} \tag{B.6}$$

Finally, dividing these two equations we get an expression for the optimal relative prices,

$$\frac{p_p}{p_g} = \frac{1+\lambda\phi_g}{1+\lambda\phi_p} \frac{(\sigma_g - 1)/\sigma_g}{(\sigma_p - 1)/\sigma_p} \tag{B.7}$$

Note that whenever $\sigma_p = \sigma_g$, $p_g/p_p = 1$ for firms without binding financial frictions ($\lambda = 0$). For firms with binding financial frictions ($\lambda > 0$) $p_g/p_p < 1$ ($p_g/p_p > 1$) whenever $\phi_g > \phi_p$ ($\phi_g < \phi_p$) because production is shifted towards the sector that provides better collateral, and $p_g/p_p = 1$ whenever $\phi_g = \phi_p$.

B.1 Some preliminary results

Lemma 1 *The terms* $\frac{r+\delta+\lambda}{1+\lambda\phi_p}$ *and* $\frac{r+\delta+\lambda}{1+\lambda\phi_g}$ *describing the cost of capital for the production of the private sector and the public sector goods respectively, are (a) strictly below* $1/\phi_p$ *and* $1/\phi_g$ *respectively, (b) increasing in* λ*, and (c) strictly above* $r+\delta$ *when* $\lambda > 0$*, if and only if* $\phi_p < (\delta + r)^{-1}$ *and* $\phi_g < (\delta + r)^{-1}$ *respectively.*

Proof: Part (a) is straightforward:

$$\frac{r+\delta+\lambda}{1+\lambda\phi_p} < \frac{1}{\phi_p} \Leftrightarrow \phi_p (r+\delta+\lambda) < (1+\lambda\phi_p) \Leftrightarrow \phi_p (r+\delta) < 1 \Leftrightarrow \phi_p < (r+\delta)^{-1}$$

For part (b) note that

$$\frac{d}{d\lambda}\left(\frac{r+\delta+\lambda}{1+\lambda\phi_p}\right) \propto (1+\lambda\phi_p) - \phi_p (r+\delta+\lambda) > 0 \Leftrightarrow \phi_p (r+\delta) < 1 \Leftrightarrow \phi_p < (r+\delta)^{-1}$$

Finally, part (c) is proved by noting that $\frac{r+\delta+\lambda}{1+\lambda\phi_p}$ equals $r+\delta$ whenever $\lambda = 0$ and its derivative w.r.t. λ is positive, see part (b). The same arguments apply for $\frac{r+\delta+\lambda}{1+\lambda\phi_g}$. ■

Proposition 1 *Holding* s *constant, more constrained firms sell less to the private sector, sell less to the public sector, and demand less capital if both* $\phi_p < (\delta + r)^{-1}$ *and* $\phi_g < (\delta + r)^{-1}$*.*

Proof: Let's combine the FOC (B.3) with the demand equation (7) to produce the expression,

$$y_p = \left(\frac{\sigma_p - 1}{\sigma_p} B_p s \frac{1+\lambda\phi_p}{r+\delta+\lambda}\right)^{\sigma_p}$$

Then, by virtue of Lemma 1 y_p falls with λ whenever $\phi_p < (\delta + r)^{-1}$. The case for y_g is analogous. Finally, note that total output is split between private sector and public sector sales, that is, $y_p + y_g = f(s,k) = sk$, so the derivative of capital with respect to λ is just,

$$\frac{dk}{d\lambda} = \frac{1}{s}\left(\frac{dy_p}{d\lambda} + \frac{dy_g}{d\lambda}\right)$$

which is negative given the previous results in this Proposition. ■

Lemma 2 *The optimal unconstrained capital for the private and the public sector respectively cannot be self-financed through its own revenues if and only if* $\phi_p \frac{\sigma_p}{\sigma_p - 1}(r+\delta) < 1$ *and* $\phi_g \frac{\sigma_g}{\sigma_g - 1}(r+\delta) <$ 1 *respectively.*

Proof: The optimal unconstrained solution for the private sector capital is given by equation (B.3) when $\lambda = 0$, which implies $\frac{p_p y_p}{k}\frac{1}{u} = \frac{\sigma_p}{\sigma_p - 1}(r+\delta)$. When $\phi_p \frac{\sigma_p}{\sigma_p - 1}(r+\delta) < 1 \Leftrightarrow \frac{\sigma_p}{\sigma_p - 1}(r+\delta) < \phi_p^{-1}$ this leads to $\frac{p_p y_p}{k}\frac{1}{u} < \phi_p^{-1} \Leftrightarrow \phi_p p_p y_p < uk$, that is, the optimal unconstrained capital for the private sector, uk, cannot be self-financed through its own revenues. The proof for the public sector capital is analogous by use of the FOC (B.4) ■

Proposition 2 *Entrepreneurs with zero net worth are financially constrained if both* $\phi_p \frac{\sigma_p}{\sigma_p - 1}(r + \delta) < 1$ *and* $\phi_g \frac{\sigma_g}{\sigma_g - 1}(r + \delta) < 1$.

Proof: Note that if both $\phi_p \frac{\sigma_p}{\sigma_p - 1}(r + \delta) < 1$ and $\phi_g \frac{\sigma_g}{\sigma_g - 1}(r + \delta) < 1$, then following Lemma 2 both $\phi_p p_p y_p < uk$ and $\phi_g p_g y_g < (1 - u)k$. Adding them up leads to $\phi_p p_p y_p + \phi_g p_g y_g < k$, which implies that the capital of the unconstrained solution cannot be financed through revenue based constraints and hence entrepreneurs with zero net worth are constrained. ■

Lemma 3 *The term* $\phi_p \frac{\partial p_p y_p}{\partial k} + \phi_g \frac{\partial p_g y_g}{\partial k}$ *describing the share of capital that can be self-financed through revenues is positive and strictly smaller than one for constrained firms.*

Proof: That this term is positive is straightforward. To show that it is lower than one, note that for constrained firms the borrowing constrain in (15) holds with equality. Hence, for $a \geq 0$ it must be that $k \geq \phi_p p_p y_p + \phi_g p_g y_g$ or $\phi_p \frac{p_p y_p}{k} + \phi_g \frac{p_g y_g}{k} \leq 1$ (with strict equality for $a = 0$). Given our revenue function, the marginal products are proportional to the average products $\frac{\partial p_p y_p}{\partial k} = \left(\frac{\sigma_p - 1}{\sigma_p}\right) \frac{y_p p_p}{k}$ and $\frac{\partial p_g y_g}{\partial k} = \left(\frac{\sigma_g - 1}{\sigma_g}\right) \frac{y_g p_g}{k}$, so we can rewrite

$$\phi_p \frac{\partial p_p y_p}{\partial k} + \phi_g \frac{\partial p_g y_g}{\partial k} = \phi_p \left(\frac{\sigma_p - 1}{\sigma_p}\right) \frac{p_p y_p}{k} + \phi_g \left(\frac{\sigma_g - 1}{\sigma_g}\right) \frac{p_g y_g}{k}$$

Note that $\sigma_p > 1$ and $\sigma_g > 1$ implies that $\frac{\sigma_p - 1}{\sigma_p} < 1$ and $\frac{\sigma_g - 1}{\sigma_g} < 1$ (the marginal products are below the average products), and hence it is the case that $\phi_p \frac{\partial p_p y_p}{\partial k} + \phi_g \frac{\partial p_g y_g}{\partial k} < \phi_p \frac{p_p y_p}{k} + \phi_g \frac{p_g y_g}{k} \leq 1$ ■

Lemma 4 *The term* $\phi_p \frac{\partial p_p y_p}{\partial u} + \phi_g \frac{\partial p_g y_g}{\partial u}$ *describing the increase in credit that can be achieved by reallocation output to the private sector has the sign of* $(\phi_p - \phi_g)$ *for constrained firms.*

Proof: Using equation (17) we can write:

$$\phi_p \frac{\partial p_p y_p}{\partial u} + \phi_g \frac{\partial p_g y_g}{\partial u} = \frac{\partial p_p y_p}{\partial u} \left[\phi_p - \phi_g \frac{1 + \lambda \phi_p}{1 + \lambda \phi_g}\right] = \frac{\partial p_p y_p}{\partial u} \phi_g \left[\frac{\phi_p}{\phi_g} - \frac{\lambda^{-1} + \phi_p}{\lambda^{-1} + \phi_g}\right]$$

Note that with $\phi_p > \phi_g$ ($\phi_p < \phi_g$), this expression is positive (negative) when λ tends to zero, it decreases (increases) monotonically with λ, and tends to zero when λ tends to infinity. ■

B.2 Firms without procurement

We start analyzing the production problem for firms without procurement, that is, firms with $d = 0$.

