Online Appendix

State Dependence of Fiscal Multipliers: The Source of Fluctuations Matters

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Appendix A. Proofs of results in main text

Lemma 1. *In a competitive equilibrium, the comparative statics of tightness* (x)*, sales* (y) *and the price* (p) *are:* $\frac{dx}{d\chi} = 0, \frac{dy}{d\chi} > 0, \frac{dp}{d\chi} > 0;$ $\quad \frac{dx}{da} = 0, \frac{dy}{da} > 0, \frac{dp}{da} < 0.$

Proof. Combining the labor supply function $l(w)$ and the labor demand function $n(p, x, w)$ with the labor market clearing condition delivers the following expression for equilibrium employment:

$$l = n = (\alpha p f(x) a)^{\frac{1}{1-\alpha+\psi}} (1+\tau)^{-\frac{1}{1-\alpha+\psi}}. \tag{A.1}$$

Inserting equilibrium employment level into goods market clearing condition:

$$\frac{f(x)}{1+\gamma(x)} a \left[(\alpha p f(x) a)^{\frac{1}{1-\alpha+\psi}} (1+\tau)^{-\frac{1}{1-\alpha+\psi}} \right]^{\alpha} = c(p, x) + G, \tag{A.2}$$

$$\frac{f(x)}{1+\gamma(x)} a \left[(\alpha p f(x) a)^{\frac{1}{1-\alpha+\psi}} (1+\tau)^{-\frac{1}{1-\alpha+\psi}} \right]^{\alpha} = \frac{\chi}{p[1+\gamma(x)]} + G, \tag{A.3}$$

$$p f(x) a \alpha^{\frac{\alpha}{1-\alpha+\psi}} (p f(x) a)^{\frac{\alpha}{1-\alpha+\psi}} (1+\tau)^{-\frac{\alpha}{1-\alpha+\psi}} = \chi + p[1+\gamma(x)]G \tag{A.4}$$

$$\alpha^{\frac{\alpha}{1-\alpha+\psi}} (p f(x) a)^{\frac{1+\psi}{1-\alpha+\psi}} (1+\tau)^{-\frac{\alpha}{1-\alpha+\psi}} = \chi + p[1+\gamma(x)]G. \tag{A.5}$$

By definition of the competitive equilibrium, $x = x^*$, and so $\frac{dx}{d\chi} = \frac{dx}{da} = 0$. The latter implies the following comparative statics for p (for simplicity, evaluated at $G = \tau = 0$):

$$\alpha^{\frac{\alpha}{1-\alpha+\psi}} (f(x)a)^{\frac{1+\psi}{1-\alpha+\psi}} \frac{1+\psi}{1-\alpha+\psi} p^{\frac{\alpha}{1-\alpha+\psi}} \frac{dp}{d\chi} = 1 \tag{A.6}$$

$$\frac{dp}{d\chi} = \alpha^{-\frac{\alpha}{1-\alpha+\psi}} (f(x)a)^{-\frac{1+\psi}{1-\alpha+\psi}} \frac{1-\alpha+\psi}{1+\psi} p^{-\frac{\alpha}{1-\alpha+\psi}} = \frac{1-\alpha+\psi}{1+\psi} \frac{p}{\chi} > 0. \tag{A.7}$$

$$\tag{A.8}$$

$$\alpha^{\frac{\alpha}{1-\alpha+\psi}} (f(x))^{\frac{1+\psi}{1-\alpha+\psi}} \frac{1+\psi}{1-\alpha+\psi} (pa)^{\frac{\alpha}{1-\alpha+\psi}} (p + a\frac{dp}{da}) = 0 \tag{A.9}$$

$$\frac{dp}{da} = -\frac{p}{a} < 0. \tag{A.10}$$

Finally, the above implies the following comparative statics for sales $y = [1+\gamma(x)](c(p, x) + G) =$

$\chi/p + G[1 + \gamma(x)]$ (also evaluated at $G = \tau = 0$):

$$py = \chi + p[1 + \gamma(x)]G \tag{A.11}$$

$$\frac{dp}{d\chi}y + p\frac{dy}{d\chi} = 1 \tag{A.12}$$

$$\frac{dy}{d\chi} = \frac{1}{p}\left(1 - \frac{dp}{d\chi}y\right) = \frac{1}{p}\frac{\alpha}{1+\psi} > 0. \tag{A.13}$$

$$\frac{dp}{da}y + p\frac{dy}{da} = 0 \tag{A.14}$$

$$\frac{dy}{da} = -\frac{dp}{da}\frac{y}{p} > 0. \tag{A.15}$$

□

Lemma 2. *In a fixprice equilibrium, the comparative statics of tightness* (x)*, sales* (y)*, and the price* (p) *are:* $\frac{dx}{d\chi} > 0, \frac{dy}{d\chi} > 0, \frac{dp}{d\chi} = 0; \qquad \frac{dx}{da} < 0, \frac{dy}{da} = 0, \frac{dp}{da} = 0.$

Proof. Condition (A.5) remains unchanged in a fixprice equilibrium. However, now the price is a parameter, so that $\frac{dp}{d\chi} = \frac{dp}{da} = 0$. The latter implies the following comparative statics for x (for simplicity, evaluated at $G = \tau = 0$):

$$\alpha^{\frac{\alpha}{1-\alpha+\psi}}(pa)^{\frac{1+\psi}{1-\alpha+\psi}}\frac{1+\psi}{1-\alpha+\psi}f(x)^{\frac{\alpha}{1-\alpha+\psi}}f'(x)\frac{dx}{d\chi} = 1 \tag{A.16}$$

$$\frac{dx}{d\chi} = \alpha^{-\frac{\alpha}{1-\alpha+\psi}}(f(x)a)^{-\frac{1+\psi}{1-\alpha+\psi}}\frac{1-\alpha+\psi}{1+\psi}f(x)^{-\frac{\alpha}{1-\alpha+\psi}}\frac{1}{f'(x)} > 0. \tag{A.17}$$

$$\alpha^{\frac{\alpha}{1-\alpha+\psi}}(p)^{\frac{1+\psi}{1-\alpha+\psi}}\frac{1+\psi}{1-\alpha+\psi}(f(x)a)^{\frac{\alpha}{1-\alpha+\psi}}(f(x) + af'(x)\frac{dx}{da}) = 0 \tag{A.18}$$

$$\frac{dx}{da} = -\frac{f(x)}{a}\frac{1}{f'(x)} < 0. \tag{A.19}$$

The above implies the following comparative statics for sales $y = [1 + \gamma(x)](c(p, x) + G) = \chi/p +$

$G[1+\gamma(x)]$ (also evaluated at $G=\tau=0$):

$$py = \chi + p[1+\gamma(x)]G \tag{A.20}$$

$$p\frac{dy}{d\chi} = 1 \tag{A.21}$$

$$\frac{dy}{d\chi} = \frac{1}{p} > 0. \tag{A.22}$$

$$p\frac{dy}{da} = 0 \tag{A.23}$$

$$\frac{dy}{da} = 0. \tag{A.24}$$

□

Proposition 1. *In a competitive equilibrium, the demand- and supply-side fiscal multipliers are equal and given by:*

$$\varphi^* \equiv \frac{\alpha}{1+\psi} = \frac{1-\frac{1}{|\epsilon^d|}}{1+\frac{1}{\epsilon^s}}, \tag{A.25}$$

where $|\epsilon^d| = \frac{1}{1-\alpha}$ *and* $\epsilon^s = \frac{1}{\psi}$ *are (absolute) elasticities of labor demand and labor supply.*

Proof. First differentiate (A.5) with respect to G (evaluated at $G=\tau=0$):

$$\alpha^{\frac{\alpha}{1-\alpha+\psi}} \frac{1+\psi}{1-\alpha+\psi} (pf(x)a)^{\frac{1+\psi}{1-\alpha+\psi}-1} \left[\frac{dp}{dG} f(x)a + pf'(x)\frac{dx}{dG}a\right] = p[1+\gamma(x)], \tag{A.26}$$

$$\alpha^{\frac{\alpha}{1-\alpha+\psi}} \frac{1+\psi}{1-\alpha+\psi} (pf(x)a)^{\frac{1+\psi}{1-\alpha+\psi}} \left[\frac{1}{p}\frac{dp}{dG} + \frac{f'(x)}{f(x)}\frac{dx}{dG}\right] = p[1+\gamma(x)], \tag{A.27}$$

$$\left[\frac{1}{p}\frac{dp}{dG} + \frac{f'(x)}{f(x)}\frac{dx}{dG}\right] = \frac{p[1+\gamma(x)]}{\frac{1+\psi}{1-\alpha+\psi}\underbrace{\alpha^{\frac{\alpha}{1-\alpha+\psi}}(pf(x)a)^{\frac{1+\psi}{1-\alpha+\psi}}}_{=\chi (\text{by (67) under } G=\tau=0)}}, \tag{A.28}$$

$$\left[\frac{1}{p}\frac{dp}{dG} + \frac{f'(x)}{f(x)}\frac{dx}{dG}\right] = \frac{p[1+\gamma(x)]}{\frac{1+\psi}{1-\alpha+\psi}\chi} \tag{A.29}$$

$$\left[\frac{1}{p}\frac{dp}{dG} + \frac{f'(x)}{f(x)}\frac{dx}{dG}\right] = \frac{1-\alpha+\psi}{1+\psi}\frac{1}{c(p,x)}. \tag{A.30}$$

From definition of demand-side fiscal multiplier:

$$\varphi^d = \frac{d\{c+G\}}{dG} = \frac{dc}{dG} + 1 = \frac{\partial c}{\partial p}\frac{dp}{dG} + \frac{\partial c}{\partial x}\frac{dx}{dG} + 1 \tag{A.31}$$

In a competitive equilibrium, $x = x^*$, so that $\frac{dx}{dG} = 0$, which combined with (A.29) implies the following:

$$\varphi^d = \frac{\partial c}{\partial p}\frac{dp}{dG} + 1 = -\frac{\chi}{p[1+\gamma(x)]}\frac{1}{p}\frac{dp}{dG} + 1 = -c(p,x)\frac{1}{p}p\frac{1-\alpha+\psi}{1+\psi}\frac{1}{c(p,x)} + 1 \tag{A.32}$$

$$= -\frac{1-\alpha+\psi}{1+\psi} + 1 \tag{A.33}$$

$$= \frac{\alpha}{1+\psi} \equiv \varphi^*. \tag{A.34}$$

Similarly, differentiate (A.5) with respect to τ (evaluated at $G = \tau = 0$):

$$\alpha^{\frac{\alpha}{1-\alpha+\psi}}\left[\frac{1+\psi}{1-\alpha+\psi}(pf(x)a)^{\frac{1+\psi}{1-\alpha+\psi}-1}\left(\frac{dp}{d\tau}f(x)a + pf'(x)\frac{dx}{d\tau}a\right) - \frac{\alpha}{1-\alpha+\psi}(-1)(pf(x)a)^{\frac{1+\psi}{1-\alpha+\psi}}\right] = 0 \tag{A.35}$$

$$\left[\frac{1}{p}\frac{dp}{d\tau} + \frac{f'(x)}{f(x)}\frac{dx}{d\tau}\right] = \frac{\alpha}{1+\psi}. \tag{A.36}$$

From definition of supply-side fiscal multiplier:

$$\varphi^s = \frac{d\{c+G\}/\{c+G\}}{d[-\tau]} = -\frac{1}{c}\frac{dc}{d\tau} = -\frac{1}{c}\left[\frac{\partial c}{\partial p}\frac{dp}{d\tau} + \frac{\partial c}{\partial x}\frac{dx}{d\tau}\right]. \tag{A.37}$$

In a competitive equilibrium, $x = x^*$, so that $\frac{dx}{d\tau} = 0$, which combined with (A.36) implies the following:

$$\varphi^s = -\frac{1}{c}\frac{\partial c}{\partial p}\frac{dp}{d\tau} = -\frac{1}{c}\left[-\frac{\chi}{p[1+\gamma(x)]}\frac{1}{p}\right]\frac{dp}{d\tau} = \frac{1}{c}c\frac{1}{p}p\frac{\alpha}{1+\psi} \tag{A.38}$$

$$= \frac{\alpha}{1+\psi} = \varphi^d = \varphi^*. \tag{A.39}$$

□

Corollary 1. *In a competitive equilibrium, both demand- and supply-side multipliers are acyclical.*

Proof. A trivial consequence of Proposition 1: in a competitive equilibrium, both multipliers are equal to $\varphi^* = \frac{\alpha}{1+\psi}$ and do not change when either preference χ or technology a varies. □

Lemma 3. *Define the fixed capacity fiscal multiplier $\theta(x)$ to be the demand-side fiscal multiplier in a fixprice equilibrium under fixed labor supply, so that*

$$\theta(x) \equiv \frac{d\{c+G\}}{dG}|_{\psi\to\infty}$$

then $\theta(x)$ has the following properties:

$$\theta(x) = \begin{cases} (-\infty, 0), & \text{if} \quad x \in (x^*, x_m) \\ 0, & \text{if} \quad x = x^* \\ (0,1), & \text{if} \quad x \in (0, x^*) \end{cases}$$

$$\theta'(x) < 0, \qquad \forall x \in (0, x_m),$$

where x_m is given by $f(x_m) = \rho x_m$.

Proof. Under $\psi \to \infty$, (A.30) can be written as:

$$\lim_{\psi\to\infty}\left[\frac{1}{p}\frac{dp}{dG} + \frac{f'(x)}{f(x)}\frac{dx}{dG}\right] = \lim_{\psi\to\infty}\frac{1-\alpha+\psi}{1+\psi}\frac{1}{c(p,x)}. \tag{A.40}$$

$$\lim_{\psi\to\infty}\left[\frac{1}{p}\frac{dp}{dG} + \frac{f'(x)}{f(x)}\frac{dx}{dG}\right] = \lim_{\psi\to\infty}\frac{1}{c(p,x)} = \frac{1}{\lim_{\psi\to\infty} c(p,x)}. \tag{A.41}$$

From definition of $\theta(x)$:

$$\theta(x) \equiv \frac{d\{c+G\}}{dG}|_{\psi\to\infty} = \lim_{\psi\to\infty}\frac{dc}{dG} + 1 = \lim_{\psi\to\infty}\left[\frac{\partial c}{\partial p}\frac{dp}{dG} + \frac{\partial c}{\partial x}\frac{dx}{dG}\right] + 1 \tag{A.42}$$

In a fixprice equilibrium $p = p_0$ is a parameter, so that $\frac{dp}{dG} = 0$, which combined with (A.42) implies the following:

$$\theta(x) = \lim_{\psi\to\infty}\frac{\partial c}{\partial x}\frac{dx}{dG} + 1 = \lim_{\psi\to\infty}\frac{\partial c}{\partial x}\lim_{\psi\to\infty}\frac{dx}{dG} + 1 \tag{A.43}$$

$$= -\lim_{\psi\to\infty}\frac{\chi}{p[1+\gamma(x)]}\frac{\gamma'(x)}{[1+\gamma(x)]}\frac{f(x)}{f'(x)}\frac{1}{\lim_{\psi\to\infty} c(p,x)} + 1 \tag{A.44}$$

$$= 1 - \frac{\gamma'(x)}{[1+\gamma(x)]}\frac{f(x)}{f'(x)}\frac{\lim_{\psi\to\infty} c(p,x)}{\lim_{\psi\to\infty} c(p,x)} \tag{A.45}$$

$$= 1 - \frac{\gamma'(x)}{[1+\gamma(x)]}\frac{f(x)}{f'(x)}. \tag{A.46}$$

Recall that $\gamma(x) \equiv \frac{\rho x}{f(x)-\rho x}$, so that $\gamma'(x) = \frac{\rho(f(x)-\rho x)-(f'(x)-\rho)\rho x}{(f(x)-\rho x)^2} = \frac{\rho(f(x)-f'(x)x)}{(f(x)-\rho x)^2}$, and $\theta(x)$ may be rewritten as:

$$\theta(x) = 1 - \frac{\frac{\rho(f(x)-f'(x)x)}{(f(x)-\rho x)^2}}{\frac{f(x)}{f(x)-\rho(x)}}\frac{f(x)}{f'(x)} = 1 - \frac{\rho(f(x)-f'(x)x)}{(f(x)-\rho x)f(x)}\frac{f(x)}{f'(x)} \tag{A.47}$$

$$= 1 - \frac{\rho(f(x)-f'(x)x)}{f'(x)(f(x)-\rho x)} = \frac{f'(x)f(x)-\rho f(x)}{f'(x)f(x)-f'(x)\rho x} = \frac{1-\frac{\rho}{f'(x)}}{1-\frac{\rho x}{f(x)}}. \tag{A.48}$$

We can now show that $\theta(x)$ possesses several convenient properties. Firstly, $\theta'(x) < 0, \quad \forall x \in (0, x_m)$, where x_m is given by $f(x_m) = \rho x_m$. In order to show this, notice that $q(x) = \frac{f(x)}{x}$ and $f'(x) = q(x)^{1+\delta}$, which allows us to rewrite $\theta(x)$ as follows:

$$\theta(x) = \frac{1 - \frac{\rho}{q(x)^{1+\delta}}}{1 - \frac{\rho}{q(x)}} = \frac{q(x)^{1+\delta} - \rho}{q(x)^{1+\delta} - \rho q(x)^{\delta}}, \tag{A.49}$$

and $\theta'(x)$ is now given by:

$$\theta'(x) = \frac{(1+\delta)q(x)^{\delta}q'(x)[q(x)^{1+\delta} - \rho q(x)^{\delta}] - [(1+\delta)q(x)^{\delta}q'(x) - \delta\rho q(x)^{\delta-1}q'(x)](q(x)^{1+\delta} - \rho)}{(q(x)^{1+\delta} - \rho)^2}. \tag{A.50}$$

Given that $q(x) > 0, q'(x) < 0, (q(x)^{1+\delta} - \rho)^2 > 0, \quad \forall x \in (0, \infty)$, a sufficient condition for $\theta'(x) < 0$ is:

$$(1+\delta)q(x)^{\delta}[q(x)^{1+\delta} - \rho q(x)^{\delta}] - [(1+\delta)q(x)^{\delta} - \delta\rho q(x)^{\delta-1}](q(x)^{1+\delta} - \rho) > 0 \tag{A.51}$$

$$-\rho q(x)^{2\delta} + \rho(1+\delta)q(x)^{\delta} - \delta\rho^2 q(x)^{\delta-1} > 0 \tag{A.52}$$

$$\rho q(x)^{\delta-1}[q(x) - q(x)^{\delta+1}] + \delta[q(x) - \rho] > 0. \tag{A.53}$$

Finally, $q(0) = 1$ and $q(x_m) = \rho$, and since $q'(x) < 0, \forall x \in (0, \infty)$ it follows that $q(x) \in (\rho, 1), \quad \forall x \in (0, x_m)$; it is clear that for all $q(x) \in (\rho, 1)$ the sufficient condition above is satisfied. Hence, $\theta'(x) < 0, \quad \forall x \in (0, x_m)$.

Secondly, it follows directly from (107) that $\theta(x^*) = 0$, since $f'(x^*) = \rho$. At the extremes:

$$\theta(0) = \frac{q(0)^{1+\delta} - \rho}{q(0)^{1+\delta} - \rho q(0)^{\delta}} = \frac{1^{1+\delta} - \rho}{1^{1+\delta} - \rho 1^{\delta}} = 1, \tag{A.54}$$

$$\lim_{x \to x_m^-} \theta(x) = \lim_{h \to 0} \frac{q(x_m - h)^{1+\delta} - \rho}{q(x_m - h)^{1+\delta} - \rho q(x_m - h)^{\delta}} \tag{A.55}$$

$$= \frac{\rho^{1+\delta} - \rho}{\rho^{1+\delta} - \rho\rho^{\delta}} = \frac{\rho(\rho^{\delta} - 1)}{0} = -\infty. \tag{A.56}$$

Since $\theta(0) = 1, \theta(x^*) = 0$ and $\lim_{x \to x_m^-} \theta(x) = -\infty$, and $\theta'(x) < 0, \forall x \in (0, x_m)$ it follows that $\theta(x) \in (0, 1), \forall x \in (0, x^*)$ and $\theta(x) \in (-\infty, 0), \forall x \in (x^*, x_m)$. □

Proposition 2. *In a fixprice equilibrium, the demand-side fiscal multiplier $\varphi^d(x)$ is given by*

$$\varphi^d(x) = \underbrace{\varphi^*}_{\text{State-invariant component}} + \underbrace{\theta(x) \times (1 - \varphi^*)}_{\text{State-dependent component}},$$

where φ^* is the competitive equilibrium multiplier. Hence, $\varphi^d(x) \in (-\infty, 1)$ and $\frac{d\varphi^d(x)}{dx} < 0, \forall x \in (0, x_m)$.

Proof. From (A.30) we know that:

$$\left[\frac{1}{p}\frac{dp}{dG}+\frac{f'(x)}{f(x)}\frac{dx}{dG}\right]=\frac{1-\alpha+\psi}{1+\psi}\frac{1}{c(p,x)}=(1-\varphi^*)\frac{1}{c(p,x)}. \tag{A.57}$$

Further, in a fixprice equilibrium $p=p_0$ is a parameter, so that $\frac{dp}{dG}=0$ and it follows that:

$$\frac{dx}{dG}=(1-\varphi^*)\frac{f(x)}{f'(x)}\frac{1}{c(p,x)}. \tag{A.58}$$

From the definition of the demand-side fiscal multiplier:

$$\varphi^d(x)=\frac{d\{c+G\}}{dG}=\frac{dc}{dG}+1=\frac{\partial c}{\partial x}\frac{dx}{dG}+1=-\frac{\chi}{p[1+\gamma(x)]}\frac{\gamma'(x)}{[1+\gamma(x)]}(1-\varphi^*)\frac{f(x)}{f'(x)}\frac{1}{c(p,x)}+1, \tag{A.59}$$

$$=1-(1-\varphi^*)\underbrace{\frac{\gamma'(x)}{[1+\gamma(x)]}\frac{f(x)}{f'(x)}}_{1-\theta(x)}\frac{c(p,x)}{c(p,x)}=1-(1-\varphi^*)(1-\theta(x)), \tag{A.60}$$

$$=\varphi^*+\theta(x)\times(1-\varphi^*). \tag{A.61}$$

Since $\frac{d\varphi^d(x)}{dx}=\theta'(x)(1-\varphi^*)$ and $\theta'(x)<0, \forall x\in(0,x_m)$ it follows that $\frac{d\varphi^d(x)}{dx}<0, \forall x\in(0,x_m)$. Further, $\varphi^d(0)=\varphi^*+\theta(0)\times(1-\varphi^*)=1$ and $\lim_{x\to x_m^-}\varphi^d(x)=\varphi^*+\lim_{x\to x_m^-}\theta(x)\times(1-\varphi^*)=-\infty$, so that $\varphi^d(x)\in(-\infty,1), \forall x\in(0,x_m)$. □

Corollary 2. *There always exists tightness $\hat{x}\in(x^*,x_m)$, such that $\varphi^d(\hat{x})=0$ and $\varphi^d(x)<0, \forall x\in(\hat{x},x_m)$, and it is equal to:* $\hat{x}=\theta^{-1}\left(-\frac{\varphi^*}{1-\varphi^*}\right)$, *where* $\frac{d\hat{x}}{d\varphi^*}>0$.

Proof. Suppose there exists $\hat{x}\in(0,x_m)$, such that $\varphi^d(\hat{x})=0$; then it should satisfy the following condition:

$$\varphi^d(\hat{x})=\varphi^*+(1-\varphi^*)\times\theta(\hat{x})=0, \tag{A.62}$$

$$\theta(\hat{x})=-\frac{\varphi^*}{1-\varphi^*}. \tag{A.63}$$

We know that $\theta(x)$ lies between $(-\infty,1)$ and is differentiable and strictly decreasing on $(0,x_m)$; hence the inverse function $\theta^{-1}(.)$ exists on $(-\infty,1)$ and returns values in $(0,x_m)$. Moreover, since $-\frac{\varphi^*}{1-\varphi^*}\in(-\infty,1), \forall\varphi^*\in(0,1)$, then $\hat{x}\in(0,x_m)$ always exists and is given by:

$$\hat{x}=\theta^{-1}\left(-\frac{\varphi^*}{1-\varphi^*}\right). \tag{A.64}$$

Since $\varphi^d(x^*)=\varphi^*\in(0,1)$ and $\frac{d\varphi^d(x)}{dx}<0, \forall x\in(0,x_m)$ it must be that $\hat{x}\in(x^*,x_m)$; further, since

$\frac{d\varphi^d(x)}{dx} < 0, \forall x \in (0, x_m)$ and $\varphi^d(\hat{x}) = 0$, it follows that $\varphi^d(x) < 0, \forall x \in (\hat{x}, x_m)$. It is also true that:

$$\theta'(\hat{x})\frac{d\hat{x}}{d\varphi^*} = \frac{d}{d\varphi^*}\left(-\frac{\varphi^*}{1-\varphi^*}\right) = -\frac{1}{(1-\varphi^*)^2}, \tag{A.65}$$

$$\frac{d\hat{x}}{d\varphi^*} = -\frac{1}{\theta'(\hat{x})}\frac{1}{(1-\varphi^*)^2} > 0. \tag{A.66}$$

□

Corollary 3. *In a fixprice equilibrium, the demand-side fiscal multiplier, $\varphi^d(x)$, is countercyclical under demand-side fluctuations and procyclical under supply-side fluctuations.*

Proof. From Lemma 2, we know that in a fixprice equilibrium $\frac{dx}{d\chi} > 0$, $\frac{dx}{da} < 0$; further, from Proposition 2, we know that in a fixprice equilibrium $\frac{d\varphi^d(x)}{dx} < 0, \forall x \in (0, x_m)$. Hence, $\frac{d\varphi^d(x)}{d\chi} = \frac{d\varphi^d(x)}{dx}\frac{dx}{d\chi} < 0, \forall x \in (0, x_m)$ and $\frac{d\varphi^d(x)}{da} = \frac{d\varphi^d(x)}{dx}\frac{dx}{da} > 0, \forall x \in (0, x_m)$. □

Proposition 3. *In a fixprice equilibrium, the supply-side fiscal multiplier $\varphi^s(x)$ is given by*

$$\varphi^s(x) = \underbrace{\varphi^*}_{\text{State-invariant component}} - \underbrace{\theta(x) \times \varphi^*}_{\text{State-dependent component}},$$

where φ^* is the competitive equilibrium multiplier. Hence, $\varphi^d(x) \in (0, +\infty)$ and $\frac{d\varphi^d(x)}{dx} > 0, \forall x \in (0, x_m)$.

Proof. From (A.36) we know that:

$$\left[\frac{1}{p}\frac{dp}{d\tau} + \frac{f'(x)}{f(x)}\frac{dx}{d\tau}\right] = \frac{\alpha}{1+\psi} = \varphi^*. \tag{A.67}$$

In a fixprice equilibrium, $p = p_0$ is a parameter, so that $\frac{dp}{d\tau} = 0$, and it follows that:

$$\frac{dx}{d\tau} = \varphi^* \frac{f(x)}{f'(x)}. \tag{A.68}$$

From the definition of the supply-side multiplier:

$$\varphi^s(x) = \frac{d\{c+G\}/\{c+G\}}{d[-\tau]} = -\frac{1}{c}\frac{dc}{d\tau} = -\frac{1}{c}\frac{\partial c}{\partial x}\frac{dx}{d\tau} = \frac{1}{c}\frac{\chi}{p[1+\gamma(x)]}\underbrace{\frac{\gamma'(x)}{[1+\gamma(x)]}\frac{f(x)}{f'(x)}}_{1-\theta(x)}\varphi^* \tag{A.69}$$

$$= \frac{c}{c}(1-\theta(x))\varphi^* = \varphi^* - \theta(x) \times \varphi^*. \tag{A.70}$$

Since $\frac{d\varphi^s(x)}{dx} = -\theta'(x)\varphi^*$ and $\theta'(x) < 0, \forall x \in (0, x_m)$ it follows that $\frac{d\varphi^s(x)}{dx} > 0, \forall x \in (0, x_m)$. Further, $\varphi^s(0) = \varphi^* - \theta(0) \times \varphi^* = 0$ and $\lim_{x \to x_m^-} \varphi^s(x) = \varphi^* - \lim_{x \to x_m^-} \theta(x) \times \varphi^* = \infty$, so that $\varphi^s(x) \in (0, \infty), \forall x \in (0, x_m)$. □

Corollary 4. *In a fixprice equilibrium, the supply-side fiscal multiplier, $\varphi^s(x)$, is procyclical under demand-side fluctuations and countercyclical under supply-side fluctuations.*

Proof. From Lemma 2, we know that in a fixprice equilibrium $\frac{dx}{d\chi} > 0, \frac{dx}{da} < 0$; further, from Proposition 3, we know that in a fixprice equilibrium $\frac{d\varphi^s(x)}{dx} > 0$. Hence, $\frac{d\varphi^s(x)}{d\chi} = \frac{d\varphi^s(x)}{dx}\frac{dx}{d\chi} > 0, \forall x \in (0, x_m)$ and $\frac{d\varphi^s(x)}{da} = \frac{d\varphi^s(x)}{dx}\frac{dx}{da} < 0, \forall x \in (0, x_m)$. □

Corollary 5. *In a fixprice equilibrium, the demand-side and supply-side fiscal multipliers are related as*

$$\underbrace{\varphi^d(x)}_{\text{Demand-side multiplier}} = \underbrace{\theta(x)}_{\text{Fixed capacity multiplier}} + \underbrace{\varphi^s(x)}_{\text{Supply-side multiplier}},$$

so that the demand-side multiplier is higher in slack equilibria, lower in tight equilibria and exactly equal to the supply-side multiplier in an efficient fixprice equilibrium.

Proof. From the expression for the demand-side fiscal multiplier is a fixprice equilibrium in Proposition 2:

$$\varphi^d(x) = \varphi^* + \theta(x)(1 - \varphi^*) = \theta(x) + \underbrace{\varphi^* - \theta(x)\varphi^*}_{\varphi^s(x)}, \tag{A.71}$$

$$\varphi^d(x) = \theta(x) + \varphi^s(x). \tag{A.72}$$

□

Corollary 6. *For sufficiently low elasticities of labor demand and labor supply such that $\varphi^* < 0.5$, an* ***Austerity Threshold*** *$\tilde{x} \in [\hat{x}, x_m)$ exists such that:*

$$-\varphi^d(x) > \varphi^s(x) > \varphi^d(x), \quad \forall x \in (\tilde{x}, x_m). \tag{A.73}$$

Furthermore, $\tilde{x} = \theta^{-1}\left(-\frac{2\varphi^*}{1-2\varphi^*}\right), \varphi^* < 0.5$, and hence $\frac{d\tilde{x}}{d\varphi^*} > 0$.

Proof. It is apparent that the austerity threshold for tightness cannot be below $\hat{x}$, as in that case $\varphi^d(x) > 0 > -\varphi^d(x), \forall x \in (0, \hat{x})$. However, suppose there exists $\tilde{x} \in (\hat{x}, x_m)$ such that $-\varphi^d(\tilde{x}) = \varphi^s(\tilde{x}) > \varphi^d(\tilde{x})$. Then it must satisfy the following:

$$-\varphi^* - \theta(\tilde{x})(1 - \varphi^*) = \varphi^* - \theta(\tilde{x})\varphi^*, \tag{A.74}$$

$$\theta(\tilde{x}) = -\frac{2\varphi^*}{1 - 2\varphi^*}. \tag{A.75}$$

As established earlier, $\theta(x)$ is differentiable and strictly decreasing on $(0, x_m)$, taking values in $(-\infty, 1)$. Therefore, the inverse function $\theta^{-1}(.)$ exists on the domain $(-\infty, 1)$. Hence, as long as

$\varphi < 0.5$, $-\frac{2\varphi^*}{1-2\varphi^*} \in (-\infty, 1)$, the austerity threshold $\tilde{x}$ exists and is given by:

$$\tilde{x} = \theta^{-1}\left(-\frac{2\varphi^*}{1-2\varphi^*}\right), \quad \varphi^* < 0.5. \tag{A.76}$$

Further, if $\varphi^* < 0.5$, $\frac{d[-\varphi^d(x)-\varphi^s(x)]}{dx} = -\theta'(x)(1-2\varphi^*) > 0$, so that $-\varphi^d(x) > \varphi^s(x) > \varphi^d(x), \forall x \in (\tilde{x}, x_m)$. It also follows that:

$$\theta'(\tilde{x})\frac{d\tilde{x}}{d\varphi^*} = \frac{d}{d\varphi^*}\left[-\frac{2\varphi^*}{1-2\varphi^*}\right], \tag{A.77}$$

$$\frac{d\tilde{x}}{d\varphi^*} = -\frac{1}{\theta'(\tilde{x})}\frac{2}{(1-2\varphi^*)^2} > 0. \tag{A.78}$$

□

Appendix B. Fiscal multipliers: (more) general cases

In this section we show that the results derived earlier hold in much more general settings. In particular, we introduce the class of *flexible* equilibria, which is a superset of the competitive equilibria. We then show that in any flexible equilibrium that has tightness fixed over the business cycle, both demand-side and supply-side multipliers are equal and acyclical, just like in the competitive equilibrium. On the other hand, we show that the cyclicality results established under fixprice equilibria extend to the more general class of *frictional* equilibria, where part of the adjustment happens via tightness.

Appendix B.1. Flexible equilibria multipliers

In the previous section we started off be considering a competitive equilibrium, where tightness was fixed at the efficient level x^* and all adjustment happened via prices and wages. However, this is not the only way to pin down tightness. Below we consider two common alternatives found in search-and-matching literature (Nash bargaining, fixed markup pricing), before introducing a much more general *Tightness Determination Mapping (TDM).*

Appendix B.1.1. Nash bargaining

One alternative, very common in the search-and-matching literature, is to consider Nash bargaining over the price between consumers and firms in order to get an extra equilibrium condition needed to close the model. In our case, the surplus to consumers from buying an additional unit of the produced good at price $\tilde{p}$ after a match is made is given by:

$$\mathcal{B}(\tilde{p}) = \frac{\chi}{c} - \tilde{p}, \tag{B.1}$$

whereas the firms' surplus from selling an extra unit at price $\tilde{p}$ is

$$\mathcal{S}(\tilde{p}) = \tilde{p} - pf(x). \tag{B.2}$$

Assuming the consumers' bargaining power is given by $\beta \in (0,1)$, the solution to Nash bargaining is given by:

$$(1-\beta)\mathcal{S}(p) = \beta\mathcal{B}(p). \tag{B.3}$$

Combining the above with agents' optimality conditions obtained earlier, one gets:

$$\frac{1-\beta}{\beta} = \frac{\gamma(x^L)}{1-f(x^L)}, \quad \frac{dx^L}{d\beta} < 0. \tag{B.4}$$

As one can see, the condition above pins down tightness at $x = x^L$, and we can even get the equivalent of the Hosios (1990) condition for the bargaining power β^* that delivers the socially efficient allocation, $\beta^* = \frac{1}{1+\frac{\gamma(x^*)}{1-f(x^*)}}$.

Appendix B.1.2. Fixed markup pricing

An alternative way to pin down tightness is to assume that the equilibrium price p is set as a fixed markup over the marginal cost, so that:

$$p = \mu \times mc, \tag{B.5}$$

where $\mu \geq 1$ is a markup parameter and mc is the marginal cost. From firms' optimisation problem one gets that the effective selling price $pf(x)$ is set equal to the marginal cost:

$$pf(x) = mc. \tag{B.6}$$

Combining the above two equations one gets the following condition for pinning down the level of tightness:

$$f(x^L) = \frac{1}{\mu}, \quad \frac{dx^L}{d\mu} < 0. \tag{B.7}$$

As before, the equivalent of the Hosios (1990) condition here is the markup μ^* that delivers the socially efficient allocation, n amely $\mu^* = \frac{1}{f(x^*)}$.

Appendix B.1.3. Generalization: Tightness Determination Mapping

In fact, the above approaches to pinning down tightness can be generalized by introducing the notion of a Tightness Determination Mapping (TDM):

Definition 3. *A Tightness Determination Mapping (TDM) $\mathcal{M}$ is given by:*

$$\mathcal{M}: \quad \left\{\Omega^M, \Omega^S, \Omega^T\right\} \to x^L, \tag{B.8}$$

where $\Omega^M = \{\rho, \gamma, \psi, \alpha\}$ *is the set of model structural parameters,* $\Omega^S = \{\chi, a, G, \tau\}$ *is the set of shock parameters,* Ω^T *is the set of parameters specific to the TDM and* x^L *is the resulting tightness. Further, a TDM* $\mathcal{M}$ *is said to be **shock invariant** if and only if*

$$\frac{d\mathcal{M}\left(\Omega^M, \Omega^S, \Omega^T\right)}{d\tilde{s}} = 0, \quad \forall \tilde{s} \in \Omega^S. \tag{B.9}$$

so that changes in shock parameters do not affect the determination of tightness.

It is easy to see that the TDM used in the competitive equilibrium is simply $x^L = x^*$. Also, Nash bargaining is a particular TDM with $\Omega^T = \{\beta\}$, which pins down tightness according to $\frac{1-\beta}{\beta} = \frac{\gamma(x^L)}{1-f(x^L)}$; similarly, fixed markup pricing is a TDM with $\Omega^T = \{\mu\}$, which pins down tightness according to $f(x^L) = \frac{1}{\mu}$. Note that all of the above are also shock invariant TDMs, as none of the shock parameters enter the conditions that pin down the level of tightness.