B.2.1 Unconstrained firms

With $\lambda = 0$ the FOC for k in (18) becomes,

$$\frac{\partial p_p y_p}{\partial k} = r + \delta$$

which states that firms must equalize the marginal revenue product of capital to the cost of capital. This equation defines the optimal demand of capital $k^*(s,a,0)$ for every entrepreneur of type $(s,a,d=0)$. In particular, one gets $\frac{\sigma-1}{\sigma}\frac{p_p y_p}{k} = r+\delta$ and substituting for the revenue function yields the optimal demand for capital

$$k^*(s,a,0) = \left[\left(\frac{\sigma_p - 1}{\sigma_p}\right)\frac{B_p}{r+\delta}\right]^{\sigma} s^{\sigma-1} \tag{B.8}$$

Next, note that profits are given by $\pi = p_p y_p - (r+\delta)k$, which given the optimal choice of capital can be written as $\pi = \frac{1}{\sigma_p} p_p y_p$ or $\pi = \frac{1}{\sigma_p - 1}(r+\delta)k$. Substituting optimal capital demand to the revenue function gives $p_p y_p = B_p\left[\left(\frac{\sigma_p-1}{\sigma_p}\right)\frac{B_p}{r+\delta}\right]^{\sigma_p-1} s^{\sigma_p-1}$, which can be substituted back to the profit function to obtain:

$$\pi^*(s,a,0) = \frac{1}{\sigma_p}\left[\left(\frac{\sigma_p - 1}{\sigma_p}\right)\frac{1}{r+\delta}\right]^{\sigma_p-1} B_p^{\sigma} s^{\sigma_p-1} \tag{B.9}$$

Hence, capital demand and profits increase monotonically with the shock s and are independent from net worth a.

B.2.2 Constrained firms

If the firm is constrained, then $\lambda > 0$ and the FOC of the problem are:

$$(1+\lambda\phi_p)\frac{\partial p_p y_p}{\partial k} = r+\delta+\lambda \tag{B.10}$$

$$k = \phi_a a + \phi_p p_p y_p \tag{B.11}$$

which determine k and λ. In particular, the borrowing constraint, equation (B.11), defines the capital demand $k(s,a,0)$, the FOC, equation (B.10), delivers the shadow value of the constrain $\lambda(s,a,0)$, and the objective function delivers the profit function $\pi(s,a,0)$. The next propositions chacarcterize the derivatives of these three functions with respect to the state variables a and s.

Let's start by totally differentiating equation (B.11) in turns with respect to a and s to obtain,

$$\frac{\partial k(s,a,0)}{\partial a} = \phi_a \left(1-\phi_p\frac{\partial p_p y_p}{\partial k}\right)^{-1} \tag{B.12}$$

$$\frac{\partial k(s,a,0)}{\partial s} = \phi_p\frac{\partial p_p y_p}{\partial s}\left(1-\phi_p\frac{\partial p_p y_p}{\partial k}\right)^{-1} \tag{B.13}$$

With $\phi_p = 0$ we are in the case without earnings-based collateral constraints and these derivatives are just equal to ϕ_a and 0 respectively: higher net worth allows to operate with more capital but higher productivity does not. With $\phi_p > 0$ both derivatives are positive, that is, constrained firms with more net worth or higher productivity operate with more capital. Indeed, in this case $\frac{\partial k(s,a,0)}{\partial a} > \phi_a$ because an increase in net worth has a multiplier effect through the increase in revenues and the easing of the earnings-based financial constraint (see Lemma 3). This is stated in the next proposition:

Proposition 3 *The derivative of $k(s,a,0)$ with respect to a is positive, while the derivative of $k(s,a,0)$ with respect to s is positive as long as $\phi_p > 0$ (and zero othwewise).*

Proof: The derivatives of $k(s,a,0)$ with respect to a and s are given by equation (B.12) and (B.13). $\phi_a \geq 1$ and Lemma 3 states that $\phi_p \frac{\partial p_p y_p}{\partial k} < 1$, so the derivative with respect to a is strictly positive. For the derivative with respect to s, note additionally that $\frac{\partial p_p y_p}{\partial s} > 0$. Hence, this derivative is strictly positive (zero) if $\phi_p > 0$ ($\phi_p = 0$). ■

Note also that the derivatives of capital with respect to a and s are higher for more constrained firms (higher λ) because the multiplier effect of the earnings-based constraints is larger for firms with higher marginal product of capital, that is, the increase in capital demand with net worth a or productivity s is larger for more financially constrained firms. This is stated in the next corollary:

Corollary 1 *The derivatives of $k(s,a,0)$ with respect to a and with respect to s increase with λ*

Proof: The derivatives are characterized by equations (B.12) and (B.13). Using the FOC (B.10) and the fact that $\frac{\partial p_p y_p}{\partial s} = \frac{k}{s}\frac{\partial p_p y_p}{\partial k}$ we can further rewrite them as

$$\frac{\partial k(s,a,0)}{\partial a} = \phi_a \left(1 - \phi_p \frac{r+\delta+\lambda}{1+\lambda\phi_p}\right)^{-1} \tag{B.14}$$

$$\frac{\partial k(s,a,0)}{\partial s} = \phi_p \frac{k}{s}\frac{r+\delta+\lambda}{1+\lambda\phi_p} \left(1 - \phi_p \frac{r+\delta+\lambda}{1+\lambda\phi_p}\right)^{-1} \tag{B.15}$$

To prove this corollary it is enough to show that the term $(r+\delta+\lambda)/(1+\lambda\phi_p)$ in equations (B.32) and (B.33) increases with λ, which is proved in Lemma 1. ■

Next, equation (B.10) allows to recover $\lambda(s,a,0)$. It can be shown that $\lambda(s,a,0)$ declines with a —wealthier entrepreneurs can finance larger amounts of capital and are hence less constrained— and increases with s —s increases optimal capital by more than it increases the amount of capital that can be self-financed through revenues. This is stated formally in Proposition 4.

Proposition 4 *The derivative of $\lambda(s,a,0)$ with respect to a is always negative, while the derivative of $\lambda(s,a,0)$ with respect to s is always positive as long as $a > 0$ (and zero otherwise).*

Proof: Equation (B.10) can be rewritten as

$$\frac{\partial p_p y_p}{\partial k} = \frac{r+\delta+\lambda}{1+\lambda\phi_p}$$

The r.h.s, the cost of capital, increases with λ, see Lemma 1. Hence, the sign of the derivative of $\lambda(s,a,0)$ with respect to a or s is equal to the sign of the derivative of $\frac{\partial p_p y_p}{\partial k}$ with respect to a or s. We start by obtaining an expression of the marginal revenue product of capital by use of the revenue function:

$$\frac{\partial p_p y_p}{\partial k} = \frac{\sigma_p - 1}{\sigma_p}\frac{p_p y_p}{k} = \frac{\sigma_p - 1}{\sigma_p} B_p s^{\frac{\sigma_p - 1}{\sigma_p}} k^{-\frac{1}{\sigma_p}}$$

where $\frac{\partial p_p y_p}{\partial k}$ declines with $k(s,a,0)$. For net worth a it is straightforward to see that $\lambda(s,a,0)$ declines with a because $k(s,a,0)$ increases with a, see Proposition 3. For the shock s we take the derivative of the marginal revenue product of capital w.r.t. s, and asking it to be non-negative delivers:

$$\frac{\partial^2 p_p y_p}{\partial k \partial s} \propto \left[(\sigma_p - 1) - \frac{\partial k}{\partial s}\frac{s}{k}\right] \geq 0$$

where the first term reflects the positive direct effect of s on the marginal revenue product of capital for fixed capital, while the second term reflects the negative indirect effect of s on the marginal revenue product of capital through its induced increase in the choice of capital. Using $\frac{\partial p_p y_p}{\partial s} = \frac{k}{s}\frac{\partial p_p y_p}{\partial k}$, equation (B.13) shows that

$$\frac{\partial k}{\partial s}\frac{s}{k} = \phi_p \frac{\partial p_p y_p}{\partial k}\left(1 - \phi_p \frac{\partial p_p y_p}{\partial k}\right)^{-1}$$

Then, we can rewrite

$$(\sigma_p - 1) - \frac{\partial k}{\partial s}\frac{s}{k} \geq 0 \quad \Leftrightarrow \quad \phi_p \frac{\partial p_p y_p}{\partial k} \leq \frac{\sigma_p - 1}{\sigma_p} \quad \Leftrightarrow \quad k \geq \phi_p p_p y_p$$

where the last step uses the fact that $\frac{\partial p_p y_p}{\partial k} = \frac{\sigma_p - 1}{\sigma_p}\frac{p_p y_p}{k}$. Note that whenever a firm has zero net worth it will be able to self-finance capital up to the point $k = \phi_p p_p y_p$. In this case the derivative of $\lambda(s,a,0)$ with respect to s will be zero. Whenever a firm owns $a > 0$ then capital k is going to be above $\phi_p p_p y_p$ and the derivative of $\lambda(s,a,0)$ with respect to s will be positive.

■

Next, with Corollary 1 and Proposition 4, one can also show that $\frac{\partial^2 k(s,a,0)}{\partial a^2} < 0$ (the increase in capital due to an increase in net worth is larger for firms with less net worth) and that $\frac{\partial^2 k(s,a,0)}{\partial a \partial s} > 0$ (the increase in capital due to an increase in net worth is larger for firms with higher productivity), see Corollary 2.