We can now define a class of *flexible* equilibria that is a superset of the competitive equilibrium considered before:

Definition 4. *A **flexible** equilibrium is a vector* $(p^L, w^L, \mathcal{M})$*, and associated allocations, such that the agents' optimality conditions and the market clearing conditions are satisfied with tightness pinned down at a level* $x^L = \mathcal{M}\left(\Omega^M, \Omega^S, \Omega^T\right)$*.*

Clearly, the competitive equilibrium considered earlier is just a flexible equilibrium with $x^L = x^*$ as the TDM. In fact, it can be shown that all of the comparative statics results established for the competitive equilibrium in Lemma 1 hold in the exact same way for any flexible equilibrium with a shock invariant TDM:

Lemma 4. *In any flexible equilibrium generated by a shock-invariant TDM, the following are the comparative statics of tightness* (x)*, sales* (y) *and the price* (p)*:*

$$\frac{dx}{d\chi} = 0, \frac{dy}{d\chi} > 0, \frac{dp}{d\chi} > 0; \qquad \frac{dx}{da} = 0, \frac{dy}{da} > 0, \frac{dp}{da} < 0 \tag{B.10}$$

Proof. Note that in this more generalized setting, condition (A.5) still holds:

$$\alpha^{\frac{\alpha}{1-\alpha+\psi}} \left(pf(x)a\right)^{\frac{1+\psi}{1-\alpha+\psi}} (1+\tau)^{-\frac{\alpha}{1-\alpha+\psi}} = \chi + p[1+\gamma(x)]G. \tag{B.11}$$

In a flexible equilibrium, $x = x^L = \mathcal{M}(\Omega^M, \Omega^S, \Omega^T)$; further, since the TDM $\mathcal{M}$ is shock-invariant it follows that $\frac{dx^L}{d\chi} = \frac{dx^L}{da} = 0$. The latter implies the following comparative statics for p (for simplicity, evaluated at $G = \tau = 0$):

$$\alpha^{\frac{\alpha}{1-\alpha+\psi}} (f(x)a)^{\frac{1+\psi}{1-\alpha+\psi}} \frac{1+\psi}{1-\alpha+\psi} p^{\frac{\alpha}{1-\alpha+\psi}} \frac{dp}{d\chi} = 1 \tag{B.12}$$

$$\frac{dp}{d\chi} = \alpha^{-\frac{\alpha}{1-\alpha+\psi}} (f(x)a)^{-\frac{1+\psi}{1-\alpha+\psi}} \frac{1-\alpha+\psi}{1+\psi} p^{-\frac{\alpha}{1-\alpha+\psi}} = \frac{1-\alpha+\psi}{1+\psi} \frac{p}{\chi} > 0. \tag{B.13}$$

(B.14)

$$\alpha^{\frac{\alpha}{1-\alpha+\psi}} (f(x))^{\frac{1+\psi}{1-\alpha+\psi}} \frac{1+\psi}{1-\alpha+\psi} (pa)^{\frac{\alpha}{1-\alpha+\psi}} (p + a\frac{dp}{da}) = 0$$

$$\frac{dp}{da} = -\frac{p}{a} < 0. \tag{B.15}$$

Finally, the above implies the following comparative statics for sales $y = [1+\gamma(x)](c(p,x)+G) = \chi/p + G[1+\gamma(x)]$ (also evaluated at $G = \tau = 0$):

$$py = \chi + p[1+\gamma(x)]G \tag{B.16}$$

$$\frac{dp}{d\chi} y + p \frac{dy}{d\chi} = 1 \tag{B.17}$$

$$\frac{dy}{d\chi} = \frac{1}{p}\left(1 - \frac{dp}{d\chi} y\right) = \frac{1}{p} \frac{\alpha}{1+\psi} > 0. \tag{B.18}$$

$$\frac{dp}{da} y + p \frac{dy}{da} = 0 \tag{B.19}$$

$$\frac{dy}{da} = -\frac{dp}{da} \frac{y}{p} > 0. \tag{B.20}$$

□

Of our particular interest, however, is the fact that all the results that we established for demand-side and supply-side fiscal multipliers under the competitive equilibrium remain true for any flexible equilibrium with a shock invariant TDM:

Proposition 4. *In any flexible equilibrium generated by a shock-invariant TDM, the demand-side fiscal multiplier and the supply-side fiscal multiplier are equal and given by:*

$$\varphi^* \equiv \frac{\alpha}{1+\psi} = \frac{1 - \frac{1}{|\epsilon^d|}}{1 + \frac{1}{\epsilon^s}}, \tag{B.21}$$

where $\alpha \in (0,1]$ and $\psi > 0$ are, respectively, returns to labor and inverse Frisch elasticity, whereas $|\epsilon^d| = \frac{1}{1-\alpha}$ and $\epsilon^s = \frac{1}{\psi}$ are (absolute) elasticities of labor demand and labor supply. Hence $\varphi^* \in (0,1]$ and it is pinned down by elasticities of labor demand and labor supply.

Proof. Note that in this more generalized setting, condition (A.29) still holds, so that:

$$\left[\frac{1}{p}\frac{dp}{dG} + \frac{f'(x)}{f(x)}\frac{dx}{dG}\right] = \frac{1-\alpha+\psi}{1+\psi}\frac{1}{c(p,x)}. \tag{B.22}$$

Further, the definition of demand-side fiscal multiplier also remains unchanged:

$$\varphi^d = \frac{d\{c+G\}}{dG} = \frac{dc}{dG} + 1 = \frac{\partial c}{\partial p}\frac{dp}{dG} + \frac{\partial c}{\partial x}\frac{dx}{dG} + 1 \tag{B.23}$$

In a flexible equilibrium, $x = x^L = \mathcal{M}(\Omega^M, \Omega^S, \Omega^T)$; further, since the TDM $\mathcal{M}$ is shock-invariant it follows that $\frac{dx^L}{dG} = 0$, which combined with (B.23) implies the following:

$$\begin{aligned}
\varphi^d &= \frac{\partial c}{\partial p}\frac{dp}{dG} + 1 = -\frac{\chi}{p[1+\gamma(x)]}\frac{1}{p}\frac{dp}{dG} + 1 = -c(p,x)\frac{1}{p}p\frac{1-\alpha+\psi}{1+\psi}\frac{1}{c(p,x)} + 1 && \text{(B.24)}\\
&= -\frac{1-\alpha+\psi}{1+\psi} + 1 && \text{(B.25)}\\
&= \frac{\alpha}{1+\psi} \equiv \varphi^*. && \text{(B.26)}
\end{aligned}$$

Similarly, condition (A.36) also holds in this more generalized setting

$$\left[\frac{1}{p}\frac{dp}{d\tau} + \frac{f'(x)}{f(x)}\frac{dx}{d\tau}\right] = \frac{\alpha}{1+\psi}. \tag{B.27}$$

And the definition of supply-side fiscal multiplier also stays the same:

$$\varphi^s = \frac{d\{c+G\}/\{c+G\}}{d[-\tau]} = -\frac{1}{c}\frac{dc}{d\tau} = -\frac{1}{c}\left[\frac{\partial c}{\partial p}\frac{dp}{d\tau} + \frac{\partial c}{\partial x}\frac{dx}{d\tau}\right]. \tag{B.28}$$

In a flexible equilibrium, $x = x^L = \mathcal{M}(\Omega^M, \Omega^S, \Omega^T)$; further, since the TDM $\mathcal{M}$ is shock-invariant it follows that $\frac{dx^L}{d\tau} = 0$, which combined with (B.27) implies the following:

$$\begin{aligned}
\varphi^s &= -\frac{1}{c}\frac{\partial c}{\partial p}\frac{dp}{d\tau} = -\frac{1}{c}\left[-\frac{\chi}{p[1+\gamma(x)]}\frac{1}{p}\right]\frac{dp}{d\tau} = \frac{1}{c}c\frac{1}{p}p\frac{\alpha}{1+\psi} && \text{(B.29)}\\
&= \frac{\alpha}{1+\psi} = \varphi^d = \varphi^*. && \text{(B.30)}
\end{aligned}$$

□

In other words, any equilibrium where tightness remains fixed over the business cycle will have demand-side and supply-side fiscal multipliers both fixed at φ^* and acyclical.

Appendix B.2. Frictional equilibria multipliers

As an alternative to the competitive equilibrium, last section considered a fixprice equilibrium, where all adjustment is happening via tightness and wages. Here we start off by considering a slightly more general *rigid price* equilibrium, that allows for an arbitrary degree of price rigidity and nests fixprice equilibrium as a special case. Subsequently, we introduce a much more general notion of a *Frictional Mapping (FM).*

Appendix B.2.1. Rigid price equilibrium

A rigid price equilibrium is formally introduced as:

Definition 5. *A* ***rigid price*** *equilibrium is a vector* $(p_0, x, w, \varepsilon, \mathcal{M})$*, and associated allocations, such that the agents' optimality conditions and the market clearing conditions are satisfied with price given by:*

$$p = (p_0)^{\varepsilon}(p^L)^{1-\varepsilon}, \quad \varepsilon \in (0,1] \tag{B.31}$$

where ε *is the degree of price rigidity and* p_0 *is a parameter and* p^L *is the price from the flexible equilibrium* $(p^L, w^L, \mathcal{M})$*.*

Clearly, the fixprice equilibrium is just a special case under $\varepsilon = 1$. In fact, a non-fixprice rigid price equilibrium $(p_0, x, w, \varepsilon, \mathcal{M})$ where $\mathcal{M}$ is a shock invariant TDM shares a lot in common with the corresponding fixprice equilibrium. Particularly, the comparative statics to demand-side and supply-side shocks are given by:

Lemma 5. *In a non-fixprice* $(\varepsilon \in (0,1))$ *rigid price equilibrium* $(p_0, x, w, \varepsilon, \mathcal{M})$ *where* $\mathcal{M}$ *is a shock invariant TDM, the following are the comparative statics of tightness* (x)*, sales* (y) *and the price* (p)*:*

$$\frac{dx}{d\chi} > 0, \frac{dy}{d\chi} > 0, \frac{dp}{d\chi} > 0; \qquad \frac{dx}{da} < 0, \frac{dy}{da} > 0, \frac{dp}{da} < 0 \tag{B.32}$$

Proof. Special case of Lemma 6 under $\mathcal{T}(p^L; \{p_0, \varepsilon\}) = (p_0)^{\varepsilon}(p^L)^{1-\varepsilon}$. □

As one can see, the only difference compared to a fixprice equilibrium is that the price co-moves with tightness and supply-side shocks have an effect on the level of sales.

Of a greater interest to us, however, are the properties of fiscal multipliers under rigid price equilibria. The following proposition establishes the demand-side multiplier is a rigid price equilibrium:

Proposition 5. *In a rigid price equilibrium* $(p_0, x, w, \varepsilon, \mathcal{M})$*, where* $\mathcal{M}$ *is a shock invariant TDM, the demand-side fiscal multiplier* $\varphi^d(x)$ *is given by*

$$\varphi^d(x) = \varphi^* + \theta(x) \times \left[(1-\varphi^*)\{1-(1-\varepsilon)g(x, x^L)\}\right] \tag{B.33}$$

where $\varphi^* = \frac{\alpha}{1+\psi}$ *is the flexible equilibrium multiplier and the function* $g(x, x^L)$ *is given by:*

$$g(x, x^L) = \frac{f(x) - \rho x}{f(x^L) - \rho x^L}. \tag{B.34}$$

Hence, $\varphi^d(x) \in (-\infty, 1]$ *and* $\frac{d\varphi^d(x)}{dx}|_{x=x^L} < 0$*.*

Proof. Special case of Proposition 7 under $\mathcal{T}(p^L; \{p_0, \varepsilon\}) = (p_0)^{\varepsilon}(p^L)^{1-\varepsilon}$. □

Note that for $\varepsilon = 1$ the expression above collapses back to the fixprice equilibrium demand-side multiplier from Proposition 2. We can also see that the rigid price equilibrium demand-side multiplier above maintains a lot of the properties of its fixprice equilibrium counterpart. In particular, it also collapses back to φ^* if the equilibrium tightness happens to coincide with the socially efficient one ($x = x^*$), it also lies between $-\infty$ and one, so that consumption always gets crowded out; it is also guaranteed to fall in tightness, although only in the neighbourhood of the corresponding flexible equilibrium allocation. The role played by ε here is in determining the relative magnitude of the state-dependent component. The upper panel of Figure B.7 (drawn for simplicity under $x^L = x^*$) shows that as the degree of price rigidity ε falls, the multiplier becomes flatter around x^* at the level equal to φ^* suggesting that the degree of state-dependence falls as well.

Similarly, one can derive the supply-side multiplier under rigid price equilibrium:

Proposition 6. *In a rigid price equilibrium $(p_0, x, w, \varepsilon, \mathcal{M})$, where $\mathcal{M}$ is a shock invariant TDM, the supply-side fiscal multiplier $\varphi^s(x)$ is given by*

$$\varphi^s(x) = \varphi^* - \theta(x) \times \varepsilon\varphi^*, \tag{B.35}$$

where $\varphi^* = \frac{\alpha}{1+\psi}$ is the long-run equilibrium multiplier. Hence, $\varphi^d(x) \in (0, +\infty)$ and $\frac{d\varphi^d(x)}{dx} > 0, \forall x \in (0, x_m)$.

Proof. Special case of Proposition 8 under $\mathcal{T}(p^L; \{p_0, \varepsilon\}) = (p_0)^{\varepsilon}(p^L)^{1-\varepsilon}$. □

Again, under $\varepsilon = 1$ the expression above collapses back to the fixprice equilibrium supply-side multiplier. One can see that the expression above shares every single property with the fixprice equilibrium counterpart, the only difference being the magnitude of state-dependence, which increases in the degree of price rigidity ε, as shown in the bottom panel of Figure B.7 (again, for simplicity drawn for $x^L = x^*$).

Appendix B.2.2. Generalization: Frictional Mapping

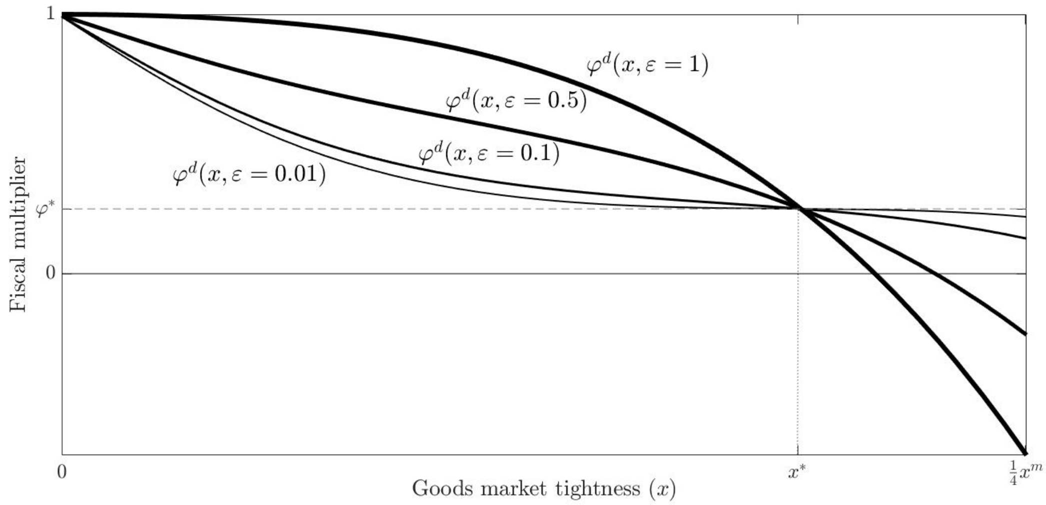
One can in fact show that the properties of fiscal multipliers that we have established for the rigid price equilibrium hold more generally, and not for the particular parametric form of frictions that we have considered so far. We generalize our findings by introducing the notion of a Frictional Mapping:

Definition 6. *For a given flexible equilibrium $(p^L, w^L, \mathcal{M})$, a Frictional Mapping (FM) $\mathcal{T}$ is given by:*

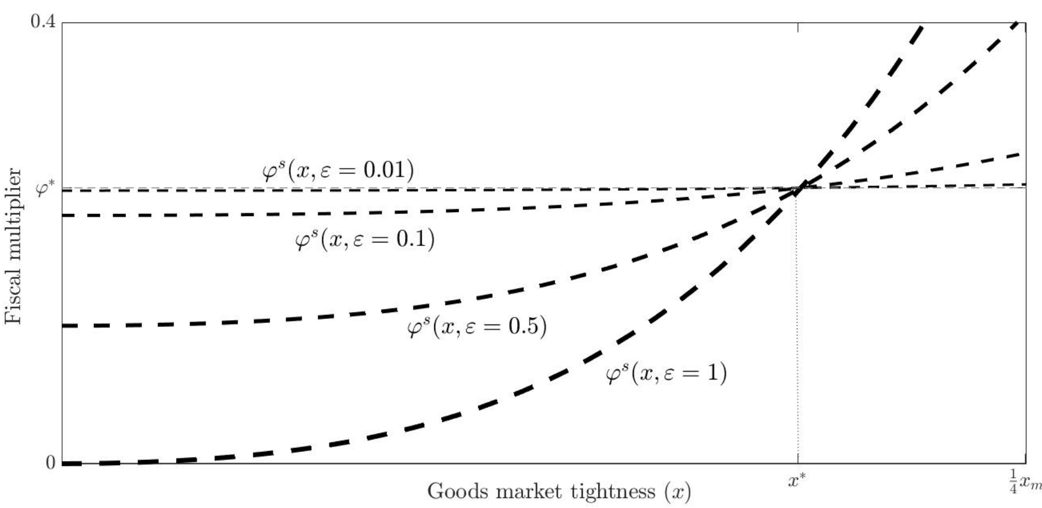
$$\mathcal{T}: \quad \left\{p^L, \Omega^F\right\} \to p^F, \tag{B.36}$$

Figure B.7: Fiscal multipliers in a rigid price equilibrium

(a) Demand-side fiscal multiplier $\varphi^d(x)$



(b) Supply-side fiscal multiplier $\varphi^s(x)$



Notes*: Panels (a) and (b) show demand-side and supply-side fiscal multipliers in a rigid price equilibrium of a calibrated version of our model ($\alpha = 0.3, \delta = 2, \rho = 0.1, \psi = 0.2, x^L = x^*$). Panel (a) shows demand-side fiscal multipliers for different values of the price rigidity parameter ε – one case see that $\varphi^d(x)$ strictly falls in tightness for all considered values of ε, but the degree of state-dependence rises in the degree of price rigidity; in Panel (b) we can see that the supply-side fiscal multiplier strictly rises in tightness for all values of ε considered, but again the degree if state-dependence falls as we allow for more price flexibility.*

where Ω^F is the set of parameters specific to the FM and p^F is the resulting price. Moreover, the Frictional Mapping $\mathcal{T}(p^L;\Omega^F)$ is said to be ***contractionary*** *if and only if*

$$\frac{d\ln p^F}{d\ln p^L} = \frac{d\mathcal{T}(p^L;\Omega^F)}{dp^L}\frac{p^L}{p^F} \in [0,1). \tag{B.37}$$

Having defined a Frictional Mapping (FM), one can now define a *frictional* equilibrium:

Definition 7. *A* ***frictional*** *equilibrium is a vector $(p^F, x^F, w^F, \mathcal{T}, \mathcal{M})$, and associated allocations, such that the agents' optimality conditions and the market clearing conditions are satisfied with price given by:*

$$p^F = \mathcal{T}(p^L) \tag{B.38}$$

where $\mathcal{T}$ is the Frictional Mapping and p^L is the price from the flexible equilibrium $(p^L, w^L, \mathcal{M})$.

Rigid price equilibrium is a special case of a frictional equilibrium for $\mathcal{T}(z) = (p_0)^{\varepsilon}(z)^{1-\varepsilon}$, $\Omega^F = \{p_0, \varepsilon\}$, $\varepsilon \in (0,1]$. Further, the above frictional mapping associated with a rigid price equilibrium is indeed contractionary, since

$$\frac{d\mathcal{T}(z;\Omega^F)}{dz}\frac{z}{p^F} = (1-\varepsilon) \in [0,1), \tag{B.39}$$

as $\varepsilon \in (0,1]$.