Corollary 2 *The derivative of $\partial k(s,a,0)/\partial a$ with respect to a is always negative, while the derivative of $\partial k(s,a,0)/\partial a$ with respect to s is positive as long as $a > 0$ (and zero otherwise).*

Proof: By the chain rule we can write

$$\begin{aligned}\frac{\partial^2 k(s,a,0)}{\partial a^2} &= \frac{\partial^2 k(s,a,0)}{\partial a \partial \lambda}\frac{\partial \lambda(s,a,0)}{\partial a} \\ \frac{\partial^2 k(s,a,0)}{\partial a \partial s} &= \frac{\partial^2 k(s,a,0)}{\partial a \partial \lambda}\frac{\partial \lambda(s,a,0)}{\partial s}\end{aligned}$$

The first derivative in the r.h.s. of these expressions is positive by Corollary 1. Hence, the sign of the derivatives $\frac{\partial^2 k(s,a,0)}{\partial a^2}$ and $\frac{\partial^2 k(s,a,0)}{\partial a \partial s}$ is the same as the sign of the derivatives $\frac{\partial \lambda(s,a,0)}{\partial a}$ and $\frac{\partial \lambda(s,a,0)}{\partial s}$ described in Proposition 4. ■

Finally, we can also characterize the derivatives of the profit function $\pi(s,a,0)$, which are given by

$$\frac{\partial \pi(s,a,0)}{\partial a} = \left[\frac{\partial p_p y_p}{\partial k} - (r+\delta)\right]\frac{\partial k(s,a,0)}{\partial a} \tag{B.16}$$
$$\frac{\partial \pi(s,a,0)}{\partial s} = \left[\frac{\partial p_p y_p}{\partial k} - (r+\delta)\right]\frac{\partial k(s,a,0)}{\partial s} + \frac{\partial p_p y_p}{\partial s} \tag{B.17}$$

We can substitute the partial derivatives of capital w.r.t. a and s described by (B.12) and (B.13) into equations (B.16) and (B.17) respectively. Then, using the FOC in (B.11) we obtain

$$\frac{\partial \pi(s,a,0)}{\partial a} = \phi_a \lambda(s,a,0) \tag{B.18}$$
$$\frac{\partial \pi(s,a,0)}{\partial s} = (1+\phi_p \lambda(s,a,0))\frac{\partial p_p y_p}{\partial s} \tag{B.19}$$

Profits increase with a because more net worth allows to increase capital and hence profits. Profits increase with s because two reasons. First, there is the direct increase of revenues with s for given capital.

Second, if $\phi_p > 0$ a larger s implies higher revenues and hence more capital can be borrowed. Second, the increase in revenues with s allows to increase capital, which in turn increases profits. This is proved in the next Proposition:

Proposition 5 *The derivatives of $\pi(s,a,0)$ with respect to a and s are always positive.*

Proof: The derivatives of the profit function with respect to a and s are given by (B.18) and (B.19). These derivatives are positive because $\lambda(s,a,0) > 0$ for constrained agents and $\frac{\partial p_p y_p}{\partial s} > 0$ (see the revenue function). ■

Finally, we can also characterize the second derivatives of the profit function:

Corollary 3 *The derivative of $\partial \pi(s,a,0)/\partial a$ with respect to a is always negative, while the derivative of $\partial \pi(s,a,0)/\partial s$ with respect to s is always positive as long as $a > 0$ (and zero otherwise).*

Proof: Using equation (B.18) we can write the second derivatives as,

$$\frac{\partial^2 \pi(s,a,0)}{\partial a^2} = \phi_a \frac{\partial \lambda(s,a,0)}{\partial a}$$
$$\frac{\partial^2 \pi(s,a,0)}{\partial a \partial s} = \phi_a \frac{\partial \lambda(s,a,0)}{\partial s}$$

Then, one only needs to check the signs of the derivatives of λ in Proposition 4. ■

B.2.3 Binding constraints

Finally, we need to characterize the set of entrepreneurs that are financially constrained. Under Assumption 1, Proposition 2 says that $k(s,0,0) < k^*(s,0,0)$, and we have shown that $\frac{\partial k(s,a,0)}{\partial a} > 0$

and that $k^*(s,a,0)$ is invariant in a. Hence, for every s there will be a unique threshold $\underline{a}(s,0)$ satisfying $k(s,a,0) = k^*(s,a,0)$ such that for every s entrepreneurs with $a \geq \underline{a}(s,0)$ are unconstrained while entrepreneurs with $a < \underline{a}(s,0)$ are constrained.

B.3 Firms with procurement

We now analyze the production problem for firms with procurement, that is, firms with $d = 1$ for the case $\sigma_p = \sigma_g = \sigma$.

B.3.1 Unconstrained firms

With $\lambda = 0$ the FOC for k and u in (18) and (17) become

$$\begin{aligned} \frac{\partial p_p y_p}{\partial u} + \frac{\partial p_g y_g}{\partial u} &= 0 \\ \frac{\partial p_p y_p}{\partial k} + \frac{\partial p_g y_g}{\partial k} &= r + \delta \end{aligned}$$

which states that unconstrained firms allocate output between the two sectors to equalize the marginal revenues and choose capital such that the marginal revenue product of capital equals the capital costs. These two equations determine the optimal capital demand $k^*(s,a,1)$ and allocation of output in the private sector $u^*(s,a,1)$ for entrepreneurs of type $(s,a,d=1)$. In particular, the FOC for k can be written as $\frac{\sigma-1}{\sigma}\frac{p_p y_p + p_g y_g}{k} = r + \delta$. Substituting for the revenue functions yields the optimal demand for capital:

$$k^*(s,a,1) = \left[\left(\frac{\sigma-1}{\sigma}\right)\frac{1}{r+\delta}\right]^{\sigma} \left(B_p^{\sigma} + B_g^{\sigma}\right) s^{\sigma-1} \tag{B.20}$$

Using the FOC for u one gets $\frac{p_p y_p}{ku} = \frac{p_g y_g}{k(1-u)}$ where again we can substitute the revenue functions to obtain:

$$u^*(s,a,1) = \left(1 + \left(\frac{B_g}{B_p}\right)^{\sigma}\right)^{-1} \tag{B.21}$$

Clearly $k^*(s,a,1)$ increases monotonically with the shock s and is invariant with the net worth a, while $u^*(s,a,1)$ is independent from both s and a and is only determined by the relative demands B_p/B_g. Next, note that profits are given by $\pi = p_p y_p + p_g y_g - (r+\delta)k$, which given the condition for the optimal choice of capital can be written as $\pi = \frac{1}{\sigma}(p_p y_p + p_g y_g)$ or $\pi = \frac{1}{\sigma-1}(r+\delta)k$. Substituting the optimal capital demand into the revenue function gives total revenues as $p_p y_p + p_g y_g = \left[\left(\frac{\sigma-1}{\sigma}\right)\frac{1}{r+\delta}\right]^{\sigma-1}\left(B_p^{\sigma} + B_g^{\sigma}\right)s^{\sigma-1}$, which can be substituted back into the profit function to obtain

$$\pi^*(s,a,1) = \frac{1}{\sigma}\left[\left(\frac{\sigma-1}{\sigma}\right)\frac{1}{r+\delta}\right]^{\sigma-1} \left(B_p^{\sigma} + B_g^{\sigma}\right) s^{\sigma-1} \tag{B.22}$$

The profit function increases with productivity s and is invariant with assets a.

B.3.2 Constrained firms.

For constrained firms with procurement, equations (17)-(19) jointly determine $k(s,a,1)$, $u(s,a,1)$, and $\lambda(s,a,1)$. The characerization of these functions is simple whenever $\phi_g = \phi_p$ and more involved when not. To characterize $u(s,a,1)$ let's start by noting that the FOC for u, given by equation (17), can be rewritten as in (B.7) and that after substituting prices we obtain,

$$\frac{u}{1-u} = \left(\frac{B_p}{B_g}\right)^{\sigma}\left(\frac{1+\lambda\phi_p}{1+\lambda\phi_g}\right)^{\sigma} \tag{B.23}$$

To characterize $k(s,a,1)$ we totally differentiate equation (19) with respect to a and s in turn, which gives,

$$\frac{\partial k}{\partial a} = \left[\phi_a + \left(\phi_p\frac{\partial p_p y_p}{\partial u} + \phi_g\frac{\partial p_g y_g}{\partial u}\right)\frac{du}{da}\right]\left[1-\left(\phi_p\frac{\partial p_p y_p}{\partial k} + \phi_g\frac{\partial p_g y_g}{\partial k}\right)\right]^{-1} \tag{B.24}$$

$$\frac{\partial k}{\partial s} = \left[\left(\phi_p\frac{\partial p_p y_p}{\partial s} + \phi_g\frac{\partial p_g y_g}{\partial s}\right) + \left(\phi_p\frac{\partial p_p y_p}{\partial u} + \phi_g\frac{\partial p_g y_g}{\partial u}\right)\frac{du}{ds}\right]\left[1-\left(\phi_p\frac{\partial p_p y_p}{\partial k} + \phi_g\frac{\partial p_g y_g}{\partial k}\right)\right]^{-1} \tag{B.25}$$

Finally, the derivatives of the profit function $\pi(s,a,1)$ are given by

$$\begin{aligned}\frac{\partial \pi(s,a,1)}{\partial a} &= \left[\frac{\partial p_p y_p}{\partial k} + \frac{\partial p_g y_g}{\partial k} - (r+\delta)\right]\frac{\partial k(s,a,1)}{\partial a} \\ &+ \left[\frac{\partial p_p y_p}{\partial u} + \frac{\partial p_g y_g}{\partial u}\right]\frac{\partial u(s,a,1)}{\partial a}\end{aligned} \tag{B.26}$$

$$\begin{aligned}\frac{\partial \pi(s,a,1)}{\partial s} &= \left[\frac{\partial p_p y_p}{\partial k} + \frac{\partial p_g y_g}{\partial k} - (r+\delta)\right]\frac{\partial k(s,a,1)}{\partial s} \\ &+ \left[\frac{\partial p_p y_p}{\partial k} + \frac{\partial p_g y_g}{\partial k}\right]\frac{\partial u(s,a,1)}{\partial s} + \frac{\partial p_p y_p}{\partial s}\end{aligned} \tag{B.27}$$

Now, substituting (B.1), (B.2), and (B.24) into (B.26) and using (17) we obtain

$$\frac{\partial \pi(s,a,1)}{\partial a} = \phi_a \lambda(s,a,1) \tag{B.28}$$

while substituting (B.1), (B.2), and (B.25) into (B.27) and using (17) we obtain

$$\frac{\partial \pi(s,a,1)}{\partial s} = (1+\phi_p\lambda(s,a,1))\frac{\partial p_p y_p}{\partial s} + (1+\phi_g\lambda(s,a,1))\frac{\partial p_g y_g}{\partial s} \tag{B.29}$$

Profits increase with a because more net worth allows to increase capital and hence profits. Profits increase with s because two reasons. First, there is the direct increase of revenues with s for given capital. Second, if $\phi_p > 0$ and/or $\phi_g > 0$ the increase in revenues with s allows to increase capital, which in turn increases profits.

For the case $\phi_g = \phi_p$ it can be shown that $u(s,a,1) = u^*(s,a,1)$ —as revenues from both sectors are equally pledgeable— and hence $u(s,a,1)$ is invariant in a and s. This makes the problem analogous to the case without procurement ($d=0$), and hence the derivatives of $k(s,a,1)$, $\lambda(s,a,1)$, and $\pi(s,a,1)$ with respect to a and s are as in the $d=0$ case. This can be seen in the next propositions.

Proposition 6 *When $\phi_g = \phi_p$, the optimal choice of $u(s,a,1)$ is as in the unconstrained case and it is hence independent from a and s*

Proof: Equation (B.23) clearly shows that whenever $\phi_g = \phi_p$ the optimal solution for u for constrained firms is equal to the one for unconstrained firms, see equation (B.21). This means that $u(s,a,1)$ is independent from s and a and only determined by the relative demands B_p/B_g of each sector. ∎

Proposition 7 *When $\phi_g = \phi_p$, the derivative of $k(s,a,1)$ with respect to a is positive, while the derivative of $k(s,a,1)$ with respect to s is positive as long as $\phi_p > 0$ (and zero othwewise).*

Proof: Note that with $\phi_g = \phi_p$ the optimality condition (17) implies that $\frac{\partial p_g y_g}{\partial u} = -\frac{\partial p_p y_p}{\partial u}$ and hence we can rewrite equations (B.24) and (B.25) as follows,

$$\frac{\partial k}{\partial a} = \phi_a \left[1 - \phi_p\left(\frac{\partial p_p y_p}{\partial k} + \frac{\partial p_g y_g}{\partial k}\right)\right]^{-1} \quad \text{(B.30)}$$

$$\frac{\partial k}{\partial s} = \phi_p\left(\frac{\partial p_p y_p}{\partial s} + \frac{\partial p_g y_g}{\partial s}\right)\left[1 - \phi_p\left(\frac{\partial p_p y_p}{\partial k} + \frac{\partial p_g y_g}{\partial k}\right)\right]^{-1} \quad \text{(B.31)}$$

Given $\phi_a \geq 1$ and $\phi_p > 0$ both $\partial k/\partial a$ and $\partial k/\partial s$ are positive because of Lemma 3. If $\phi_p = 0$ then $\partial k/\partial s = 0$. ∎

Corollary 4 *When $\phi_g = \phi_p$, the derivatives of $k(s,a,1)$ with respect to a and with respect to s increase with λ*

Proof: Equation (18) can be written as,

$$\frac{\partial p_p y_p}{\partial k} + \frac{\partial p_g y_g}{\partial k} = \frac{r + \delta + \lambda}{1 + \lambda\phi_p}$$

Then, using the fact that $\frac{\partial p_p y_p}{\partial s} = \frac{k}{s}\frac{\partial p_p y_p}{\partial k}$ we can rewrite equations (B.30) and (B.31) as

$$\frac{\partial k(s,a,1)}{\partial a} = \phi_a \left(1 - \phi_p \frac{r + \delta + \lambda}{1 + \lambda\phi_p}\right)^{-1} \quad \text{(B.32)}$$

$$\frac{\partial k(s,a,1)}{\partial s} = \phi_p \frac{k}{s}\frac{r + \delta + \lambda}{1 + \lambda\phi_p}\left(1 - \phi_p \frac{r + \delta + \lambda}{1 + \lambda\phi_p}\right)^{-1} \quad \text{(B.33)}$$

To prove this corollary it is enough to show that the term $(r + \delta + \lambda)/(1 + \lambda\phi_p)$ in equations (B.32) and (B.33) increases with λ, which is proved in Lemma 1. ∎

Proposition 8 *When $\phi_g = \phi_p$, the derivative of $\lambda(s,a,1)$ with respect to a is always negative, while the derivative of $\lambda(s,a,1)$ with respect to s is always positive as long as $a > 0$ (and zero otherwise).*

Proof: Note that the FOC for k_p is given by equation (B.1). Because u is invariant in a and s, see Proposition 6, the proof of Proposition 8 for the case $d = 0$ carries over. ∎

Corollary 5 *When* $\phi_g = \phi_p$*, the derivative of* $\partial k(s,a,1)/\partial a$ *with respect to* a *is always negative, while the derivative of* $\partial k(s,a,1)/\partial a$ *with respect to* s *is positive as long as* $a > 0$ *(and zero otherwise).*

Proof: By the chain rule we can write

$$\frac{\partial^2 k(s,a,1)}{\partial a^2} = \frac{\partial^2 k(s,a,1)}{\partial a \partial \lambda}\frac{\partial \lambda(s,a,1)}{\partial a}$$
$$\frac{\partial^2 k(s,a,1)}{\partial a \partial s} = \frac{\partial^2 k(s,a,1)}{\partial a \partial \lambda}\frac{\partial \lambda(s,a,1)}{\partial s}$$

The first derivative in the r.h.s. of these expressions is positive by Corollary 4. Hence, the sign of the derivatives $\frac{\partial^2 k(s,a,1)}{\partial a^2}$ and $\frac{\partial^2 k(s,a,1)}{\partial a \partial s}$ is the same as the sign of the derivatives $\frac{\partial \lambda(s,a,1)}{\partial a}$ and $\frac{\partial \lambda(s,a,1)}{\partial s}$ described in Proposition 8. ■

Proposition 9 *When* $\phi_g = \phi_p$*, the derivatives of* $\pi(s,a,1)$ *with respect to* a *and* s *are always positive.*

Proof: The derivatives of the profit function with respect to a and s are given by (B.28) and (B.29). These derivatives are positive because $\lambda(s,a,1) > 0$ for constrained agents and $\frac{\partial p_p y_p}{\partial s} > 0$ and $\frac{\partial p_g y_g}{\partial s} > 0$ (see the revenue functions). ■

Corollary 6 *When* $\phi_g = \phi_p$*, the derivative of* $\partial \pi(s,a,1)/\partial a$ *with respect to* a *is always negative, while the derivative of* $\partial \pi(s,a,1)/\partial s$ *with respect to* s *is always positive as long as* $a > 0$ *(and zero otherwise).*

Proof: Using equation (B.28) we can write the second derivatives as,

$$\frac{\partial^2 \pi(s,a,1)}{\partial a^2} = \phi_a \frac{\partial \lambda(s,a,1)}{\partial a}$$
$$\frac{\partial^2 \pi(s,a,1)}{\partial a \partial s} = \phi_a \frac{\partial \lambda(s,a,1)}{\partial s}$$

Then, one only needs to check the signs of the derivatives of λ in Proposition 8. ■

The case $\phi_g > \phi_p$ is more involved because $u(s,a,1)$ changes with a and s. It can be shown that firms with more net worth are less constrained and hence run larger firms and sell a higher fraction of output to the private sector, which offers lower collateral value. More productive firms are able to run larger firms thanks to the earnings-based constraints but are more constrained —because their optimal capital is even larger— and hence sell a lower fraction of output to the private sector. This which means that firms with larger s sell a larger quantity to the pubic sector but they may either sell a larger or smaller quantity to the private sector. This is proved in the following propositions.