We can now derive and discuss properties of demand-side and supply-side multipliers in a generic frictional equilibrium. Firstly, note that the comparative statics to demand-side and supply-side shocks established in a rigid price equilibrium extend to a generic frictional equilibrium generated by a contractionary frictional mapping:

Lemma 6. *In a non-fixprice $\left(\frac{d\mathcal{T}(p^L;\Omega^F)}{dp^L}\frac{p^L}{p^F} \neq 0\right)$ frictional equilibrium $(p^F, x^F, w^F, \mathcal{T}, \mathcal{M})$ where $\mathcal{T}$ is a contractionary FM and $\mathcal{M}$ is a shock invariant TDM, the following are the comparative statics of tightness (x), sales (y) and the price (p):*

$$\frac{dx}{d\chi} > 0, \frac{dy}{d\chi} > 0, \frac{dp}{d\chi} > 0; \qquad \frac{dx}{da} < 0, \frac{dy}{da} > 0, \frac{dp}{da} < 0 \tag{B.40}$$

Proof. Note that condition (A.5) still holds in this more generalized setting:

$$\alpha^{\frac{\alpha}{1-\alpha+\psi}}\left(pf(x)a\right)^{\frac{1+\psi}{1-\alpha+\psi}}(1+\tau)^{-\frac{\alpha}{1-\alpha+\psi}} = \chi + p[1+\gamma(x)]G. \tag{B.41}$$

Differentiate both sides with respect to χ (evaluated at $G = \tau = 0$):

$$\alpha^{\frac{\alpha}{1-\alpha+\psi}} \frac{1+\psi}{1-\alpha+\psi} [pf(x)a]^{\frac{1+\psi}{1-\alpha+\psi}} \left[\frac{1}{p}\frac{dp}{d\chi} + \frac{f'(x)}{f(x)}\frac{dx}{d\chi}\right] = 1, \tag{B.42}$$

$$\frac{dx}{d\chi} = \frac{f(x)}{f'(x)} \left[\alpha^{-\frac{\alpha}{1-\alpha+\psi}} \frac{1-\alpha+\psi}{1+\psi} [pf(x)a]^{-\frac{1+\psi}{1-\alpha+\psi}} - \frac{1}{p}\frac{dp}{d\chi}\right], \tag{B.43}$$

$$\frac{dx}{d\chi} = \frac{f(x)}{f'(x)} \left[\frac{1-\alpha+\psi}{1+\psi}\frac{1}{\chi} - \frac{1}{p}\frac{dp}{d\chi}\right]. \tag{B.44}$$

Since $p = \mathcal{T}(p^L)$, it follows that $\frac{dp}{d\chi} = \frac{d\mathcal{T}(p^L)}{dp^L}\frac{dp^L}{d\chi}$; further, from Lemma 4 we know that $\frac{dp^L}{d\chi} = \frac{1-\alpha+\psi}{1+\psi}\frac{p^L}{\chi}$ and it follows that:

$$\frac{dp}{d\chi} = \underbrace{\frac{d\mathcal{T}(p^L)}{dp^L}\frac{p^L}{p}}_{\in(0,1) \text{ as } \mathcal{T} \text{ contractionary FM}} \frac{1-\alpha+\psi}{1+\psi}\frac{p}{\chi} > 0. \tag{B.45}$$

$$\frac{dx}{d\chi} = \frac{f(x)}{f'(x)} \left[\frac{1-\alpha+\psi}{1+\psi}\frac{1}{\chi} \underbrace{\left(1 - \frac{d\mathcal{T}(p^L)}{dp^L}\frac{p^L}{p}\right)}_{\in(0,1) \text{ as } \mathcal{T} \text{ contractionary FM}}\right] > 0 \tag{B.46}$$

Given that $py = \chi$, it follows that $\frac{dy}{d\chi} = \frac{1}{p}\left[1 - \frac{dp}{d\chi}\frac{\chi}{p}\right]$ and hence:

$$\frac{dy}{d\chi} = \frac{1}{p}\left[1 - \underbrace{\frac{d\mathcal{T}(p^L)}{dp^L}\frac{p^L}{p}\frac{1-\alpha+\psi}{1+\psi}}_{\in(0,1) \text{ as } \mathcal{T} \text{ contractionary FM}}\right] > 0. \tag{B.47}$$

Similarly, differentiate both sides of (B.41) with respect to a (evaluated at $G = \tau = 0$):

$$\alpha^{\frac{\alpha}{1-\alpha+\psi}} \frac{1+\psi}{1-\alpha+\psi} [pf(x)a]^{\frac{1+\psi}{1-\alpha+\psi}} \left[\frac{1}{p}\frac{dp}{da} + \frac{f'(x)}{f(x)}\frac{dx}{da} + \frac{1}{a}\right] = 0, \tag{B.48}$$

$$\frac{dx}{da} = -\frac{f(x)}{f'(x)}\left[\frac{1}{a} + \frac{1}{p}\frac{dp}{da}\right] = 0. \tag{B.49}$$

Since $p = \mathcal{T}(p^L)$, it follows that $\frac{dp}{da} = \frac{d\mathcal{T}(p^L)}{dp^L}\frac{dp^L}{da}$; further, from Lemma 4 we know that $\frac{dp^L}{da} = -\frac{p^L}{a}$

and it follows that:

$$\frac{dp}{da} = -\underbrace{\frac{d\mathcal{T}(p^L)}{dp^L}\frac{p^L}{p}}_{\in(0,1)\text{ as }\mathcal{T}\text{ contractionary FM}}\frac{p}{a} < 0. \tag{B.50}$$

$$\frac{dx}{da} = -\frac{f(x)}{f'(x)}\left[\frac{1}{a}\underbrace{\left(1 - \frac{d\mathcal{T}(p^L)}{dp^L}\frac{p^L}{p}\right)}_{\in(0,1)\text{ as }\mathcal{T}\text{ contractionary FM}}\right] < 0 \tag{B.51}$$

Given that $py = \chi$, it follows that:

$$\frac{dy}{da} = -\frac{dp}{da}\frac{y}{p} > 0. \tag{B.52}$$

□

Moreover, one can also solve for the demand-side fiscal multiplier under a generic frictional equilibrium and see that under certain properties it also falls in tightness:

Proposition 7. *In a frictional equilibrium $(p^F, x^F, w^F, \mathcal{T}, \mathcal{M})$, where $\mathcal{T}$ is a contractionary FM and $\mathcal{M}$ is a shock invariant TDM, the demand-side fiscal multiplier $\varphi^d(x)$ is given by*

$$\varphi^d(x) = \varphi^* + \theta(x) \times \left[(1-\varphi^*)\left\{1 - \frac{\mathcal{T}'(p^L)p^L}{\mathcal{T}(p^L)}g(x, x^L)\right\}\right] \tag{B.53}$$

where $\varphi^* = \frac{\alpha}{1+\psi}$ is the flexible equilibrium multiplier and the function $g(x, x^L)$ is given by:

$$g(x, x^L) = \frac{f(x) - \rho x}{f(x^L) - \rho x^L}. \tag{B.54}$$

Hence, $\varphi^d(x) \in (-\infty, 1)$ and $\frac{d\varphi^d(x)}{dx}|_{x=x^L} < 0$.

Proof. Note that (A.30) still holds in this more general setting:

$$\left[\frac{1}{p}\frac{dp}{dG} + \frac{f'(x)}{f(x)}\frac{dx}{dG}\right] = \frac{1-\alpha+\psi}{1+\psi}\frac{1}{c(p,x)} = (1-\varphi^*)\frac{1}{c(p,x)}. \tag{B.55}$$

From the definition of the demand-side fiscal multiplier:

$$\varphi^d(x) = \frac{d\{c+G\}}{dG} = \frac{dc}{dG} + 1 = \frac{\partial c}{\partial p}\frac{dp}{dG} + \frac{\partial c}{\partial x}\frac{dx}{dG} + 1 = \tag{B.56}$$

$$= -c(p,x)\left[\frac{1}{p}\frac{dp}{dp^L}\frac{dp^L}{dG}\right] - c(p,x)\underbrace{\frac{\gamma'(x)}{1+\gamma(x)}\frac{f(x)}{f'(x)}}_{1-\theta(x)}\left[(1-\varphi^*)\frac{1}{c(p,x)} - \frac{1}{p}\frac{dp}{dp^L}\frac{dp^L}{dG}\right]. \tag{B.57}$$

From Proposition 4 we know that $\frac{dp^L}{dG} = p^L(1-\varphi^*)\frac{1}{c(p^L,x^L)}$, hence:

$$\varphi^d(x) = -c(p,x)\frac{dp}{dp^L}\frac{p^L}{p}(1-\varphi^*)c(p^L,x^L) - c(p,x)(1-\theta(x))\left[(1-\varphi^*)\frac{1}{c(p,x)} - \frac{dp}{dp^L}\frac{p^L}{p}(1-\varphi^*)\frac{1}{c(p^L,x^L)}\right], \tag{B.58}$$

$$= 1-(1-\varphi^*)(1-\theta(x)) - \theta(x)(1-\varphi^*)\frac{dp}{dp^L}\frac{p^L}{p}\frac{c(p,x)}{c(p^L,x^L)} \tag{B.59}$$

$$= \varphi^* + \theta(x)(1-\varphi^*)\left[1 - \frac{dp}{dp^L}\frac{p^L}{p}\frac{c(p,x)}{c(p^L,x^L)}\right] \tag{B.60}$$

$$= \varphi^* + \theta(x)(1-\varphi^*)\left[1 - \frac{dp}{dp^L}\frac{p^L}{p}\frac{p^L[1+\gamma(x^L)]}{p[1+\gamma(x)]}\right] \tag{B.61}$$

$$= \varphi^* + \theta(x)(1-\varphi^*)\left[1 - \frac{dp}{dp^L}\frac{p^L}{p}\frac{\frac{p^L f(x^L)}{f(x^L)-\rho x^L}}{\frac{p f(x)}{f(x)-\rho x}}\right] \tag{B.62}$$

$$= \varphi^* + \theta(x)(1-\varphi^*)\left[1 - \frac{dp}{dp^L}\frac{p^L}{p}\frac{f(x)-\rho x}{f(x^L)-\rho x^L}\right] \tag{B.63}$$

$$= \varphi^* + \theta(x)(1-\varphi^*)\left[1 - \frac{\mathcal{T}'(p^L)p^L}{\mathcal{T}(p^L)}g(x,x^L)\right], \tag{B.64}$$

where $g(x,x^L) = \frac{f(x)-\rho x}{f(x^L)-\rho x^L}$. Further, notice that:

$$\frac{d\varphi^d(x)}{dx}|_{x=x^L} = \theta'(x^L)(1-\varphi^*)\left[1 - \frac{\mathcal{T}'(p^L)p^L}{\mathcal{T}(p^L)}\right] - \theta(x^L)(1-\varphi^*)\frac{\mathcal{T}'(p^L)p^L}{\mathcal{T}(p^L)}\frac{f'(x^L)-\rho}{f(x^L)-\rho x^L} < 0, \forall x^L \in (0,x_m) \tag{B.65}$$

since $\theta'(x^L) < 0, \theta(x^L)(f'(x^L)-\rho) > 0, \forall x \in (0,x_,)$. Also, it follows that $\varphi^d(0) = \varphi^* + \theta(0)(1-\varphi^*)[1-0] = 1$, and $\lim_{x\to x_m^-}\varphi^d(x) = \varphi^* + \lim_{x\to x_m^-}(1-\varphi^*)[1-\frac{\mathcal{T}'(p^L)p^L}{\mathcal{T}(p^L)}] = -\infty$, so that $\varphi^d(x) \in (-\infty,1), \forall x \in (0,x_m)$. □

Similarly, one can also solve for the supply-side fiscal multiplier in a generic frictional equilibrium and establish its properties:

Proposition 8. *In a frictional equilibrium $(p^F, x^F, w^F, \mathcal{T}, \mathcal{M})$, where $\mathcal{T}$ is a contractionary FM and*

$\mathcal{M}$ *is a shock-invariant TDM, the supply-side fiscal multiplier* $\varphi^s(x)$ *is given by*

$$\varphi^s(x) = \varphi^* - \theta(x) \times \left(1 - \frac{\mathcal{T}'(p^L)p^L}{\mathcal{T}(p^L)}\right)\varphi^*, \tag{B.66}$$

where $\varphi^* = \frac{\alpha}{1+\psi}$ *is the flexible equilibrium multiplier. Hence,* $\varphi^d(x) \in (0, +\infty)$ *and* $\frac{d\varphi^d(x)}{dx} > 0, \forall x \in (0, x_m)$.

Proof. Note that (A.36) still holds in this more general setting:

$$\left[\frac{1}{p}\frac{dp}{d\tau} + \frac{f'(x)}{f(x)}\frac{dx}{d\tau}\right] = \varphi^*. \tag{B.67}$$

From the definition of the supply-side fiscal multiplier:

$$\varphi^s(x) = \frac{d\{c+G\}/\{c+G\}}{d[-\tau]} = -\frac{1}{c(p,x)}\frac{dc}{d\tau} = -\frac{1}{c(p,x)}\left[\frac{\partial c}{\partial p}\frac{dp}{d\tau} + \frac{\partial c}{\partial x}\frac{dx}{d\tau}\right] \tag{B.68}$$

$$= \frac{1}{c(p,x)}\left[c(p,x)\frac{1}{p}\frac{dp}{d\tau} + c(p,x)\frac{\gamma'(x)}{1+\gamma(x)}\frac{f(x)}{f'(x)}\left(\varphi^* - \frac{1}{p}\frac{dp}{d\tau}\right)\right] \tag{B.69}$$

$$= (1-\theta(x))\varphi^* + \frac{1}{p}\frac{d\mathcal{T}(p^L)}{dp^L}\frac{dp^L}{d\tau}\theta(x). \tag{B.70}$$

From Lemma 4 we know that $\frac{dp^L}{d\tau} = p^L\varphi^*$:

$$\varphi^s(x) = \varphi^* - \theta(x)\left(1 - \frac{\mathcal{T}'(p^L)p^L}{\mathcal{T}(p^L)}\right)\varphi^*. \tag{B.71}$$

Further, $\frac{d\varphi^s(x)}{dx} = -\theta(x)\left(1 - \frac{\mathcal{T}'(p^L)p^L}{\mathcal{T}(p^L)}\right)\varphi^* > 0, \forall x \in (0, x_m)$ since $\theta'(x) < 0, \forall x \in (0, x_m)$. Also, $\varphi^s(0) = \varphi^*\frac{\mathcal{T}'(p^L)p^L}{\mathcal{T}(p^L)} \in [0,1)$, and $\lim_{x\to x_m^-}\varphi^s(x) = \varphi^* - \lim_{x\to x_m}\theta(x)\left(1 - \frac{\mathcal{T}'(p^L)p^L}{\mathcal{T}(p^L)}\right) = +\infty$, so that $\varphi^s(x) \in (0, +\infty), \forall x \in (0, x_m)$. □

Appendix B.3. Cyclicality of fiscal multipliers

Firstly , note that our result of equal and acyclical demand-side and supply-side multipliers in a competitive equilibrium extends more generally to *any* flexible equilibrium generated by a shock-invariant TDM:

Corollary 13. *In any flexible equilibrium generated by policy-invariant TDM both demand-side and supply-side multipliers are acyclical.*

Proof. Trivial consequence of Proposition 4: in any flexible equilibrium generated by a shock-invariant TDM, both multipliers are equal to $\varphi^* = \frac{\alpha}{1+\psi}$ and do not change as either preference χ or technology a varies. □

In the words, any equilibrium that sees tightness fixed over the business cycle, will see both multipliers fixed at φ^* and hence acyclical.

Further, our state-dependence result for the demand-side multiplier in a fixprice equilibrium still holds in *any* frictional equilibrium as long as the elasticity between frictional and flexible price is in $[0,1)$ and the flexible equilibrium around which the friction is defined is generated by a shock-inariant TDM:

Corollary 14. *In any frictional equilibrium generated by a contractionary frictional mapping and a shock-invariant TDM, in the local neighbourhood of the flexible equilibrium allocation, the demand-side multiplier is countercyclical under demand-driven fluctuations, and procyclical under supply-driven fluctuations.*

Proof. From Lemma 6 we know that in any frictional equilibrium generated by a contractionary frictional mapping and a shock-invariant TDM, $\frac{dx}{d\chi} > 0, \frac{dx}{da} < 0$; further, from Proposition 7 we know that in a frictional equilibrium generated by a contractionary frictional mapping and a shock-invariant TDM $\frac{d\varphi^d(x)}{dx}|_{x=x^L} < 0$. Hence, $\frac{d\varphi^d(x)}{d\chi}|_{x=x^L} = \frac{d\varphi^d(x)}{dx}|_{x=x^L} \frac{dx}{d\chi}|_{x=x^L} < 0$ and $\frac{d\varphi^d(x)}{da}|_{x=x^L} = \frac{d\varphi^d(x)}{dx}|_{x=x^L} \frac{dx}{da}|_{x=x^L} > 0$. □

Similarly for the supply-side multiplier:

Corollary 15. *In any frictional equilibrium generated by a contractionary frictional mapping and a shock-invariant TDM, the supply-side multiplier is procyclical under demand-driven fluctuations, and countercyclical under supply-driven fluctuations.*

Proof. From Lemma 6 we know that in any frictional equilibrium generated by a contractionary frictional mapping and a shock-invariant TDM, $\frac{dx}{d\chi} > 0, \frac{dx}{da} < 0$; further, from Proposition 8 we know that in a frictional equilibrium generated by a contractionary frictional mapping and a shock-invariant TDM $\frac{d\varphi^s(x)}{dx} > 0$. Hence, $\frac{d\varphi^s(x)}{d\chi} = \frac{d\varphi^s(x)}{dx}\frac{dx}{d\chi} > 0$ and $\frac{d\varphi^s(x)}{da} = \frac{d\varphi^s(x)}{dx}\frac{dx}{da} < 0$. □

Appendix C. Alternative fiscal instruments

Appendix C.1. Government employment

Let the government employ a fraction $h \in [0,1)$ of the households' labor supply, so that government labor demand is given by $n^G = hl$ and the government collects additional lump-sum taxes to finance public sector wages, so that $T = p[1+\gamma(x)]G - wn\tau - wn^G$. Then labor market clearing condition becomes:

$$n(w;p,x,\tau) + n^G = l(w) \tag{C.1}$$

$$n(w;p,x,\tau) = (1-h)l(w) \tag{C.2}$$

Without loss of generality, assume $\tau = 0$, and substitute our solutions for n and l:

$$[\alpha p f(x) a]^{\frac{1}{1-\alpha}} w^{-\frac{1}{1-\alpha}} = (1-h) w^{\frac{1}{\psi}} \tag{C.3}$$

$$[\alpha p f(x) a]^{\frac{1}{1-\alpha}} (1-h)^{-1} = w^{\frac{1-\alpha+\psi}{\psi(1-\alpha)}} \tag{C.4}$$

$$w = [\alpha p f(x) a]^{\frac{\psi}{1-\alpha+\psi}} (1-h)^{-\frac{\psi(1-\alpha)}{1-\alpha+\psi}} \tag{C.5}$$

$$n = (1-h) l(w) = (1-h) w^{\frac{1}{\psi}} = [\alpha p f(x) a]^{\frac{1}{1-\alpha+\psi}} (1-h)^{\frac{\psi}{1-\alpha+\psi}}. \tag{C.6}$$

Substituting the solution for n above into the goods market clearing condition:

$$\frac{f(x) a n^{\alpha}}{1+\gamma(x)} = c(p,x) + G \tag{C.7}$$

$$p f(x) a n^{\alpha} = \chi + p[1+\gamma(x)]G \tag{C.8}$$

$$p f(x) a [\alpha p f(x) a]^{\frac{\alpha}{1-\alpha+\psi}} (1-h)^{\frac{\alpha\psi}{1-\alpha+\psi}} \tag{C.9}$$

$$\alpha^{\frac{\alpha}{1-\alpha+\psi}} [a p f(x)]^{\frac{1+\psi}{1-\alpha+\psi}} (1-h)^{\frac{\alpha\psi}{1-\alpha+\psi}} = \chi + p[1+\gamma(x)]G \tag{C.10}$$

Differentiating with respect to h (for simplicity, where $G = h = 0$):

$$\frac{1+\psi}{1-\alpha+\psi} [a p f(x)]^{\frac{1+\psi}{1-\alpha+\psi}-1} \left(a \frac{dp}{dh} f(x) + a p f'(x) \frac{dx}{dh} \right) (1-h)^{\frac{\alpha\psi}{1-\alpha+\psi}} = [a p f(x)]^{\frac{1+\psi}{1-\alpha+\psi}} \frac{\alpha\psi}{1-\alpha+\psi} (1-h)^{\frac{\alpha\psi}{1-\alpha+\psi}-1} \tag{C.11}$$

$$\frac{1+\psi}{1-\alpha+\psi} [a p f(x)]^{\frac{1+\psi}{1-\alpha+\psi}} \left(\frac{1}{p} \frac{dp}{dh} + \frac{f'(x)}{f(x)} \frac{dx}{dh} \right) = [a p f(x)]^{\frac{1+\psi}{1-\alpha+\psi}} \frac{\alpha\psi}{1-\alpha+\psi} \tag{C.12}$$

$$\frac{1}{p} \frac{dp}{dh} + \frac{f'(x)}{f(x)} \frac{dx}{dh} = \frac{\alpha\psi}{1+\psi}. \tag{C.13}$$

Define the government employment multiplier:

$$\varphi^h(x) \equiv \frac{d\{c+G\}/\{c+G\}}{dh} = \frac{1}{c} \frac{c}{h}. \tag{C.14}$$

In a competitive equilibrium $x = x^*$ so that:

$$\varphi^h = \frac{1}{c} \left[-\frac{\chi}{p[1+\gamma(x)]} \frac{1}{p} \frac{dp}{dh} \right] = -\frac{1}{p} p \frac{\alpha\psi}{1+\psi} = \frac{\alpha\psi}{1+\psi}. \tag{C.15}$$

Therefore, just like the multipliers studied in the main text, the government employment multiplier is acyclical in the competitive equilibrium and is pinned down exclusively by the relative elasticities of labor demand and labor supply.

In a fixprice equilibrium, $p = p_0$ so that:

$$\varphi^h(x) = -\frac{\gamma'(x)}{1+\gamma(x)}\frac{dx}{dh} = -\underbrace{\frac{\gamma'(x)}{1+\gamma(x)}\frac{f(x)}{f'(x)}}_{1-\theta(x)}\frac{\alpha\psi}{1+\psi} = (\theta(x)-1)\frac{\alpha\psi}{1+\psi}. \tag{C.16}$$

Note that $\frac{d\varphi^h(x)}{dx} = \theta'(x)\frac{\alpha\psi}{1+\psi} < 0, \forall x \in (0, x_m)$; therefore, in a fixprice equilibrium, the government employment multiplier strictly falls in tightness and has the same cyclicality properties as the government consumption spending multiplier considered in the main text.

Appendix C.2. Distortionary taxes on consumption, labor income and firms' sales

We introduce taxes on households' consumption and labor income, so that the budget constraint of the representative households is given by:

$$p(1+\tau^c)[1+\gamma(x)] + m \leq w(1-\tau^l)l + \bar{m} + \Pi - T, \tag{C.17}$$

where τ^c is the consumption tax rate, τ^l is the labor income tax rate. The consumption function and the labor supply function become:

$$c(p,x) = \frac{\chi}{p(1+\tau^c)[1+\gamma(x)]}, \qquad l(w) = [w(1-\tau^l)]^{\frac{1}{\psi}}. \tag{C.18}$$

Further, we introduce taxes on firms' payroll, so that firms' profits are given by:

$$\Pi = p(1-\tau^s)f(x)an^\alpha - wn(1+\tau), \tag{C.19}$$

where τ^s is the rate of tax on firms' sales. The labor demand function resulting from profit maximisation is then given by:

$$n(w; p, x, \tau, \tau^s) = \left[\frac{\alpha p(1-\tau^s)f(x)a}{w(1+\tau)}\right]^{\frac{1}{1-\alpha}}. \tag{C.20}$$

Combining the labor demand function with labor market clearing condition delivers the following equilibrium employment:

$$n = [\alpha p f(x) a]^{\frac{1}{1-\alpha+\psi}}(1-\tau^s)^{\frac{1}{1-\alpha+\psi}}(1+\tau)^{-\frac{1}{1-\alpha+\psi}}(1-\tau^l)^{\frac{1}{1-\alpha+\psi}}. \tag{C.21}$$

Using the goods market clearing condition:

$$\frac{f(x)an^{\alpha}}{1+\gamma(x)} = c(p,x) + G \tag{C.22}$$

$$pf(x)an^{\alpha} = \frac{\chi}{1+\tau^c} + p[1+\gamma(x)]G \tag{C.23}$$

$$\alpha^{\frac{\alpha}{1\alpha+\psi}}[pf(x)a]^{\frac{1+\psi}{1-\alpha+\psi}}(1-\tau^s)^{\frac{\alpha}{1-\alpha+\psi}}(1-\tau)^{-\frac{\alpha}{1-\alpha+\psi}}(1-\tau^l)^{\frac{\alpha}{1-\alpha+\psi}} = \frac{\chi}{1+\tau^c} + p[1+\gamma(x)]G. \tag{C.24}$$

Appendix C.2.1. Consumption tax cut multiplier

Differentiate with respect to τ^c (at $\tau = \tau^c = \tau^l = \tau^s = G = 0$):

$$\alpha^{\frac{\alpha}{1-\alpha+\psi}}\left[\frac{1+\psi}{1-\alpha+\psi}(pf(x)a)^{\frac{1+\psi}{1-\alpha+\psi}-1}\left\{\frac{dp}{d\tau^c}f(x)a + pf'(x)\frac{dx}{d\tau^c}\right\}\right] = -\frac{\chi}{(1+\tau^c)^2} \tag{C.25}$$

$$\underbrace{\alpha^{\frac{\alpha}{1-\alpha+\psi}}[pf(x)a]^{\frac{1+\psi}{1-\alpha+\psi}}}_{\chi}\left\{\frac{1}{p}\frac{dp}{d\tau^c} + \frac{f'(x)}{f(x)}\frac{dx}{d\tau^c}\right\} = -\chi\frac{1-\alpha+\psi}{1+\psi} \tag{C.26}$$

$$\frac{1}{p}\frac{dp}{d\tau^c} + \frac{f'(x)}{f(x)}\frac{dx}{d\tau^c} = \varphi^* - 1. \tag{C.27}$$

Define the consumption tax cut multiplier:

$$\varphi^{\tau^c}(x) \equiv \frac{d\{c+G\}/\{c+G\}}{d[-\tau^c]} = -\frac{1}{c}\frac{dc}{d\tau^c}. \tag{C.28}$$

In a competitive equilibrium $x = x^*$, so that:

$$\varphi^{\tau^c} = -\frac{1}{c}\left[-c\frac{1}{p}\frac{dp}{d\tau^c} + \frac{\partial c}{\partial \tau^c}\right] = -\frac{1}{c}\left[-c\frac{1}{p}p(\varphi^*-1) - c\right] = \varphi^*, \tag{C.29}$$

so that in a competitive equilibrium the consumption tax cut multiplier is acyclical and pinned down exclusively by elasticities of labor supply and labor demand.

In a fixprice equilibrium $p = p_0$, so that:

$$\varphi^{\tau^c}(x) = -\frac{1}{c}\left[-c\frac{\gamma'(x)}{1+\gamma(x)}\frac{dx}{d\tau^c} + \frac{\partial c}{\partial \tau^c}\right] = -\frac{1}{c}\left[-c\underbrace{\frac{\gamma'(x)}{1+\gamma(x)}\frac{f(x)}{f'(x)}}_{1-\theta(x)}(\varphi^*-1) - c\right] = \varphi^* + \theta(x)(1-\varphi^*) = \varphi^d)(x), \tag{C.30}$$

so that in a fixprice equilibrium the consumption tax cut multiplier is identical to the government consumption spending multiplier, and thus shares all of the properties of the latter.

Appendix C.2.2. Labor income tax cut multiplier

Differentiate the goods market clearing condition with respect to τ^l (at $\tau = \tau^c = \tau^l = \tau^s = G = 0$):

$$\alpha^{\frac{\alpha}{1-\alpha+\psi}} \left[\frac{1+\psi}{1-\alpha+\psi} (pf(x)a)^{\frac{1+\psi}{1-\alpha+\psi}-1} \left\{ \frac{dp}{d\tau^l} f(x)a + pf'(x)\frac{dx}{d\tau^l}a \right\} - \frac{\alpha}{1-\alpha+\psi} (pf(x)a)^{\frac{1+\psi}{1-\alpha+\psi}} \right] = 0 \tag{C.31}$$

$$\frac{1}{p}\frac{dp}{d\tau^l} + \frac{f'(x)}{f(x)}\frac{dx}{d\tau^l} = \frac{\alpha}{1+\psi} = \varphi^*. \tag{C.32}$$

Define the labor income tax cut multiplier:

$$\varphi^{\tau^l}(x) \equiv \frac{d\{c+G\}/\{c+G\}}{d[-\tau^l]} = -\frac{1}{c}\frac{dc}{d\tau^l}. \tag{C.33}$$

In a competitive equilibrium $x = x^*$, so that:

$$\varphi^{\tau^l} = -\frac{1}{c}\frac{\partial c}{\partial p}\frac{dp}{d\tau^l} = -\frac{1}{c}[-c\frac{1}{p}]p\varphi^* = \varphi^*, \tag{C.34}$$

so that in a competitive equilibrium the labor income tax cut multiplier is acyclical and pinned down exclusively by the elasticities of labor demand and labor supply.

In a fixprice equilibrium $p = p_0$, so that:

$$\varphi^{\tau^l}(x) = -\frac{1}{c}\frac{\partial c}{\partial x}\frac{dx}{d\tau^l} = -\frac{1}{c}[-c\underbrace{\frac{\gamma'(x)}{1+\gamma(x)}\frac{f(x)}{f'(x)}}_{1-\theta(x)}]\varphi^* = \varphi^* - \theta(x)\varphi^* = \varphi^s(x), \tag{C.35}$$

so that in a fixprice equilibrium the labor income tax cut multiplier is identical to the payroll tax cut multiplier considered in the main text, and shares all of its properties.

Appendix C.2.3. Firms' sales tax cut multiplier

Differentiate the goods market clearing condition with respect to τ^s (at $\tau = \tau^c = \tau^l = \tau^s = G = 0$):

$$\alpha^{\frac{\alpha}{1-\alpha+\psi}} \left[\frac{1+\psi}{1-\alpha+\psi} (pf(x)a)^{\frac{1+\psi}{1-\alpha+\psi}-1} \left\{ \frac{dp}{d\tau^s} f(x)a + pf'(x)\frac{dx}{d\tau^s}a \right\} - \frac{\alpha}{1-\alpha+\psi} (pf(x)a)^{\frac{1+\psi}{1-\alpha+\psi}} \right] = 0 \tag{C.36}$$

$$\frac{1}{p}\frac{dp}{d\tau^s} + \frac{f'(x)}{f(x)}\frac{dx}{d\tau^s} = \frac{\alpha}{1+\psi} = \varphi^*. \tag{C.37}$$

Define the sales tax cut multiplier:

$$\varphi^{\tau^s}(x) \equiv \frac{d\{c+G\}/\{c+G\}}{d[-\tau^s]} = -\frac{1}{c}\frac{dc}{d\tau^s}. \tag{C.38}$$

In a competitive equilibrium $x = x^*$, so that:

$$\varphi^{\tau^s} = -\frac{1}{c}\frac{\partial c}{\partial p}\frac{dp}{d\tau^s} = -\frac{1}{c}[-c\frac{1}{p}]p\varphi^* = \varphi^*, \tag{C.39}$$

so that in a competitive equilibrium the sales tax cut multiplier is acyclical and pinned down exclusively by the elasticities of labor demand and labor supply.

In a fixprice equilibrium $p = p_0$, so that:

$$\varphi^{\tau^s}(x) = -\frac{1}{c}\frac{\partial c}{\partial x}\frac{dx}{d\tau^s} = -\frac{1}{c}[-c\underbrace{\frac{\gamma'(x)}{1+\gamma(x)}\frac{f(x)}{f'(x)}}_{1-\theta(x)}]\varphi^* = \varphi^* - \theta(x)\varphi^* = \varphi^s(x), \tag{C.40}$$

so that in a fixprice equilibrium the sales tax cut multiplier is identical to the payroll tax cut multiplier considered in the main text, and shares all of its properties.

Appendix D. Comparative statics

Appendix D.1. Competitive equilibrium

Panel (a) of Figure D.8 shows comparative statics following a positive demand-side shock, parameterized as a permanent increase in χ. The aggregate demand curve shifts out, exercising upward pressure on goods market tightness. In order to retain tightness at the socially efficient level, the price must increase to lower private consumption to offset the rise in aggregate demand. Higher price increases labor demand, which expands capacity until the goods market reaches the efficient level of tightness (x^*). The new equilibrium features tightness at the efficient level, with higher price and sales compared to the original equilibrium.

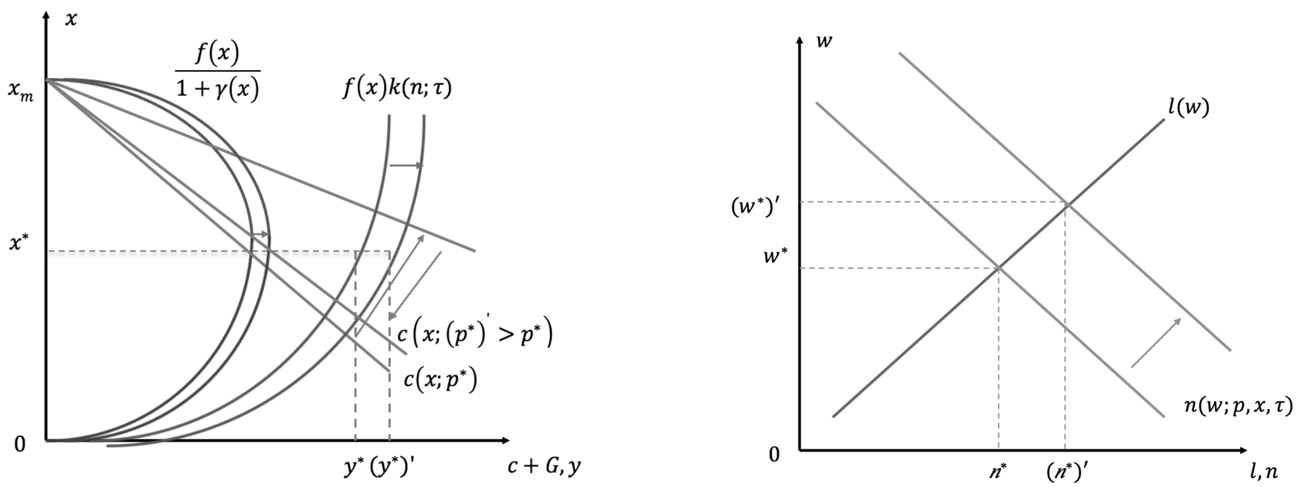
Panel (b) of Figure D.8 shows comparative statics following a positive supply shock, parameterized as a permanent increase in a. In response to the shock, aggregate supply curve shifts out, putting downward pressure on goods market tightness. In order to retain tightness at the socially efficient level, the price decreases in order to increase private consumption, until there are no more pressures on tightness to deviate from x^*. Eventually, tightness remains at the socially efficient level, sales increase and price falls.

Appendix D.2. Fixprice equilibrium

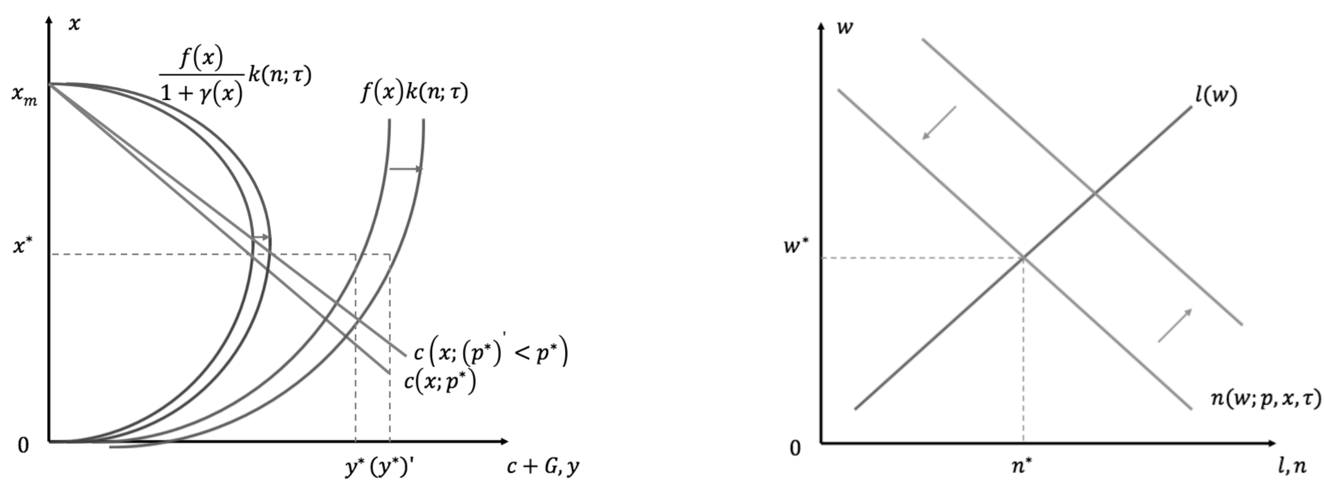
Panel (a) of Figure D.9 shows comparative statics following a positive demand-side shock, parameterized as a permanent increase in χ. The aggregate demand curve shifts out, creating excess demand that under the fixed price is cleared out by rising tightness increase the cost of search and decreasing private consumption; higher tightness also encourages more labor demand as the effective price from the firms' perspective increases. The latter effect causes an outward shift of the aggregate supply curve, but tightness remains above the initial level, similarly to sales. By construction, the price remains unchanged.

Figure D.8: Comparative statics in a competitive equilibrium

(a) Positive demand-side shock (increase in χ)



(b) Positive supply-side shock (increase in a)

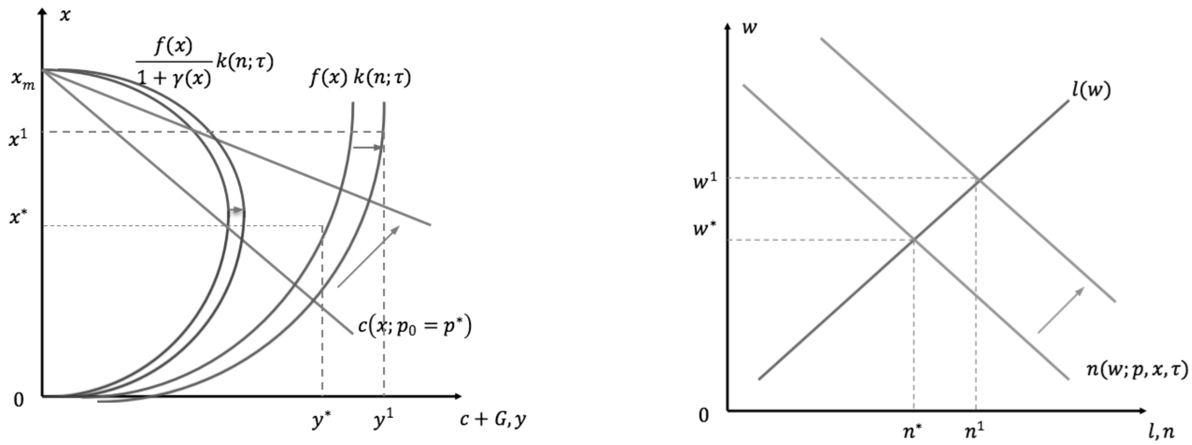


***Notes**: Panel (a) shows comparative statics in a competitive equilibrium, following a positive demand-side shock, parameterised as an increase in the preference parameter χ; following the shock, price increases to clear excess demand created by the shock and keep tightness at x^*, and equilibrium labor and sales also increase.*

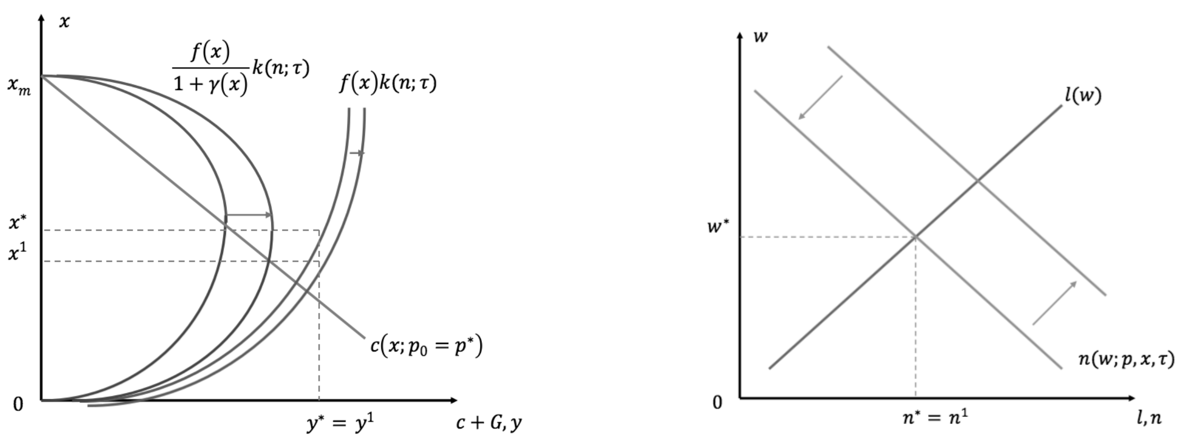
Panel (b) shows comparative statics in a competitive equilibrium, following a positive supply-side shock, parameterised as an increase in the technology parameter a; following the shock, price falls to clear excess supply caused by the shock and keep tightness at x^*, equilibrium labor remains unchanged and sales increase, as every unit of labor is now more productive, leading to higher capacity and higher sales, due to unchanged level of tightness.