Lemma 5 *The sign of the derivative of u with respect to λ is the same as the sign of $(\phi_p - \phi_g)$, that is, more constrained firms shift their output relatively towards the sector whose revenues provide better colateral.*

Proof: Simply note that equation (B.23) implies that $du/d\lambda < 0$ when $\phi_g > \phi_p$ and the opposite when $\phi_g < \phi_p$. ■

Proposition 10 *When $\phi_g > \phi_p$, the derivatives of $u(s,a,1)$, $k(s,a,1)$, and $\lambda(s,a,1)$ with respect to a are positive, positive, and negative respectively,*

Proof: First note that, following Lemma 5, $du/d\lambda < 0$ when $\phi_g > \phi_p$ and that Proposition 1 says that $dk/d\lambda < 0$. That is, more constrained entrepreneurs tilt production towards the sector with higher collateral value and run smaller firms. Next, using the FOC (B.3) and (B.4), the demand equations (7) and (8), and the production function we can write,

$$k_p = \left(\frac{\sigma_p - 1}{\sigma_p} B_p \frac{1 + \lambda\phi_p}{r + \delta + \lambda}\right)^{\sigma_p} s^{\sigma_p - 1} \quad \text{and} \quad k_g = \left(\frac{\sigma_p - 1}{\sigma_p} B_g \frac{1 + \lambda\phi_g}{r + \delta + \lambda}\right)^{\sigma_p} s^{\sigma_p - 1}$$

Adding them up, and using the chain rule, let us express $\frac{\partial k}{\partial a}$

$$\frac{\partial k}{\partial a} = \frac{\partial k}{\partial \lambda}\frac{\partial \lambda}{\partial a}$$

Also, using equation (B.23) and the chain rule we can write

$$\frac{\partial u}{\partial a} = \frac{\partial u}{\partial \lambda}\frac{\partial \lambda}{\partial a}$$

These two expressions state that $\frac{\partial k}{\partial a}$ and $\frac{\partial u}{\partial a}$ should have the same sign because both k and u fall with λ. Given this, equation (B.24) implies that $\frac{\partial k}{\partial a} > 0$ and $\frac{\partial u}{\partial a} > 0$. To see why, recall that by Lemma 3 the denominator is positive. In addition, the term $\phi_p \frac{\partial p_p y_p}{\partial u} + \phi_g \frac{\partial p_g y_g}{\partial u}$ is negative whenever $\phi_g > \phi_p$ see Lemma 4. Hence, for $\frac{\partial k}{\partial a} < 0$ we would need $\frac{\partial k}{\partial u} > 0$. That is, given that higher a allows to increase capital through ϕ_a, for higher a to lead to lower capital it must be that entrepreneurs with higher a tilt production towards the sector with lower collateral value. But this would require the signs of $\frac{\partial k}{\partial a}$ and $\frac{\partial u}{\partial a}$ to be different. Instead, $\frac{\partial k}{\partial a} > 0$ can be obtained with $\frac{\partial u}{\partial a} > 0$. It follows that, because $\frac{\partial k}{\partial \lambda} < 0$ and $\frac{\partial k}{\partial a} > 0$, it must be the case that $\frac{\partial \lambda}{\partial a} < 0$. ■

Proposition 11 *When $\phi_g > \phi_p$, the derivatives of $u(s,a,1)$, $k(s,a,1)$, and $\lambda(s,a,1)$ with respect to s are negative, positive, and positive respectively,*

Proof: First note that, following Lemma 5, $du/d\lambda < 0$ when $\phi_g > \phi_p$ and that Proposition 1 says that $dk/d\lambda < 0$. That is, more constrained entrepreneurs tilt production towards the sector with higher collateral value and run smaller firms. Next, by the chain rule (see proof of Proposition 10) we can write

$$\frac{dk}{ds} = \frac{\partial k}{\partial \lambda}\frac{\partial \lambda}{\partial s} + \frac{\partial k}{\partial s} \quad \text{and} \quad \frac{du}{ds} = \frac{\partial u}{\partial \lambda}\frac{\partial \lambda}{\partial s}$$

We learn two things from here. First, $\frac{dk}{ds} \leq 0$ requires $\frac{\partial\lambda}{\partial s} > 0$ (because $\frac{\partial k}{\partial s} > 0$ and $\frac{\partial k}{\partial\lambda} < 0$). Second, $\frac{\partial\lambda}{\partial s} > 0$ requires $\frac{du}{ds} < 0$ (because $du/d\lambda < 0$). But equation (B.25) shows that if $\frac{du}{ds} < 0$ then it must be $\frac{dk}{ds} > 0$ so this enters a contradiction. Therefore, $\frac{dk}{ds} > 0$. Note that from equation (B.25) $\frac{dk}{ds} > 0$ can be achieved with any sign of $\frac{du}{ds}$. Now, regarding the derivatives of $u(s,a,1)$ and $\lambda(s,a,1)$ with respect to s, two different things can happen. If $\frac{\partial\lambda}{\partial s} \geq 0$ then $\frac{du}{ds} \leq 0$ (this is an if and only if statement), and then $\frac{dk}{ds} > 0$ according to equation (B.25). Instead, if $\frac{\partial\lambda}{\partial s} < 0$ then $\frac{du}{ds} > 0$ (again an if and only if statement) and we can have both $\frac{dk}{ds} > 0$ or $\frac{dk}{ds} < 0$ according to equation (B.25). ■

Proposition 12 *When $\phi_g > \phi_p$, the derivatives of $\pi(s,a,1)$ with respect to a and s are always positive.*

Proof: The derivatives of the profit function with respect to a and s are given by (B.28) and (B.29). These derivatives are positive because $\lambda(s,a,1) > 0$ for constrained agents and $\frac{\partial p_p y_p}{\partial s} > 0$ and $\frac{\partial p_g y_g}{\partial s} > 0$ (see the revenue functions). ■

Corollary 7 *When $\phi_g > \phi_p$, the derivative of $\partial\pi(s,a,1)/\partial a$ with respect to a is always negative, while the derivative of $\partial\pi(s,a,1)/\partial s$ with respect to s is always positive as long as $a > 0$ (and zero otherwise).*

Proof: Using equation (B.28) we can write the second derivatives as,

$$\frac{\partial^2\pi(s,a,1)}{\partial a^2} = \phi_a \frac{\partial\lambda(s,a,1)}{\partial a}$$
$$\frac{\partial^2\pi(s,a,1)}{\partial a\partial s} = \phi_a \frac{\partial\lambda(s,a,1)}{\partial s}$$

Then, one only needs to check the signs of the derivatives of λ in Proposition 10 and 11. ■

B.4 A procurement shock

Finally, in this Section we analyze how firm choices change upon arrival of a procurement project for the case $\sigma_p = \sigma_g = \sigma$. To do so, we compare the choices of firms in the $(s,a,1)$ state with firms in the $(s,a,0)$ state.

B.4.1 Unconstrained firms

For unconstrained firms, the increase in total capital is given by,

$$\frac{k^*(s,a,1)}{k^*(s,a,0)} = 1 + \left(\frac{B_g}{B_p}\right)^\sigma = \frac{1}{u^*(s,a,1)}$$

which implies that $u^*(s,a,1)\,k^*(s,a,1) = k^*(s,a,0)$. Hence, the amount of capital used in the private sector for the unconstrained firm with a procurement project equals the capital stock it was using without procurement. This means that unconstrained firms do not change their private sector operations and increase their capital stock to meet the extra demand. The increase in capital $k^*(s,a,1) - k^*(s,a,0)$ is given by $\left(\frac{B_g}{B_p}\right)^{\sigma} k^*(s,a,0)$. Because $k^*(s,a,0)$ increases with s and is independent from a, so does the capital increase with procurement.

We can also see that the value of a procurement contract increases with firm productivity s and is independent from firm net worth a. This can be seen by use of the expression $\pi = \frac{1}{\sigma-1}(r+\delta)k$, which implies that $\pi^*(s,a,1) - \pi^*(s,a,0)$ is proportional to the capital increase $k^*(s,a,1) - k^*(s,a,0)$. This could have also be seen by combining equations (B.9) and (B.22), which allows to express

$$\pi^*(s,a,1) - \pi^*(s,a,0) = \frac{1}{\sigma}\left[\left(\frac{\sigma-1}{\sigma}\right)\frac{1}{r+\delta}\right]^{\sigma-1} B_g^{\sigma} s^{\sigma-1}$$

B.4.2 Constrained firms

The first thing to note is that a procurement shock worsens the financial situation of firms when $\phi_g \leq \phi_p$. With $\phi_g = \phi_p$ this is because the firm with $d = 1$ has two demands to serve, they are equally pledgeable, and has the same net worth a to finance capital in the two different markets. As a result the firm scales down the operations in the private sector to free up colateral for the production in the public sector, which generates a negative within-firm private sector spillover of the procurement contract, that is, $k_p(s,a,1) \equiv u(s,a,1)\,k(s,a,1) < k(s,a,0)$. When $\phi_g < \phi_p$ the situation is aggravated because the public sector demand can be self-financed to a lesser extent. When $\phi_g > \phi_p$ it could happen otherwise: the public sector demand can be self-financed to a larger extent, which means that for firms with small net worth it could happen that they are less constrained and use the extra financing capacity coming from the public sector to scale up operations in the private sector. This is stated in Proposition 13 below, but we first look at two preliminary results in Lemma 6 and 7.