Figure D.9: Comparative statics in a fixprice equilibrium

(a) Positive demand-side shock (increase in χ)



(b) Positive supply-side shock (increase in a)



***Notes**: Panel (a) shows comparative statics in a fixprice equilibrium that initially coincides with the socially efficient allocation, and is hit by a positive demand-side shock, parameterised as an increase in the preference parameter χ; following the shock, goods market tightness increases to clear excess demand created by the shock, and equilibrium labor and sales also increase.*

Panel (b) shows comparative statics in a fixprice equilibrium that initially coincides with the socially efficient allocation, and is hit by a positive supply-side shock, parameterised as an increase in the technology parameter a; following the shock, goods market tightness falls to clear excess supply caused by the shock, whereas equilibrium labor and sales remain unchanged, since the effects of lower tightness and higher level of technology exactly offset each other.

Panel (b) of Figure D.9 shows comparative statics for a positive supply-side shock, parameterized as a permanent increase in a. The aggregate supply curve shifts out, putting downward pressure on tightness via excess supply, and under the fixed price tightness falls to clear the market by lowering the cost of search for households and increasing private consumption. In equilibrium, tightness fall and sales remain unchanged.[29] By construction, the price also remains unchanged.

Appendix E. Results under general CRRA utility of consumption

Appendix E.1. Comparative statics

Goods market clearing under general CRRA utility is given by:

$$\frac{f(x)an^{\alpha}}{1+\gamma(x)} = \frac{\chi^{\frac{1}{\sigma}}}{(p[1+\gamma(x)])^{\frac{1}{\sigma}}} + G. \tag{E.1}$$

Combining with the equilibrium labor expression:

$$\alpha^{\frac{\alpha}{1-\alpha+\psi}}(pf(x)a)^{\frac{1+\psi}{1-\alpha+\psi}}(1+\tau)^{-\frac{\alpha}{1-\alpha+\psi}} = \chi^{\frac{1}{\sigma}}(p[1+\gamma(x)])^{1-\frac{1}{\sigma}} + p[1+\gamma(x)]G. \tag{E.2}$$

[29] Note that sales remain the same due to two countervailing forces: on the one hand, productivity increases, expanding capacity and leading to more sales, *ceteris paribus*; on the other hand, tightness falls, lowering $f(x)$, which decreases sales, *ceteris paribus*. In the special case of log utility of consumption and a fixprice equilibrium these two effects exactly offset each other. However, once one considers equilibria with rigid, but not fully fixed prices, as we do in Appendix B, the first effect dominates and sales rise following a positive technology shock.

Appendix E.1.1. Competitive equilibrium

In a competitive equilibrium $x = x^*$, so that $\frac{dx}{d\chi} = 0$; differentiate (E.2) (at $G = \tau = 0$) with respect to χ to find the comparative statics to a demand shock in a competitive equilibrium:

$$\alpha^{\frac{\alpha}{1-\alpha+\psi}} \frac{1+\psi}{1-\alpha+\psi} [pf(x)a]^{\frac{1+\psi}{1-\alpha+\psi}-1} \frac{dp}{d\chi} f(x)a = \frac{1}{\sigma}\chi^{\frac{1}{\sigma}-1}(p[1+\gamma(x)])^{1-\frac{1}{\sigma}} + \left(1-\frac{1}{\sigma}\right)\chi^{\frac{1}{\sigma}}(p[1+\gamma(x)])^{-\frac{1}{\sigma}}\frac{dp}{d\chi}[1+\gamma(x)] \tag{E.3}$$

$$\frac{dp}{d\chi} = \left[\frac{\alpha}{1-\alpha+\psi} + \frac{1}{\sigma}\right]^{-1} \frac{p}{\sigma\chi} > 0. \tag{E.4}$$

$$\frac{dc}{d\chi} = \frac{d}{d\chi}\left[\chi^{\frac{1}{\chi}} p^{-\frac{1}{\sigma}}[1+\gamma(x)]^{-\frac{1}{\sigma}}\right] = \frac{1}{\sigma}\chi^{\frac{1}{\sigma}-1}p^{-\frac{1}{\sigma}}[1+\gamma(x)]^{-\frac{1}{\sigma}} - \frac{1}{\sigma}\chi^{\frac{1}{\sigma}}p^{-\frac{1}{\sigma}-1}\frac{dp}{d\chi}[1+\gamma(x)]^{-\frac{1}{\sigma}} \tag{E.5}$$

$$\frac{dc}{d\chi} = \frac{c}{\sigma\chi}\left[\frac{1}{\sigma}\left(\frac{\alpha}{1+\psi}\right)^{-1} + \left(1-\frac{1}{\sigma}\right)\right]^{-1} > 0. \tag{E.6}$$

$$\frac{dy}{d\chi} = \frac{d}{d\chi}(c[1+\gamma(x)]) = [1+\gamma(x)]\frac{dc}{d\chi} > 0. \tag{E.7}$$

Similarly, differentiate (E.2) (at $G = \tau = 0$) with respect to a to find the comparative statics to a supply shock in a competitive equilibrium:

$$\frac{1+\psi}{1-\alpha+\psi}\alpha^{\frac{\alpha}{1-\alpha+\psi}} [pf(x)a]^{\frac{1+\psi}{1-\alpha+\psi}-1} \left[\frac{dp}{da}f(x)a + pf(x)\right] = \left(1-\frac{1}{\sigma}\right)\chi^{\frac{1}{\sigma}} (p[1+\gamma(x)])^{-\frac{1}{\sigma}} \frac{dp}{da}[1+\gamma(x)] \tag{E.8}$$

$$\frac{dp}{da} = -\frac{p}{a}\frac{\sigma(1+\psi)}{1+\psi+(\sigma-1)\alpha} < 0. \tag{E.9}$$

$$\frac{dc}{da} = \frac{d}{da}\left[\chi^{\frac{1}{\chi}} p^{-\frac{1}{\sigma}}[1+\gamma(x)]^{-\frac{1}{\sigma}}\right] = -\frac{1}{\sigma}\chi^{\frac{1}{\sigma}}p^{-\frac{1}{\sigma}-1}\frac{dp}{da}[1+\gamma(x)]^{-\frac{1}{\sigma}} \tag{E.10}$$

$$\frac{dc}{da} = \frac{c}{a}\frac{\frac{1}{\sigma}}{\frac{1}{\sigma} + \left(1-\frac{1}{\sigma}\right)\frac{\alpha}{1+\psi}} > 0. \tag{E.11}$$

$$\frac{dy}{da} = \frac{d}{da}(c[1+\gamma(x)]) = [1+\gamma(x)]\frac{dc}{da} > 0. \tag{E.12}$$

Appendix E.1.2. Fixprice equilibrium

In a fixprice equilibrium $p = p_0$ is a parameter, so that $\frac{dp}{d\chi} = 0$; differentiate (E.2) (at $G = \tau = 0$) with respect to χ to find the comparative statics to a demand shock in a fixprice equilibrium:

$$\alpha^{\frac{\alpha}{1-\alpha+\psi}} \frac{1+\psi}{1-\alpha+\psi} [pf(x)a]^{\frac{1+\psi}{1-\alpha+\psi}-1} pf'(x) \frac{dx}{d\chi} a = \frac{1}{\sigma} \chi^{\frac{1}{\sigma}-1} (p[1+\gamma(x)])^{1-\frac{1}{\sigma}} + \left(1 - \frac{1}{\sigma}\right) \chi^{\frac{1}{\sigma}} (p[1+\gamma(x)])^{-\frac{1}{\sigma}} p\gamma'(x) \frac{dx}{d\chi} \tag{E.13}$$

$$\left[\frac{1+\psi}{1-\alpha+\psi} - \left(1 - \frac{1}{\sigma}\right) \frac{\gamma'(x)}{1+\gamma(x)} \frac{f(x)}{f'(x)}\right] \frac{f'(x)}{f(x)} \frac{dx}{d\chi} = \frac{1}{\sigma\chi} \tag{E.14}$$

$$\frac{dx}{d\chi} = \frac{1}{\sigma\chi} \left[\frac{1+\psi+(\sigma-1)\alpha}{\sigma(1-\alpha+\psi)} + \left(1 - \frac{1}{\sigma}\right) \theta_{\sigma=1}(x)\right]^{-1} \frac{f(x)}{f'(x)} \tag{E.15}$$

$$\frac{dx}{d\chi}|_{x=x^*} = \frac{1}{\sigma\chi} \left[\frac{1+\psi+(\sigma-1)\alpha}{\sigma(1-\alpha+\psi)}\right]^{-1} \frac{f(x^*)}{f'(x^*)} > 0. \tag{E.16}$$

$$\frac{dc}{d\chi} = \frac{d}{d\chi} \left[\chi^{\frac{1}{\chi}} p^{-\frac{1}{\sigma}} [1+\gamma(x)]^{-\frac{1}{\sigma}}\right] = \frac{1}{\sigma} \chi^{\frac{1}{\sigma}-1} p^{-\frac{1}{\sigma}} [1+\gamma(x)]^{-\frac{1}{\sigma}} - \frac{1}{\sigma} \chi^{\frac{1}{\sigma}} p^{-\frac{1}{\sigma}} [1+\gamma(x)]^{-\frac{1}{\sigma}-1} \gamma'(x) \frac{dx}{d\chi} \tag{E.17}$$

$$\frac{dc}{d\chi} = \frac{c}{\sigma\chi} \left[\frac{1}{\sigma} [\varphi^d_{\sigma=1}(x)]^{-1} + \left(1 - \frac{1}{\sigma}\right)\right]^{-1}. \tag{E.18}$$

$$\frac{dc}{d\chi}|_{x=x^*} = \frac{c}{\sigma\chi} \frac{\sigma\alpha}{1+\psi+(\sigma-1)\alpha} > 0. \tag{E.19}$$

$$\frac{dy}{d\chi} = \frac{d}{d\chi}(c[1+\gamma(x)]) = \frac{dc}{dx}[1+\gamma(x)] + c\gamma'(x) \frac{dx}{d\chi} \tag{E.20}$$

$$\frac{dy}{d\chi}|_{x=x^*} = \frac{dc}{dx}|_{x=x^*} [1+\gamma(x^*)] + c\gamma'(x^*) \frac{dx}{d\chi}|_{x=x^*} > 0. \tag{E.21}$$

Similarly, differentiate (E.2) (at $G = \tau = 0$) with respect to a to find the comparative statics to a supply shock in a fixprice equilibrium:

$$\alpha^{\frac{\alpha}{1-\alpha+\psi}} \frac{1+\psi}{1-\alpha+\psi}[pf(x)a]^{\frac{1+\psi}{1-\alpha+\psi}-1}\left[pf'(x)\frac{dx}{dx}a + pf(x)\right] = \left(1-\frac{1}{\sigma}\right)\chi^{\frac{1}{\sigma}}(p[1+\gamma(x)])^{-\frac{1}{\sigma}}p\gamma'(x)\frac{dx}{da} \tag{E.22}$$

$$\frac{dx}{da} = -\frac{1}{a}\frac{1+\psi}{1-\alpha+\psi}\frac{f(x)}{f'(x)}\left[\frac{1+\psi+(\sigma-1)\alpha}{\sigma(1-\alpha+\psi)} + \left(1-\frac{1}{\sigma}\theta_{\sigma=1}(x)\right)\right]^{-1} \tag{E.23}$$

$$\frac{dx}{da}|_{x=x^*} = -\frac{1}{a}\frac{1+\psi}{1-\alpha+\psi}\frac{f(x^*)}{f'(x^*)}\left[\frac{1+\psi+(\sigma-1)\alpha}{\sigma(1-\alpha+\psi)}\right]^{-1} < 0. \tag{E.24}$$

$$\frac{dc}{da} = \frac{d}{da}\left[\chi^{\frac{1}{\chi}}p^{-\frac{1}{\sigma}}[1+\gamma(x)]^{-\frac{1}{\sigma}}\right] = -\frac{1}{\sigma}\chi^{\frac{1}{\sigma}}p^{-\frac{1}{\sigma}}[1+\gamma(x)]^{-\frac{1}{\sigma}-1}\gamma'(x)\frac{dx}{da} \tag{E.25}$$

$$\frac{dc}{da} = \frac{c}{a}\frac{1+\psi}{\alpha}\frac{\frac{1}{\sigma}\varphi^s_{\sigma=1}(x)}{\frac{1}{\sigma}+\left(1-\frac{1}{\sigma}\right)\varphi^d_{\sigma=1}(x)} \tag{E.26}$$

$$\frac{dc}{da}|_{x=x^*} = \frac{c}{a}\left[1+(\sigma-1)\frac{\alpha}{1+\psi}\right]^{-1} > 0. \tag{E.27}$$

$$\frac{dy}{da} = \frac{dc}{da}[1+\gamma(x)] + c\gamma'(x)\frac{dx}{da} \tag{E.28}$$

$$\frac{dy}{da} = (1-\sigma)\frac{dc}{da}[1+\gamma(x)] \tag{E.29}$$

$$\frac{dy}{da}|_{x=x^*} = (1-\sigma)\frac{dc}{da}_{x=x^*}[1+\gamma(x^*)]. \tag{E.30}$$

Appendix E.2. Demand-side fiscal multiplier

Differentiate (E.2) with respect to G (at $G = \tau = 0$):

$$\alpha^{\frac{\alpha}{1-\alpha+\psi}}\frac{1+\psi}{1-\alpha+\psi}\left(\frac{dp}{dG}f(x)a + \frac{f'(x)}{f(x)}\frac{dx}{dG}pa\right)(pf(x)a)^{\frac{1+\psi}{1-\alpha+\psi}-1} = \tag{E.31}$$

$$\chi^{\frac{1}{\sigma}}(p[1+\gamma(x)])^{-\frac{1}{\sigma}}(1-\frac{1}{\sigma})\left([1+\gamma(x)]\frac{dp}{dG} + p\gamma'(x)\frac{dx}{dG}\right) + p[1+\gamma(x)]. \tag{E.32}$$

In a competitive equilibrium $x = x^*$, so that $\frac{dx}{dG} = 0$; the demand-side multiplier is thus given by:

$$(\varphi^d_\sigma)^* = 1 + \frac{\partial c}{\partial p}\frac{dp}{dG} = \frac{\frac{1+\psi}{1-\alpha+\psi} - 1}{\frac{1+\psi}{1-\alpha+1} - (1-\frac{1}{\sigma})} = \frac{\frac{\alpha}{1+\psi}}{\frac{1}{\sigma} + (1-\frac{1}{\sigma})\frac{\alpha}{1+\psi}} = \left[\frac{1}{\sigma} \times \left(\frac{\alpha}{1+\psi}\right)^{-1} + \left(1 - \frac{1}{\sigma}\right) \times 1^{-1}\right]^{-1}. \tag{E.33}$$

(E.34)

or just a weighted harmonic average between $\frac{\alpha}{1+\psi}$ and one, with weights given by $\left(\frac{1}{\sigma}, 1-\frac{1}{\sigma}\right)$. Hence, in a competitive equilibrium the demand-side multiplier remains acyclical under a general CRRA utility of consumption.

In a fixprice equilibrium $p = p_0$, so that $\frac{dp}{dG} = 0$; the demand-side multiplier is thus given by:

$$\varphi^d_\sigma(x) = 1 + \frac{\partial c}{\partial x}\frac{dx}{dG} = \frac{\frac{1+\psi}{1-\alpha+\psi} - \frac{\gamma'(x)}{1+\gamma(x)}\frac{f(x)}{f'(x)}}{\frac{1+\psi}{1-\alpha+\psi} - \left(1-\frac{1}{\sigma}\right)\frac{\gamma'(x)}{1+\gamma(x)}\frac{f(x)}{f'(x)}} = \frac{\varphi^d_{\sigma=1}(x)}{\frac{1}{\sigma} + \left(1-\frac{1}{\sigma}\right)\varphi^d_{\sigma=1}(x)}$$
$$= \left[\frac{1}{\sigma} \times \left\{\varphi^d_{\sigma=1}(x)\right\}^{-1} + \left(1-\frac{1}{\sigma}\right) \times 1^{-1}\right]^{-1} \tag{E.35}$$

or just a weighted harmonic average between $\varphi^d_{\sigma=1}(x)$, which is the demand-side multiplier under $\sigma = 1$ considered in the main text, and one, with weights given by $\left(\frac{1}{\sigma}, 1-\frac{1}{\sigma}\right)$. Hence, $\varphi^d_\sigma(0) = 1$ and $\frac{d\varphi^d_\sigma(x)}{dx} = \frac{\frac{1}{\sigma}\frac{d\varphi^d_{\sigma=1}(x)}{dx}}{\left[\frac{1}{\sigma}+\left(1-\frac{1}{\sigma}\right)\varphi^d_{\sigma=1}(x)\right]^2} < 0, \quad \forall x \in (0, x_m)$, so that $\frac{d\varphi^d_\sigma(x)}{d\chi}|_{x=x^*} = \frac{d\varphi^d_\sigma(x)}{dx}|_{x=x^*}\frac{dx}{d\chi}|_{x=x^*} < 0$ and $\frac{d\varphi^d_\sigma(x)}{da}|_{x=x^*} = \frac{d\varphi^d_\sigma(x)}{dx}|_{x=x^*}\frac{dx}{da}|_{x=x^*} > 0$, establishing that the cyclicality properties of the demand-side multiplier found in the main text are preserved under a general CRRA utility of consumption in the local neighborhood of the efficient allocation.

Appendix E.3. Supply-side fiscal multiplier

Differentiate (E.2) with respect to τ (at $G = \tau = 0$):

$$\left[\frac{1+\psi}{1-\alpha+\psi}\frac{f'(x)}{f(x)} - \left(1-\frac{1}{\sigma}\right)\frac{\gamma'(x)}{1+\gamma(x)}\right]\frac{dx}{d\tau} + \left[\frac{1+\psi}{1-\alpha+\psi}\frac{1}{p} - \left(1-\frac{1}{\sigma}\right)\frac{1}{p}\right]\frac{dp}{d\tau} = \frac{\alpha}{1-\alpha+\psi}. \tag{E.36}$$

In a competitive equilibrium $x = x^*$, so that $\frac{dx}{d\tau} = 0$; the supply-side multiplier is thus given by:

$$(\varphi^s_\sigma)^* = -\frac{1}{c}\frac{\partial c}{\partial p}\frac{dp}{d\tau} = \frac{\frac{1}{\sigma}\frac{\alpha}{1-\alpha+\psi}}{\frac{1+\psi}{1-\alpha+\psi} - \left(1-\frac{1}{\sigma}\right)} = \frac{\frac{1}{\sigma}\frac{\alpha}{1+\psi}}{\frac{1}{\sigma} + \left(1-\frac{1}{\sigma}\right)\frac{\alpha}{1+\psi}} = \frac{1}{\sigma}(\varphi^d)^*, \tag{E.37}$$

so that in a competitive equilibrium the supply-side multiplier remains acyclical under a general CRRA utility of consumption.

In a fixprice equilibrium $p = p_0$, so that $\frac{dp}{d\tau} = 0$; the supply-side multiplier is thus given by:

$$\varphi_\sigma^s(x) = -\frac{1}{c}\frac{\partial c}{\partial x}\frac{dx}{d\tau} = \frac{\frac{1}{\sigma}\frac{\alpha}{1-\alpha+\psi}\frac{\gamma'(x)}{1+\gamma(x)}\frac{f(x)}{f'(x)}}{\frac{1+\psi}{1-\alpha+\psi} - \left(1-\frac{1}{\sigma}\right)\frac{\gamma'(x)}{1+\gamma(x)}\frac{f(x)}{f'(x)}} = \frac{\frac{1}{\sigma}\varphi_{\sigma=1}^s(x)}{\frac{1}{\sigma} + \left(1-\frac{1}{\sigma}\right)\varphi_{\sigma=1}^d(x)}, \tag{E.38}$$

where $\varphi_{\sigma=1}^s(x)$ is the supply-side multiplier under $\sigma = 1$ considered in the main text. Hence, $\varphi_\sigma^s(0) = 0$ and $\frac{d\varphi_\sigma^s(x)}{dx} = -\frac{\frac{1}{\sigma}\frac{\alpha}{1+\psi}\theta'(x)}{\left[\frac{1}{\sigma}+\left(1-\frac{1}{\sigma}\right)\varphi_{\sigma=1}^d(x)\right]^2} > 0, \quad \forall x \in (0, x_m)$, so that $\frac{d\varphi_\sigma^s(x)}{d\chi}|_{x=x^*} = \frac{d\varphi_\sigma^s(x)}{dx}|_{x=x^*}\frac{dx}{d\chi}|_{x=x^*} > 0$ and $\frac{d\varphi_\sigma^s(x)}{da}|_{x=x^*} = \frac{d\varphi_\sigma^s(x)}{dx}|_{x=x^*}\frac{dx}{da}|_{x=x^*} < 0$, establishing that the cyclicality properties of the supply-side multiplier found in the main text are preserved under a general CRRA utility of consumption in the local neighborhood of the efficient allocation.

Appendix F. Results under utility cost per visit

Appendix F.1. Household optimization

In this version of the model, the setup is remains unchanged compared to the baseline case in main text, except households now face a *utility* cost $\iota > 0$ per visit; given that every visit is successful with probability $q(x)$, the total number of visits required to purchase c units of the produced good is $v = c/q(x)$, and the total utility cost of search is $\iota \times v = \iota \times c/q(x)$. Households' optimization problem is now given by:

$$\max_{c,m,l}\left[\chi\frac{c^{1-\sigma}}{1-\sigma} + \zeta(m) - \frac{l^{1+\psi}}{1+\psi} - \iota\frac{c}{q(x)}\right] \qquad \text{s.t.} \tag{F.1}$$

$$pc + m \leq wl + \bar{m} + \Pi - T. \tag{F.2}$$

As before, here we normalize $\bar{m}$ so that $\zeta'(\bar{m}) = 1$, and focus on the special case of log utility of consumption ($\sigma = 1$). The solution to the above problem delivers a labor supply function identical to the one in the baseline model; however, the consumption function now takes a different form:

$$c(p, x) = \frac{\chi}{p + \varsigma(x)}, \tag{F.3}$$

where $\varsigma(x) \equiv \frac{\iota x}{f(x)} = \frac{\iota}{q(x)} > 0, \quad \varsigma'(x) > 0, \forall x \in (0, +\infty)$ summarizes the total cost of search, which is now additive to the price, and strictly increases in tightness on the whole domain. It thus follows that $\frac{\partial c}{\partial p} = -\frac{\chi}{[p+\varsigma(x)]^2} < 0$ and $\frac{\partial c}{\partial x} = -\frac{\chi\varsigma'(x)}{[p+\varsigma(x)]^2} < 0$.

The firms' problem remains unchanged, hence labor market equilibrium remains unaffected; goods market clearing can now be written as:

$$f(x)an^{\alpha} = c(p,x) + G \tag{F.4}$$

$$f(x)a\left[\alpha p f(x)a\right]^{\frac{\alpha}{1-\alpha+\psi}}(1+\tau)^{-\frac{\alpha}{1-\alpha+\psi}} = \frac{\chi}{p+\varsigma(x)} + G \tag{F.5}$$

$$\alpha^{\frac{\alpha}{1-\alpha+\psi}}\left[pf(x)a\right]^{\frac{1+\psi}{1-\alpha+\psi}}\left[p+\varsigma(x)\right] = p\chi + pG[p+\varsigma(x)]. \tag{F.6}$$

Given the evidence that fixprice equilibrium is more empirically relevant at business cycle frequencies, we will continue our analysis in this section under the assumption of fixprice equilibrium, so that $p = p_0$ is a parameter.

Appendix F.2. Comparative statics

Differentiate (F.6) with respect to χ (at $G = \tau = 0$) in order to find comparative statics after a demand shock:

$$\alpha^{\frac{\alpha}{1-\alpha+\psi}}\left[\frac{1+\psi}{1-\alpha+\psi}(pf(x)a)^{\frac{1+\psi}{1-\alpha+\psi}-1}pf'(x)\frac{dx}{d\chi}a[p+\varsigma(x)] + (pf(x)a)^{\frac{1+\psi}{1-\alpha+\psi}}\varsigma'(x)\frac{dx}{d\chi}\right] = p \tag{F.7}$$

$$\frac{dx}{d\chi} = \frac{1}{\chi}\left[\frac{1+\psi}{1-\alpha+\psi}\frac{f'(x)}{f(x)} + \frac{\varsigma'(x)}{p+\varsigma(x)}\right]^{-1} > 0. \tag{F.8}$$

$$\frac{dc}{d\chi} = \frac{1}{p+\varsigma(x)} - \frac{\chi\varsigma'(x)}{[p+\varsigma(x)]^2}\frac{dx}{d\chi} \tag{F.9}$$

$$\frac{dc}{d\chi} = \frac{\frac{1+\psi}{1-\alpha+\psi}}{\left[\frac{1+\psi}{1-\alpha+\psi} + \frac{\varsigma'(x)}{p+\varsigma(x)}\right][p+\varsigma(x)]} > 0. \tag{F.10}$$

Similarly, differentiate (F.6) with respect to a (at $G = \tau = 0$) in order to find comparative statics after a supply shock:

$$\alpha^{\frac{\alpha}{1-\alpha+\psi}}\left[\frac{1+\psi}{1-\alpha+\psi}(pf(x)a)^{\frac{1+\psi}{1-\alpha+\psi}-1}(pf'(x)\frac{dx}{da}a+pf(x))[p+\varsigma(x)]+(pf(x)a)^{\frac{1+\psi}{1-\alpha+\psi}}\varsigma'(x)\frac{dx}{da}\right]=0 \tag{F.11}$$

$$\frac{dx}{da} = -\frac{1+\psi}{1-\alpha+\psi}\frac{1}{a}\left[\frac{1+\psi}{1-\alpha+\psi}\frac{f'(x)}{f(x)}+\frac{\varsigma'(x)}{p+\varsigma(x)}\right]^{-1} < 0. \tag{F.12}$$

$$\frac{dc}{da} = -\frac{\chi\varsigma'(x)}{[p+\varsigma(x)]^2}\frac{dx}{da} \tag{F.13}$$

$$\frac{dc}{da} = \frac{\frac{\chi\varsigma'(x)}{[p+\varsigma(x)]^2}\frac{1+\psi}{1-\alpha+\psi}\frac{1}{a}}{\frac{1+\psi}{1-\alpha+\psi}\frac{f'(x)}{f(x)}+\frac{\varsigma'(x)}{p+\varsigma(x)}} > 0. \tag{F.14}$$

Appendix F.3. Demand-side fiscal multiplier

Differentiating (F.6) with respect to G (at $G = \tau = 0$) delivers the following:

$$\frac{dx}{dG} = \frac{1}{c}\left[\frac{1+\psi}{1-\alpha+\psi}\frac{f'(x)}{f(x)}\frac{\varsigma'(x)}{p+\varsigma(x)}\right]^{-1}. \tag{F.15}$$

From the definition of the demand-side fiscal multiplier:

$$\varphi^d(x) = 1+\frac{\partial c}{\partial x}\frac{dx}{dG} \tag{F.16}$$

$$= \left[1+\frac{1-\alpha+\psi}{1+\psi}\frac{\varsigma'(x)}{p+\varsigma(x)}\frac{f(x)}{f'(x)}\right]^{-1}. \tag{F.17}$$

After some algebra it can be shown that:

$$\frac{d\varphi^d(x)}{dx} = \frac{\iota\frac{1-\alpha+\psi}{1+\psi}q(x)^\delta q'(x)(\delta[p+\varsigma(x)]+p(1-q(x)^\delta))}{[[p+\varsigma(x)]q(x)^{1+\delta}+\frac{1-\alpha+\psi}{1+\psi}\iota(1-q(x)^\delta)]^2} < 0, \tag{F.18}$$

since $\varsigma(x) > 0, q(x) \in (0,1), q'(x) < 0, \forall x \in (0,+\infty)$ and $\delta > 0$. Hence, $\frac{\varphi^d(x)}{d\chi} = \frac{d\varphi^d(x)}{dx}\frac{dx}{d\chi} < 0$ and $\frac{d\varphi^d(x)}{da} = \frac{d\varphi^d(x)}{dx}\frac{dx}{da} > 0$, so the cyclicality properties of the demand-side fiscal multiplier found in the main text are preserved.

Appendix F.4. Supply-side fiscal multiplier

Differentiating (F.6) with respect to τ (at $G = \tau = 0$) delivers the following:

$$\frac{dx}{d\tau} = \frac{\alpha}{1-\alpha+\psi}\left[\frac{1+\psi}{1-\alpha+\psi}\frac{f'(x)}{f(x)} + \frac{\varsigma'(x)}{p+\varsigma(x)}\right]^{-1}. \tag{F.19}$$

From the definition of the supply-side fiscal multiplier:

$$\varphi^s(x) = -\frac{1}{c}\frac{\partial c}{\partial x}\frac{dx}{d\tau} \tag{F.20}$$

$$= \frac{\alpha}{1+\psi}\varsigma'(x)f(x)\left[1 + \frac{1-\alpha+\psi}{1+\psi}\frac{\varsigma'(x)}{p+\varsigma(x)}\frac{f(x)}{f'(x)}\right]^{-1}. \tag{F.21}$$

After some algebra it can be shown that:

$$\frac{\varphi^s(x)}{dx} = -\frac{\iota\frac{\alpha}{1+\psi}q(x)^{\delta}q'(x)(\delta(p+\varsigma(x)) + p(1-q(x)^{\delta}))}{[[p+\varsigma(x)]q(x)^{1+\delta} + \frac{1-\alpha+\psi}{1+\psi}\iota(1-q(x)^{\delta})]^2} > 0, \tag{F.22}$$

since $\varsigma(x) > 0, q(x) \in (0,1), q'(x) < 0, \forall x \in (0,+\infty)$ and $\delta > 0$. Hence, $\frac{\varphi^s(x)}{d\chi} = \frac{d\varphi^s(x)}{dx}\frac{dx}{d\chi} > 0$ and $\frac{d\varphi^s(x)}{da} = \frac{d\varphi^s(x)}{dx}\frac{dx}{da} < 0$, so the cyclicality properties of the supply-side fiscal multiplier found in the main text are preserved.

Appendix G. Social planner's allocation

Appendix G.1. Static model

The social planner's problem is given by:

$$\max_{c,l,v,m}\left[\chi\frac{c^{1-\sigma}}{1-\sigma} + \zeta(m) - \frac{l^{1+\psi}}{1+\psi}\right] \qquad \text{s.t.} \tag{G.1}$$

$$c + G + \rho v = \left[(al^{\alpha})^{-\delta} + v^{-\delta}\right]^{-\frac{1}{\delta}}, \qquad m = \bar{m}. \tag{G.2}$$

Inserting $m = \bar{m}$, the associated Lagrangian becomes:

$$\mathcal{L} = \left[\chi\frac{c^{1-\sigma}}{1-\sigma} + \zeta(\bar{m}) - \frac{l^{1+\psi}}{1+\psi}\right] + \lambda\left(\left[(al^{\alpha})^{-\delta} + v^{-\delta}\right]^{-\frac{1}{\delta}} - (c+G+\rho v)\right) \tag{G.3}$$

The first-oder conditions are:

$$\frac{\partial \mathcal{L}}{\partial c} = \chi c^{-\delta} - \lambda = 0 \tag{G.4}$$

$$\frac{\partial \mathcal{L}}{\partial v} = \lambda \left(-\frac{1}{\delta} \left[(al^{\alpha})^{-\delta} + v^{-\delta} \right]^{-\frac{1}{\delta}-1} (-\delta) v^{-\delta-1} - \rho \right) = 0 \tag{G.5}$$

$$\frac{\partial \mathcal{L}}{\partial v} = -l^{\psi} + \lambda \left(-\frac{1}{\delta} \left[(al^{\alpha})^{-\delta} + v^{-\delta} \right]^{-\frac{1}{\delta}-1} (-\delta) \alpha a l^{\alpha-1} \right) = 0 \tag{G.6}$$

Note that $x \equiv \frac{v}{al^{\alpha}}$ and $f'(x) = (1 + x^{\delta})^{-\frac{1}{\delta}-1}$, the social planner's allocation $\{c^*, l^*, v^*, m^*, x^*\}$ is given by:

$$f'(x^*) = \rho \tag{G.7}$$

$$x^* = \frac{v^*}{a(l^*)^{\alpha}} \tag{G.8}$$

$$\chi (c^*)^{-\sigma} = \frac{\left[(a(l^*)^{\alpha})^{-\delta} + (v^*)^{-\delta} \right]^{-\frac{1}{\delta}-1} \alpha a (l^*)^{\alpha-1}}{(l^*)^{\psi}} \tag{G.9}$$

$$c^* + G + \rho v^* = \left[(a(l^*)^{\alpha})^{-\delta} + (v^*)^{-\delta} \right]^{-\frac{1}{\delta}} \tag{G.10}$$

$$m^* = \bar{m}. \tag{G.11}$$

Appendix G.2. Dynamic model

The social planner's problem is given by:

$$\max_{\{c_{t+s}, l_{t+s}, m_{t+s}, v_{t+s}, y_{t+s}\}_{s=0}^{\infty}} \mathbb{E}_t \sum_{s=0}^{\infty} \beta^s \left[\chi_{t+s} \frac{c_{t+s}^{1-\sigma}}{1-\sigma} + \zeta(m_{t+s}) - \nu \frac{l_{t+s}^{1+\psi}}{1+\psi} \right] \qquad \text{s.t.} \tag{G.12}$$

$$y_t = (1-\eta) y_{t-1} + \left[v_t^{-\delta} + (a_t l_t^{\alpha} - (1-\eta) y_{t-1})^{-\delta} \right]^{-\frac{1}{\delta}}, \qquad \forall t \geq 0 \tag{G.13}$$

$$y_t = c_t + G_t + \rho v_t, \qquad m_t = \bar{m}, \qquad \forall t \geq 0. \tag{G.14}$$

The associated Lagrangian is given by:

$$\mathcal{L}_t = \mathbb{E}_t \sum_{s=0}^{\infty} \beta^s [\chi_{t+s} \frac{c_{t+s}^{1-\sigma}}{1-\sigma} + \zeta(\bar{m}) - \frac{l_{t+s}^{1+\psi}}{1+\psi} + \lambda_{t+s} \left(y_{t+s} - (1-\eta) y_{t+s-1} - \left[v_{t+s}^{-\delta} + (a_{t+s} l_{t+s}^{\alpha} - (1-\eta) y_{t+s-1})^{-\delta} \right]^{-\frac{1}{\delta}} \right)$$
$$+ \mu_{t+s} (y_{t+s} - c_{t+s} - G_{t+s} - \rho v_{t+s})]. \tag{G.15}$$

The first order conditions are given by:

$$\lambda_t + \frac{\nu l_t^{\psi}}{\left[x_t^{-\delta}+1\right]^{-\frac{1}{\delta}-1}\alpha a_t l_t^{\alpha-1}} = 0, \qquad \forall t \geq 0 \tag{G.16}$$

$$\lambda_t[1+x_t^{\delta}]^{-\frac{1}{\delta}-1} + \rho\chi_t c_t^{-\sigma} = 0, \qquad \forall t \geq 0 \tag{G.17}$$

$$\lambda_t + \chi_t c_t^{-\sigma} + \beta(1-\eta)\mathbb{E}_t\left[\lambda_{t+1}\left((x_{t+1}^{-\delta}+1)^{-\frac{1}{\delta}-1}-1\right)\right] = 0, \qquad \forall t \geq 0 \tag{G.18}$$

which together with the definition of tightness $x_t = \frac{v_t}{a_t l_t^{\alpha}-(1-\eta)y_{t-1}}$ and the feasibility constraints describe the social planner's allocation.

Appendix H. Dynamic model: further results and steady state

Appendix H.1. Decentralized equilibrium: full set of optimality conditions

$$f(x_t) \equiv (1+x_t^{-\delta})^{-\frac{1}{\delta}} \tag{H.1}$$

$$\gamma(x_t) \equiv \frac{\rho x_t}{f(x_t)-\rho x_t} \tag{H.2}$$

$$y_t^c = [1+\gamma(x_t)]c_t - (1-\eta)\gamma(x_t)y_{t-1}^c \tag{H.3}$$

$$\chi_t c_t^{-\sigma} + \beta(1-\eta)\mathbb{E}_t\left[\chi_{t+1}c_{t+1}^{-\sigma}\frac{[1+\gamma(x_t)]}{[1+\gamma(x_{t+1})]}\gamma(x_{t+1})\right] = p_t[1+\gamma(x_t)] \tag{H.4}$$

$$l_t = [w_t/\nu]^{\frac{1}{\psi}} \tag{H.5}$$

$$F_{t,t+s} = \beta^s \tag{H.6}$$

$$y_t = (1-\eta)y_{t-1} + f(x_t)\left[a_t n_t^{\alpha} - (1-\eta)y_{t-1}\right] \tag{H.7}$$

$$p_t + (1-\eta)\mathbb{E}_t\left[F_{t,t+1}\frac{w_{t+1}(1+\tau_{t+1})}{\alpha f(x_{t+1})a_{t+1}n_{t+1}^{\alpha-1}}[1-f(x_{t+1})]\right] = \frac{w_t(1+\tau_t)}{\alpha f(x_t)a_t n_t^{\alpha-1}} \tag{H.8}$$

$$y_t^G = [1+\gamma(x_t)]G_t - (1-\eta)\gamma(x_t)y_{t-1}^G \tag{H.9}$$

$$y_t = y_t^c + y_t^G \tag{H.10}$$

$$l_t = n_t \tag{H.11}$$

$$G_t = (1-\rho_G)g + \rho_G G_{t-1} + \varepsilon_t^G \tag{H.12}$$

$$\tau_t = (1-\rho_\tau)\tau + \rho_\tau\tau_{t-1} + \varepsilon_t^{\tau} \tag{H.13}$$

Appendix H.2. Alternative fiscal instruments in the dynamic model

In this subsection we consider two additional fiscal instruments in the context of the our dynamic model: distortionary taxation on consumption (τ_t^c) and households' labor income (τ_t^l). Compared to the baseline model in the main text, the representative household's per-period budget constraint becomes:

$$p_t(1+\tau_t^c)y_t^c + m_t + \mathbb{E}_t\left[F_{t,t+1}B_{t+1}\right] \leq w_t(1-\tau_t^l)l_t + \bar{m}_t + B_t + \Pi_t - T_t, \qquad \forall t \geq 0, \tag{H.14}$$

and the first order conditions for the choice of consumption and labor supply become:

$$\chi_t c_t^{-\sigma} + \beta(1-\eta)\mathbb{E}_t\left[\chi_{t+1}c_{t+1}^{-\sigma}\frac{[1+\gamma(x_t)]}{[1+\gamma(x_{t+1})]}\gamma(x_{t+1})\right] = p_t(1+\tau_t^c)[1+\gamma(x_t)], \tag{H.15}$$

$$l_t = [w_t(1-\tau_t^l)/\nu]^{\frac{1}{\psi}}. \tag{H.16}$$

We further assume that the two additional tax rates follow exogenous autoregressive processes:

$$\tau_t^i = (1-\rho_\tau)\tau^i + \rho_\tau\tau_{t-1}^i + \varepsilon_t^{\tau^i}, \qquad \forall t \geq 0, \quad \tau_t^i \in \{\tau_t^l, \tau_t^c\}, \tag{H.17}$$

and the lump sum tax raised by the government is now given by:

$$T_t = p_t y_t^G - w_t n_t \tau_t - w_t n_t \tau_t^l - p_t y_t^c \tau_t^c. \tag{H.18}$$

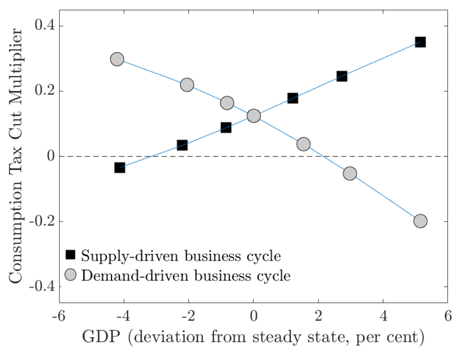
Steady state consumption and labor income taxes ($\tau^c = 0.05, \tau^l = 0.28$) follow calibrations in Trabandt and Uhlig (2011) and Zanetti (2012). The rest of the model remains unchanged.