Lemma 6 *A procurement shock generates a private sector negative spillover if and only if the procurement shock makes the firm more constrained, that is,* $k_p(s,a,1) < k(s,a,0) \Leftrightarrow \lambda(s,a,1) > \lambda(s,a,0)$

Proof: The FOC for the optimal choice of k_p for a firm with $d = 1$ is given by equation (B.1), where recall $\frac{\partial p_p y_p}{\partial k}\frac{1}{u} = \frac{\partial p_p y_p}{\partial k_p}$. The FOC for the optimal choice of k for a firm with $d = 0$ is given by the same equation (B.1) when $u = 1$. The right hand side of equation (B.1) increase with λ (see Lemma 1), so more constrained firms have higher marginal product of capital and a lower level of capital in the private sector. Hence, $k_p(s,a,1) < k(s,a,0) \Leftrightarrow \lambda(s,a,1) > \lambda(s,a,0)$ ■

Lemma 7 *A procurement shock generates a private sector negative spillover for constrained firms if and only if the chosen production for the public sector cannot be self-financed, that is, if and only if* $\phi_g p_g(s,a,1)\,y_g(s,a,1) < k_g(s,a,1)$

Proof: The demand for capital of constrained firms, with or without procurement, is given by equation (15), which allows to write,

$$\begin{aligned} k_p(s,a,0) - \phi_p p_p(s,a,0)\, y_p(s,a,0) &= \phi_a a \\ k_p(s,a,1) - \phi_p p_p(s,a,1)\, y_p(s,a,1) &= \phi_a a - \left[k_g(s,a,1) - \phi_g p_g(s,a,1)\, y_g(s,a,1)\right] \end{aligned}$$

Importantly, the left hand side of these equations increases with k_p. To see how, note that the derivative of the left hand side w.r.t. k_p is equal to $1 - \phi_p \frac{\partial p_p y_p}{\partial k_p} = 1 - \phi_p \frac{r+\delta+\lambda}{1+\lambda\phi_p}$ according to equation (B.1). Now, $\phi_p \frac{r+\delta+\lambda}{1+\lambda\phi_p} < 1$ according to Lemma 1, so the derivative is positive. Hence, if $k_p(s,a,1) < k_p(s,a,0)$ then $\left[k_p(s,a,1) - \phi_p p_p(s,a,1)\, y_p(s,a,1)\right] < \left[k_p(s,a,0) - \phi_p p_p(s,a,0)\, y_p(s,a,0)\right]$ which requires $\phi_g p_g(s,a,1)\, y_g(s,a,1) < k_g(s,a,1)$. ■

Proposition 13 *When $\phi_g \leq \phi_p$, a procurement shock for constrained firms generates a private sector negative spillover, that is, $k_p(s,a,1) < k(s,a,0)$, makes the firm more constrained, that is, $\lambda(s,a,1) > \lambda(s,a,0)$, and production in the government sector cannot be self-financed, that is, $\phi_g p_g(s,a,1)\, y_g(s,a,1) < k_g(s,a,1)$. When $\phi_g > \phi_p$ the same will happen, with the exception of firms with very small net worth for which the opposite will happen.*

Proof: To prove the first part, let's rewrite the borrowing constraint in (15) for $d = 0$ firms as

$$1 = \phi_a \frac{a}{k(s,a,0)} + \phi_p \frac{p_p(s,a,0)\, y_p(s,a,0)}{k(s,a,0)} \tag{B.34}$$

and for $d = 1$ firms as

$$\begin{aligned} 1 &= \phi_a \frac{a}{k_p(s,a,1) + k_g(s,a,1)} + \phi_p \frac{p_p(s,a,1)\, y_p(s,a,1)}{k_p(s,a,1)} \\ &+ (1 - u(s,a,1)) \left[\phi_g \frac{p_g(s,a,1)\, y_g(s,a,1)}{k_g(s,a,1)} - \phi_p \frac{p_p(s,a,1)\, y_p(s,a,1)}{k_p(s,a,1)}\right] \end{aligned} \tag{B.35}$$

If $\phi_g = \phi_p$, firms with $d = 1$ equalize the average product in the public and private sectors, see equations (B.3) and (B.4), so that the third term in equation (B.35) disappears. In this case, if $k_g(s,a,1) = 0$ then equations (B.34) and (B.35) are identical and $k_p(s,a,1) = k(s,a,0)$. However, because the marginal revenue product in the public sector goes to infinity when $k_g(s,a,1) = 0$, it must be that $k_g(s,a,1) > 0$ and hence comparison of equations (B.34) and (B.35) requires $k_p(s,a,1) < k(s,a,0)$. If $\phi_g < \phi_p$, then the third term in equation (B.35) is negative. This can be easily seen by multiplying both sides of equation (B.3) by ϕ_p and both sides of equation (B.4) by ϕ_g. Then whenever $k_g > 0$ and hence $(1-u) > 0$, equation (B.35) requires $k_p(s,a,1) < k(s,a,0)$ to hold. The second and third parts of the Proposition come from Lemma 6 and Lemma 7 respectively. Finally, for the case $\phi_g > \phi_p$ the third term in equation (B.35) is positive. If $a = 0$ this requires $k_p(s,a,1) > k(s,a,0)$ for equation (B.35) to hold as the first term in the right hand side of equation (B.35) disappears. For $a > 0$, the first term in the right hand side of equation (B.35) reappears and offsets this force. More specifically, as a increases, λ falls by Proposition 10, and thus $\frac{p_g(s,a,1)y_g(s,a,1)}{k_g(s,a,1)}$ decreases. In the limit, if a becomes sufficiently large and exceeds the net worth level $a_g^*(s)$ above which the procurement firm is unconstrained, $\frac{p_g(s,a,1)y_g(s,a,1)}{k_g(s,a,1)}$ falls to the unconstrained level of $\frac{\sigma}{\sigma-1}(r+\delta)$, which is strictly smaller than $\frac{1}{\phi_g}$ by Assumption 1. This means that there exists a cutoff level $\bar{a}_g(s)$ such that if $a \in (\bar{a}_g(s), a_g^*(s))$, then $\phi_g p_g(s,a,1)y_g(s,a,1) < k_g(s,a,1)$. And by Lemma 7, this means that the spillover is negative for a in this interval. ■

Proposition 14 *Whenever $\phi_g \geq \phi_p > 0$ having access to procurement always generates an increase in firm size, that is, $k(s,a,1) > k(s,a,0)$ $\forall a, s$. Whenever $\phi_g < \phi_p$ the opposite may happen. In the particular case that $\phi_g = \phi_p = 0$ a procurement shock does not change the size of the firm.*

Proof: We prove the $\phi_g \geq \phi_p > 0$ case by contradiction by showing that if $k(s,a,1) \leq k(s,a,0)$, then the borrowing constraint for the firm with $d = 1$ would not bind, which could not be optimal; so it must be that $k(s,a,1) > k(s,a,0)$. To se why, we start with the case $k(s,a,1) = k(s,a,0)$. In this situation, the firm with $d = 1$ optimally chooses $u(s,a,1) < 1$ because the marginal revenue product of revenues in the public sector tend to infinity as u tends to 1. This generates more revenues and because $\phi_g \geq \phi_p > 0$, Lemma 4 guarantees that this also generates more (unused) borrowing capacity, so it cannot be optimal. If $k(s,a,1) < k(s,a,0)$ and $u(s,a,1) = 1$ this again generates slack in the borrowing constraint because of Lemma 3, and cannot be optimal. But lowering u generates the same or further slack when $\phi_g \geq \phi_p > 0$, see Lemma 4. So $k(s,a,1) < k(s,a,0)$ cannot be optimal either. Note that the argument by contradiction requires that $\phi_g \geq \phi_p > 0$ such that when the firm with $d = 1$ substitutes private revenues with public revenues the borrowing capacity increases. When $\phi_g < \phi_p$, instead, the contrary happens because selling to the government limits the borrowing capacity of the firm, and the proof does not hold. For example, it can be shown that with $0 = \phi_g < \phi_p$ we will have $k(s,a,1) < k(s,a,0)$. Using the financial constraint, the difference in the capital that can be financed with $d = 1$ and $d = 0$ when $\phi_g = 0$ is given by,

$$k(s,a,1) - k(s,a,0) = \phi_p \left[p_p(s,a,1)\, y_p(s,a,1) - p_p(s,a,0)\, y_p(s,a,0)\right]$$

Proposition 13 says that there is a negative private sector spillover, that is $p_p(s,a,1)\, y_p(s,a,1) < p_p(s,a,0)\, y_p(s,a,0)$, whenever $\phi_g < \phi_p$, so we will have $k(s,a,1) < k(s,a,0)$. Finally, note that with $\phi_g = \phi_p = 0$, $k(s,a,1) = k(s,a,0)$ as capital for constrained firms is determined only by a. ■

Proposition 15 *Having access to procurement always generates extra profits, that is, $\pi(s,a,1) > \pi(s,a,0)$ $\forall s, a$. Whenever $\phi_g \leq \phi_p$, the value of procurement is increasing in net worth; whenever $\phi_g > \phi_p$, the value of procurement is generally increasing in net worth except for firms with very low net worth when the opposite will happen. The value of procurement is increasing in firm productivity whenever $\phi_g \geq \phi_p$.*

Proof: The first part is trivial. A firm with $d = 1$ has profits equal to

$$\pi(s,a,1) = p_p(s,a,1)\,y_p(s,a,1) + p_g(s,a,1)\,y_g(s,a,1) - (r+\delta)\,k(s,a,1)$$

and can always replicate the profits of a firm with $d = 0$ by choosing $u(s,a,1) = 1$. Because of our functional form assumptions, the marginal revenue product of capital in the public sector, $\partial p_g y_g / \partial k_g$, tends to infinity whenever $u(s,a,1) = 1$, so it means that it is optimal for any firm with $d = 1$ to choose $u(s,a,1) < 1$ and increase profits compared to the case $u(s,a,1) = 1$ and therefore compared to the case of no procurement. For the second part we want to show that $\frac{[\partial\pi(s,a,1) - \partial\pi(s,a,0)]}{\partial a} > 0$. Equations (B.18) and (B.28) imply

$$\frac{\partial\left[\pi(s,a,1) - \pi(s,a,0)\right]}{\partial a} = \phi_a\big[\lambda(s,a,1) - \lambda(s,a,0)\big] > 0$$

and the sign of $\lambda(s,a,1) - \lambda(s,a,0)$ is given by Proposition 13. Finally, for the third part we want to show that $\frac{[\partial\pi(s,a,1) - \partial\pi(s,a,0)]}{\partial s} > 0$ whenever $\phi_g \geq \phi_p$. Equations (B.19) and (B.29) imply

$$\frac{[\partial\pi(s,a,1) - \partial\pi(s,a,0)]}{\partial s} = (1+\phi_p\lambda(s,a,1))\frac{\partial p_p y_p}{\partial s} + (1+\phi_g\lambda(s,a,1))\frac{\partial p_g y_g}{\partial s} - (1+\phi_p\lambda(s,a,0))\frac{\partial p_p y_p}{\partial s}$$

Note that $\frac{\partial p_p y_p}{\partial s} = \frac{k_p}{s}\frac{\partial p_p y_p}{\partial k_p} = \frac{k_p}{s}\frac{r+\delta+\lambda}{1+\lambda\phi_p}$ and an analogous expression holds for the public good. Substituting these expressions in the above equation gives

$$\frac{[\partial\pi(s,a,1) - \partial\pi(s,a,0)]}{\partial s} = \frac{r+\delta+\lambda(s,a,1)}{s}\left[k_p(s,a,1) + k_g(s,a,1)\right] - \frac{r+\delta+\lambda(s,a,0)}{s}k_p(s,a,0)$$

With $\phi_g \geq \phi_p$, Proposition 14 states that $k_p(s,a,1) + k_g(s,a,1) > k_p(s,a,0)$. Therefore, whenever $\lambda(s,a,1) > \lambda(s,a,0)$ we can guarantee that $\frac{[\partial\pi(s,a,1) - \partial\pi(s,a,0)]}{\partial s} > 0$. According to Proposition 13 this will generally happen, except for very low a when $\lambda(s,a,1) < \lambda(s,a,0)$. However, in this case we can still show the statement to be true by showing that $\frac{r+\delta+\lambda(s,a,1)}{s}k_p(s,a,1) > \frac{r+\delta+\lambda(s,a,0)}{s}k_p(s,a,0)$. To show this, we take the FOC for k_p in equation (B.1) to obtain an expression for λ as:

$$\lambda = \frac{\frac{\partial p_p y_p}{\partial k_p} - (r+\delta)}{1 - \phi_p\frac{\partial p_p y_p}{\partial k_p}}$$

Then adding $(r+\delta)$ in both sides, rearranging, and multiplying by k_p in both sides we obtain

$$(r+\delta+\lambda)k_p = \left[1 - \phi_p(r+\delta)\right]\frac{\frac{\partial p_p y_p}{\partial k_p}}{1 - \phi_p\frac{\partial p_p y_p}{\partial k_p}}k_p$$

Using our functional form for the revenue function, we can rewrite the last terms as:

$$\frac{\frac{\partial p_p y_p}{\partial k_p}}{1 - \phi_p\frac{\partial p_p y_p}{\partial k_p}}k_p = \frac{\sigma-1}{\sigma}\frac{B_p s^{\frac{\sigma-1}{\sigma}}k_p^{\frac{\sigma-1}{\sigma}}}{1 - \phi_p\frac{\sigma-1}{\sigma}B_p s^{\frac{\sigma-1}{\sigma}}k_p^{\frac{-1}{\sigma}}}$$

Taking the derivative of this object w.r.t. k_p, we have:

$$\begin{aligned}
\frac{\partial}{\partial k_p}\left(\frac{\frac{\partial p_p y_p}{\partial k_p}}{1-\phi_p\frac{\partial p_p y_p}{\partial k_p}}k_p\right) &\propto \frac{\sigma-1}{\sigma}B_p s^{\frac{\sigma-1}{\sigma}}k_p^{\frac{-1}{\sigma}}\left[1-\phi_p\frac{\sigma-1}{\sigma}B_p s^{\frac{\sigma-1}{\sigma}}k_p^{\frac{-1}{\sigma}}\right]-B_p s^{\frac{\sigma-1}{\sigma}}k_p^{\frac{\sigma-1}{\sigma}}\frac{\sigma-1}{\sigma}\frac{1}{\sigma}\phi_p B_p s^{\frac{\sigma-1}{\sigma}}k_p^{\frac{-1}{\sigma}-1}\\
&= \frac{\sigma-1}{\sigma}B_p s^{\frac{\sigma-1}{\sigma}}k_p^{\frac{-1}{\sigma}}\left\{\left[1-\phi_p\frac{\sigma-1}{\sigma}B_p s^{\frac{\sigma-1}{\sigma}}k_p^{\frac{-1}{\sigma}}\right]-\phi_p\frac{1}{\sigma}B_p s^{\frac{\sigma-1}{\sigma}}k_p^{\frac{-1}{\sigma}}\right\}\\
&= \frac{\sigma-1}{\sigma}B_p s^{\frac{\sigma-1}{\sigma}}k_p^{\frac{-1}{\sigma}}\left\{1-\phi_p B_p s^{\frac{\sigma-1}{\sigma}}k_p^{\frac{-1}{\sigma}}\right\}\\
&= \frac{\sigma-1}{\sigma}B_p s^{\frac{\sigma-1}{\sigma}}k_p^{\frac{-1}{\sigma}}\left\{1-\phi_p\frac{p_p y_p}{k_p}\right\}>0
\end{aligned}$$

where the last inequality follows from the fact that by Lemma 3, $\phi_p\frac{p_p y_p}{k_p}<1$ for constrained firms. This establishes that for constrained firms, the term $(r+\delta+\lambda)k_p$ must be higher whenever k_p is higher. Therefore, if $\lambda(s,a,1)<\lambda(s,a,0)$, the k_p FOC, implies that $k_p(s,a,1)>k_p(s,a,0)$, which in turns implies $[r+\delta+\lambda(s,a,1)]k_p(s,a,1)>[r+\delta+\lambda(s,a,0)]k_p(s,a,0)$. And trivially, this implies $\frac{r+\delta+\lambda(s,a,1)}{s}\left[k_p(s,a,1)+k_g(s,a,1)\right]-\frac{r+\delta+\lambda(s,a,0)}{s}k_p(s,a,0)$, proving the statement whenever $\phi_g\geq\phi_p$ and $\lambda(s,a,1)<\lambda(s,a,0)$. ■

Appendix C Details on some aggregates

C.1 Sectorial and aggregate TFP

The TFP for the private and public sectors are given by,

$$\text{TFP}_p\equiv\frac{Y_p}{K_p}=\left[\int_{[0,1]}\left(s_i\frac{\overline{\text{MRPK}}_p}{\text{MRPK}_{ip}}\right)^{\sigma_p-1}di\right]^{\frac{1}{\sigma_p-1}},\quad \text{TFP}_g\equiv\frac{Y_g}{K_g}=\left[\int_{I_g}\frac{1}{m_g}\left(s_i\frac{\overline{\text{MRPK}}_g}{\text{MRPK}_{ig}}\right)^{\sigma_g-1}di\right]^{\frac{1}{\sigma_g-1}}\tag{C.1}$$

where

$$\frac{1}{\overline{\text{MRPK}}_p}\equiv\int_{[0,1]}\frac{p_{ip}y_{ip}}{P_pY_p}\frac{1}{\text{MRPK}_{ip}}di,\quad \frac{1}{\overline{\text{MRPK}}_g}\equiv\int_{I_g}\frac{1}{m_g}\frac{p_{ig}y_{ig}}{P_gY_g}\frac{1}{\text{MRPK}_{ig}}di\tag{C.2}$$

Then aggregate TFP $=(Y_p+P_gY_g)/(K_p+K_g)$ in units of the private sector good is given by the weighted average

$$\text{TFP}=\text{TFP}_p\frac{K_p}{K_p+K_g}+P_g\text{TFP}_g\frac{K_g}{K_p+K_g}\tag{C.3}$$

Finally, absent financial frictions there would be no heterogeneity in $\overline{\text{MRPK}}_p$ and $\overline{\text{MRPK}}_g$ and optimal TFP in the private and public sectors (conditional on selection) would be,

$$\text{TFP}_p^*=\left[\int_{[0,1]}s_i^{\sigma_p-1}di\right]^{\frac{1}{\sigma_p-1}}\quad\text{and}\quad\text{TFP}_g^*=\left[\int_{I_g}\frac{1}{m_g}s_i^{\sigma_g-1}di\right]^{\frac{1}{\sigma_g-1}}\tag{C.4}$$

C.2 Relative price of public sector good

Using the definitions of P_g and P_p in equations (9), the relative price can be written as,

$$\frac{P_g}{P_p} = \frac{\left[\int_{I_g} \frac{1}{m_g} p_{ig}^{1-\sigma_g} di\right]^{\frac{1}{1-\sigma_g}}}{\left[\int_{[0,1]} \ p_{ip}^{1-\sigma_p} di\right]^{\frac{1}{1-\sigma_p}}} = \frac{\left[\int_{I_g} \frac{1}{m_g} \left(\frac{1}{s_i}\mathrm{MRPK}_{ig}\right)^{1-\sigma_g} di\right]^{\frac{1}{1-\sigma_g}}}{\left[\int_{[0,1]} \ \left(\frac{1}{s_i}\mathrm{MRPK}_{ip}\right)^{1-\sigma_p} di\right]^{\frac{1}{1-\sigma_p}}}$$

where the last equality follows from using the definition of MRPK_{ip} and the production function as follows,

$$\mathrm{MRPK}_{ip} \equiv \frac{\partial p_{ip} y_{ip}}{\partial k_{ip}} = \frac{\sigma_p - 1}{\sigma_p}\frac{p_{ip} y_{ip}}{k_{ip}} = \frac{\sigma_p - 1}{\sigma_p} p_{ip} s_i \quad \Rightarrow \quad p_{ip} = \frac{\sigma_p}{\sigma_p - 1}\frac{1}{s_i}\mathrm{MRPK}_{ip}$$

and the same applies for MRPK_{ig}. Next multiplying and dividing by $\overline{\mathrm{MRPK}}_g$ in the numerator and by $\overline{\mathrm{MRPK}}_p$ in the denominator we obtain,

$$\frac{P_g}{P_p} = \frac{\overline{\mathrm{MRPK}}_g}{\overline{\mathrm{MRPK}}_p} \frac{\left[\int_{I_g} \frac{1}{m_g} \left(\frac{1}{s_i}\frac{\overline{\mathrm{MRPK}}_g}{\mathrm{MRPK}_{ig}}\right)^{1-\sigma_g} di\right]^{\frac{1}{\sigma_g - 1}}}{\left[\int_{[0,1]} \ \left(\frac{1}{s_i}\frac{\overline{\mathrm{MRPK}}_p}{\mathrm{MRPK}_{ip}}\right)^{\sigma_p - 1} di\right]^{\frac{1}{1-\sigma_p}}} = \frac{\overline{\mathrm{MRPK}}_p}{\overline{\mathrm{MRPK}}_g}\frac{\mathrm{TFP}_p}{\mathrm{TFP}_g}$$

C.3 Relative sectoral TFP

Given the definition of TFP_p in equation (C.1), we can write

$$\begin{aligned}
\mathrm{TFP}_p &= \left[m_g \int_{I_g} \frac{1}{m_g}\left(s_i \frac{\overline{\mathrm{MRPK}}_p}{\mathrm{MRPK}_{ip}}\right)^{\sigma_p - 1} di + (1 - m_g) \int_{I_g^c} \frac{1}{1 - m_g}\left(s_i \frac{\overline{\mathrm{MRPK}}_p}{\mathrm{MRPK}_{ip}}\right)^{\sigma_p - 1} di\right]^{\frac{1}{\sigma_p - 1}} \\
&= \left[m_g \mathrm{TFP}_{p,I_g}^{\sigma_p - 1} + (1 - m_g)\,\mathrm{TFP}_{p,I_g^c}^{\sigma_p - 1}\right]^{\frac{1}{\sigma_p - 1}}
\end{aligned}$$

where we have defined TFP_{p,I_g} and TFP_{p,I_g^c} as the average TFP in the private sector within the set of procurement (I_g) and non-procurement (I_g^c) firms respectively. Then, dividing by TFP_g in both sides we get the expression for $\mathrm{TFP}_p/\mathrm{TFP}_g$:

$$\frac{\mathrm{TFP}_p}{\mathrm{TFP}_g} = \left[m_g\left(\frac{\mathrm{TFP}_{p,I_g}}{\mathrm{TFP}_g}\right)^{\sigma_p - 1} + (1 - m_g)\left(\frac{\mathrm{TFP}_{p,I_g^c}}{\mathrm{TFP}_g}\right)^{\sigma_p - 1}\right]^{\frac{1}{\sigma_p - 1}} \tag{C.5}$$

The first term in equation (C.5) reflects the within-firm misallocation. With $\sigma_g = \sigma_p$ this term would be equal to 1 if $\phi_g = \phi_p$ or if there were no financial frictions ($\lambda_i = 0 \ \forall i$). Instead, if $\phi_g > \phi_p$ firms switch their output relatively towards the public sector and the dispersion of MRPK_{ig} declines, which makes $\mathrm{TFP}_{p,I_g}/\mathrm{TFP}_g$ fall. The second term in equation (C.5) reflects both between-firm misallocation and selection into procurement. If firms with higher s self-select into procurement, then $\mathrm{TFP}_{p,I_g^c}/\mathrm{TFP}_g$ declines. If there is more dispersion in MRPK_{ip} between non-procurement firms than in MRPK_{ig} between procurement firms, then $\text{TFP}_{p,I_g^c}/\text{TFP}_g$ is lower. In short, absent financial frictions the only reason for $\text{TFP}_p/\text{TFP}_g \neq 1$ would be the selection of firms into procurement. In the first best (no financial frictions and the government selects the firms with highest s) we would have $\text{TFP}_p/\text{TFP}_g < 1$.

C.4 Relative sectoral $\overline{\text{MRPK}}$

Given the definition of $\overline{\text{MRPK}}_p$ in equation (C.2), we can write

$$\begin{aligned}
\overline{\text{MRPK}}_p &= \left[\frac{R_{p,I_g}}{P_p Y_p} \int_{I_g} \frac{p_{ip} y_{ip}}{R_{p,I_g}} \text{MRPK}_{ip}^{-1} di + \frac{R_{p,I_g^c}}{P_p Y_p} \int_{I_g^c} \frac{p_{ip} y_{ip}}{R_{p,I_g^c}} \text{MRPK}_{ip}^{-1} di \right]^{-1} \\
&= \left[\frac{R_{p,I_g}}{P_p Y_p} \overline{\text{MRPK}}_{p,I_g}^{-1} + \frac{R_{p,I_g^c}}{P_p Y_p} \overline{\text{MRPK}}_{p,I_g^c}^{-1} \right]^{-1}
\end{aligned}$$

where R_{p,I_g} and R_{p,I_g^c} denote total revenues in the private sector by procurement firms and non-procurement firms respectively. Then, dividing by $\overline{\text{MRPK}}_g$ in both sides we obtain the expression for $\overline{\text{MRPK}}_p/\overline{\text{MRPK}}_g$

$$\frac{\overline{\text{MRPK}}_p}{\overline{\text{MRPK}}_g} = \left[\frac{R_{p,I_g}}{P_p Y_p} \left(\frac{\overline{\text{MRPK}}_{p,I_g}}{\overline{\text{MRPK}}_g} \right)^{-1} + \frac{R_{p,I_g^c}}{P_p Y_p} \left(\frac{\overline{\text{MRPK}}_{p,I_g^c}}{\overline{\text{MRPK}}_g} \right)^{-1} \right]^{-1} \tag{C.6}$$

Whenever $\overline{\text{MRPK}}_p \neq \overline{\text{MRPK}}_g$ there is misallocation of capital across sectors. The first term in equation (C.6) reflects the effects of within-firm misallocation on this between-sector misallocation. With $\sigma_g = \sigma_p$ this term would be equal to 1 if $\phi_g = \phi_p$ or if there were no financial frictions ($\lambda_i = 0 \forall i$). Instead, if $\phi_g > \phi_p$ firms switch their output relatively towards the public sector and hence $\overline{\text{MRPK}}_{p,I_g} > \overline{\text{MRPK}}_g$. The second term in equation (C.6) reflects both between-firm misallocation and selection into procurement.