We compute horizon-specific conditional state-dependent multipliers out of cuts in taxes on consumption and labor income, following the methodology used for multipliers out of payroll taxes, as detailed in the main text. Figure H.10 shows the results for impact, 2-year and 4-year horizon multipliers. As one can see, multipliers out of cuts in consumption taxes exhibit cyclicality properties that are similar to those of government consumption multipliers, as detailed in the main text; in particular, compared to the steady state, consumption tax cut multipliers rise in demand-side recessions and supply side recessions, but fall in demand-side expansions and supply-side recession, with the magnitude of state dependence falling at further horizons.

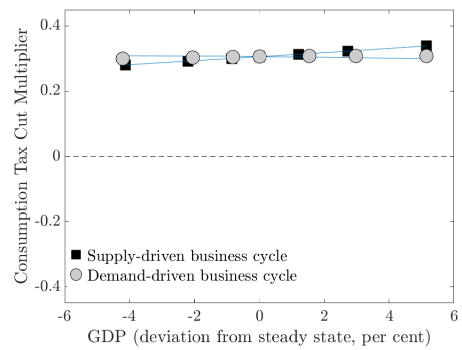
As for multipliers out of cuts in taxes on labor income, those have cyclicality properties identical to those of payroll tax cut multipliers. Indeed, compared to steady state, labor income tax cut multipliers are high in supply-side recession and demand-side expansions, whereas they are low in demand-side recessions and supply-side expansions. As before, the magnitude of state-dependence falls with the horizon considered.

Figure H.10: Conditional state-dependent fiscal multipliers

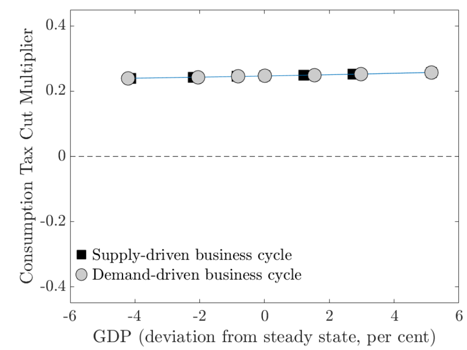
(a) Impact multiplier: cut consumption tax by 1%



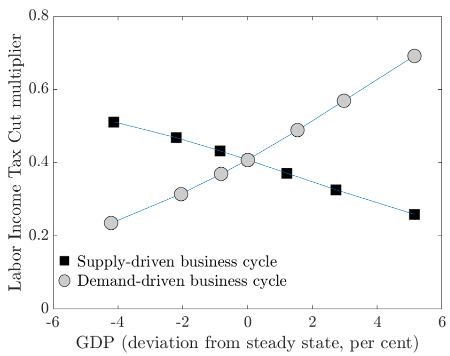
(b) 2-year hor. multiplier: cut consumption tax by 1%



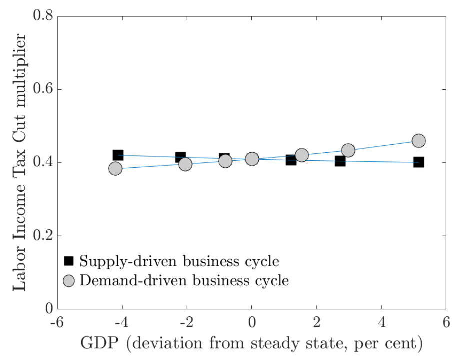
(c) 4-year hor. multiplier: cut consumption tax by 1%



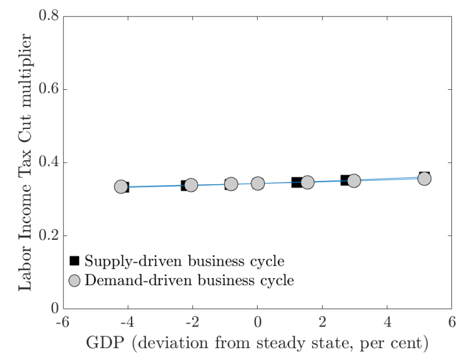
(d) Impact multiplier: cut labor income tax by 1%



(e) 2-year hor. multiplier: cut labor income tax by 1%



(f) 4-year hor. multiplier: cut labor income tax by 1%



***Notes**: Panel (a) shows impact multipliers following a one-time innovation to the consumption tax rate process equal to negative 1 percentage point, in recessionary and expansionary episodes caused by different types of shocks; Panels (b) and (c) repeat the exercise for the 2-year horizon and the 4-year horizon tax cut multipliers, respectively.*

Panel (d) shows impact multipliers following a one-time innovation to the labor income tax rate process equal to negative 1 percentage point, in recessionary and expansionary episodes caused by different types of shocks; Panels (e) and (f) repeat the exercise for the 2-year horizon and the 4-year horizon tax cut multipliers, respectively.

Appendix H.3. Decentralized equilibrium: steady state

$$c^{-\sigma} = \frac{p(1+\tau^c)[1+\gamma(x)]}{1+\beta(1-\eta)\gamma(x)} \tag{H.19}$$

$$y^c = \frac{[1+\gamma(x)]c}{1+(1-\eta)\gamma(x)} \tag{H.20}$$

$$l^\psi = w(1-\tau^l)/\nu \tag{H.21}$$

$$y = \frac{f(x)l^\alpha}{1-(1-\eta)(1-f(x))} \tag{H.22}$$

$$p = \frac{w(1+\tau)}{\alpha f(x) l^{\alpha-1}}[1-(1-\eta)\beta(1-f(x))] \tag{H.23}$$

$$y^G = \frac{[1+\gamma(x)]g}{1+(1-\eta)\gamma(x)} \tag{H.24}$$

$$y = y^c + y^G \tag{H.25}$$

$$x = \frac{v}{l^\alpha - (1-\eta)y} \tag{H.26}$$

$$m = \bar{m} \tag{H.27}$$

$$\gamma(x) = \frac{\rho x}{f(x) - \rho x} \tag{H.28}$$

$$f(x) = (1+x^{-\delta})^{-\frac{1}{\delta}}. \tag{H.29}$$

Appendix H.4. Social planner's allocation: steady state

$$c^{-\sigma} = \frac{\nu l^{1+\psi-\alpha}}{\rho}\left[\frac{1+x^\delta}{1+x^{-\delta}}\right]^{-\frac{1}{\delta}-1} \tag{H.30}$$

$$y = \frac{1}{\eta}\left[v^{-\delta} + (l^\alpha - (1-\eta)y)^{-\delta}\right]^{-\frac{1}{\delta}} \tag{H.31}$$

$$\frac{1}{\rho}[1+x^\delta]^{-\frac{1}{\delta}-1} = 1 + \beta(1-\eta)[(1+x^{-\delta})^{-\frac{1}{\delta}-1} - 1] \tag{H.32}$$

$$y = c + g + \rho v \tag{H.33}$$

$$x = \frac{v}{l^\alpha - (1-\eta)y} \tag{H.34}$$

$$m = \bar{m}. \tag{H.35}$$

Appendix I. Econometric evidence: additional results and robustness checks

Appendix I.1. Demand-side and supply-side recessions: a closer look

Panel (a) in Figure I.11 shows historical periods of demand-side recessions characterized by a negative co-movement between unemployment and the cyclical component of inflation (solid shaded area), and supply-side recession characterized by positive comovement between these variables (striped shaded area). The majority of the US Great Depression is identified as a demand-side recession; the oil shocks of the 1970s start off as a supply-side recession, evolving into a demand-side recession. In the case of the late 1970s/early 1980s recession, this could be due to Volcker disinflation that immediately followed the second wave of oil shocks. The Great Recession, on the other hand, originates as a demand-side recession, evolving into a supply-side recession. One explanation is that initial negative effect on households wealth and income evolved into a supply-side constraint as firms were unable to access capital due to the distorted financial system.

Our identification strategy relies on having enough spending and taxation shocks in each of the three states of the world considered in the baseline specification. Panel (b) in Figure I.11 plots the time series for military spending news shocks from Ramey and Zubairy (2018) and narrative tax rate shocks from Romer and Romer (2010) against our definition of states. The figure shows that spending and tax rate shocks are spread fairly evenly across expansions, demand- and supply-side recessions.[30] Formally, 28% of quarters identified as an expansion, 16% of quarters identified as a demand-side recession and 16% of quarters identified as a supply-side recession contain a non-zero military spending news shock. Similarly, 14% of quarters identified as an expansion, 31% of quarters identified as a demand-side recession and 32% of quarters identified as a supply-side recession contain a non-zero narrative tax shock.

Appendix I.2. Responses to an identified productivity shock

In this subsection we would like to test whether following an identified productivity shock, output, tightness and inflation move in the same direction as predicted by our theory. We run the following sequence of local projections:

$$variable_{t+H} = \alpha_H + \beta_H \times a_t + \gamma_H \mathbf{z}_{t-1} + \varepsilon_{t+H}, \tag{I.1}$$

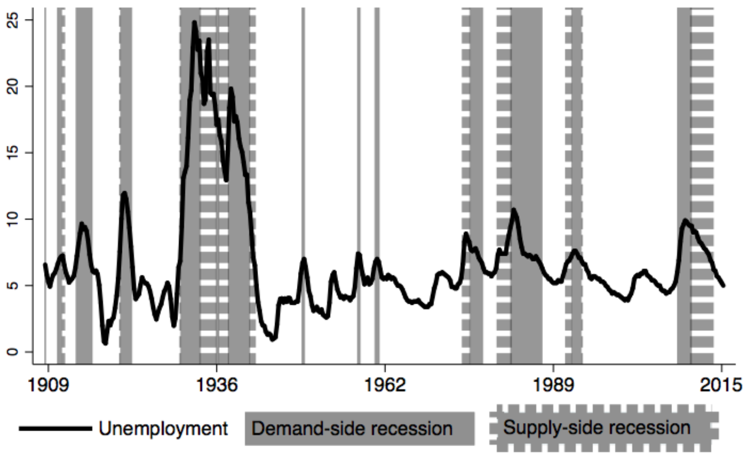
with $variable_t \in \left\{\frac{GDP_t}{GDP_t^*}, x_t, \pi_t\right\}$, where $\frac{GDP_t}{GDP_t^*}$ is real GDP over its polynomial trend, x_t is goods market tightness series from Michaillat and Saez (2015), π_t is inflation based on GDP deflator, and $a_t \equiv \ln TFP_t - \ln TFP_{t-1}$, where TFP is utilization-adjusted TFP series from Fernald (2014); the set of controls $\mathbf{z}_{t-1}$ is specified in the description to the results figure.

In Figure I.12 one can see that consistently with our theory, following a positive productivity shock, one obtains a statistically significant increase in (cyclical) output, as well as a statistically significant reduction in inflation and goods market tightness.

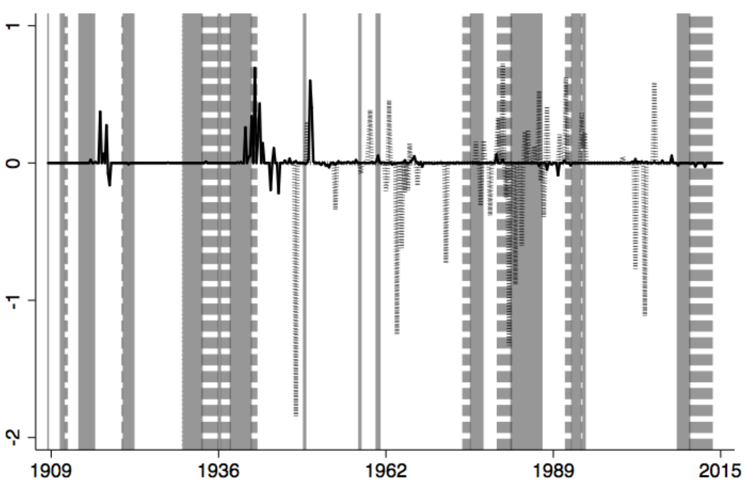
[30] A bulk of variation in our spending and tax rate shocks overlaps with periods of price controls around World War II and the Korean War. In order to check that such price controls do not pose a challenge to our strategy of using co-movement between inflation and activity, Figure I.13 in Appendix I.3 reports estimated responses of inflation to our spending and tax rate shocks. Reassuringly, we find that both a positive spending shock and a positive tax rate shock produce a statistically significant rise in inflation.

Figure I.11: Demand- and supply-side recessions and fiscal shocks

(a) Demand- and supply-side recessions



(b) Fiscal shocks and sources of recessions



Notes: *Panel (a) shows the unemployment rate in the US between 1909-2015, as well as demand-side recessions, identified by the indicator variable* $\mathbf{1}\{U_t \geq \bar{U}; \pi_t < \bar{\pi}_t\}$*, and supply-side recessions, identified by the indicator variable* $\mathbf{1}\{U_t \geq \bar{U}; \pi_t \geq \bar{\pi}_t\}$*; Panel (b) additionally plots time series of military spending news shocks from Ramey and Zubairy (2018) and narrative tax rate shocks from Romer and Romer (2010).*

Appendix I.3. Inflation responses to fiscal shocks

A bulk of variation in our spending and tax rate shocks overlaps with periods of price controls around World War II and the Korean War. In order to check that such price controls do not pose a challenge to our strategy of using co-movement between inflation and activity to identify sources of fluctuations, we estimate the response of inflation to our fiscal shocks:

$$\pi_{t+H} = \alpha_H + \beta_H \times shock_t + \gamma_H \mathbf{z}_{t-1} + \varepsilon_{t+H}, \tag{I.2}$$

where π_t is inflation based on GDP deflator, $shock_t$ is either military spending news shock from Ramey and Zubairy (2018) or tax rate shocks from Romer and Romer (2010); the set of controls $\mathbf{z}_{t-1}$ is specified in the description to the results figure.

As can be seen in Figure I.13, both a positive spending shock and a positive shock to the tax rate produce a statistically significant increase in inflation. In this sense, even though the bulk of variation in our spending and tax rate shocks overlaps with periods of price controls, they produce a response in inflation that is consistent with our theory. The latter gives extra support to our strategy of using co-movement of inflation and activity to identify periods driven by either demand or supply shocks.

Appendix I.4. Demand-side and supply-side expansions

In Table I.4 we repeat estimation of conditional state-dependent spending multipliers, but extend our baseline exercise by further splitting expansionary states, where $U_t < \bar{U}$, into those where inflation is above trend, $\pi_t \geq \bar{\pi}_t$, corresponding to demand-side expansions, and those where inflation is below trend, $\pi_t < \bar{\pi}_t$, corresponding to supply-side expansions. Consistently with our theory, we find the 2-year horizon cumulative spending multiplier in supply-side expansions (0.77) to be higher than in demand-side expansions (0.64); however, the 4-year horizon spending multiplier is very imprecisely estimated in supply-side expansions, making it hard to test our predictions. In Figure I.14 we report conditional state-dependent spending multipliers at horizons ranging from 4 to 20 quarters; Panel (b) confirms our earlier finding: our prediction of higher multipliers in supply-side expansions finds confirmation only at shorter horizons, up to 8 quarters.

In Table I.5 we lower the unemployment threshold down to $\bar{U} = 4.5\%$, so that our expansionary states, where $U_t < \bar{U}$ now pick up more severe overheating episodes, potentially making our identification sharper and helping test our theoretical predictions regarding spending multipliers in demand- and supply-side expansions. Once again, we find strong confirmation of our theory at the 2-year horizon: in supply-side expansions the multiplier is at 1.12, as opposed to 0.85 in demand-side expansions; at the 4-year horizon we still find supply-side expansion multipliers to be higher, although the demand-side expansion multiplier is very imprecisely estimated. Figure I.15 confirms that most robust confirmation of our theory for expansions is indeed found at shorter horizons, up to 8 quarters.

Table I.6 extends our analysis of conditional state-dependent tax cut multipliers to demand- and supply-side expansions. Our theory predicts that tax cut multipliers should be higher in demand-side recessions, and we find empirical support for this at the 4-year horizon, but not at the 2-year horizon; moreover Figure I.16 shows that our prediction for expansions holds at longer horizons, above 10 quarters, but not at shorter ones. One reason behind this could be income effects associated with tax cuts that our model does not capture very well.

Appendix I.5. Blanchard and Perotti (2002) shocks

In Tables I.7 and I.8 as well as Figures I.17 and I.18 we repeat the conditional state-dependent spending multiplier estimation using VAR-based spending shocks following Blanchard and Perotti (2002), for both demand- and supply-side recessions and expansions. Overall, when we set $\bar{U} = 6.5\%$, we find confirmation to our theory for both expansions and recessions at shorter horizons, up to 5-quarters, whereas the results at longer horizons are less precise and deliver mixed evidence; when we set $\bar{U} = 4.5\%$, the results are consistent with our theoretical predictions across all horizons, but still most quantitatively significant at shorter horizons, up to 8 quarters. Therefore, our theoretical predictions for spending multiplier find most robust econometric confirmation at shorter cumulation horizons, regardless of whether one performs estimation with military spending news shocks or Blanchard-Perotti (2002) shocks; this horizon-dependence is in fact consistent with our dynamic simulations: at longer horizons more firms set prices optimally, less adjustment happens via tightness, state-dependence of multipliers is weaker and hence harder to detect econometrically.

Appendix I.6. Economic activity threshold based on detrended real GDP

Our baseline analysis uses unemployment as the measure of economic activity, which is done to be consistent with Ramey and Zubairy (2018). However, our theoretical model does not feature (involuntary) unemployment, and a measure of activity most consistent with our model is the cyclical component of real GDP. In this subsection we describe the results of performing estimation with an activity threshold based on detrended real GDP. [31]

In Table I.9 we show results for 2- and 4-year horizon cumulative spending multipliers, where we define a recession as an episode where real GDP drops more than 3% below trend. Our classification of demand- and supply-driven recessions and expansions based on cyclical component of inflation remains unchanged. Consistently with our theory, we find that spending multipliers in demand-driven recessions are larger than spending multipliers in supply-driven recessions: 0.55 vs. 0.11 at the 2-year horizon, and 0.60 vs. 0.48 at the 4-year horizon. In panel (c) of Figure I.19 we show that the pattern of higher multipliers in demand-driven recessions holds consistently across horizons, with the effect most pronounced at earlier horizons, again in line with our theory.

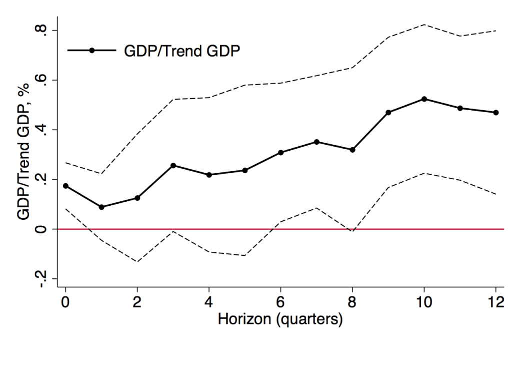
[31] We use the same polynomial trend as in Gordon and Krenn (2010)

In Table I.10 we change the threshold, so that only episodes where real GDP is more than 3% above trend counts as an expansion (and anything else counts as a recession). In this way we can focus on the most substantial episodes of overheating and have more power to test our theoretical predictions for expansions. Consistently with our theory we find that spending multipliers are higher in supply-driven expansions relative to demand-driven expansions: 0.68 vs 0.38 at the 2-year horizon and 0.69 vs 0.40 at the 4-year horizon. In panel (b) of Figure I.20 we show that the pattern of higher multipliers in supply-driven expansions holds consistently across horizons, with the effect most pronounced at earlier horizons, again in line with our theory.

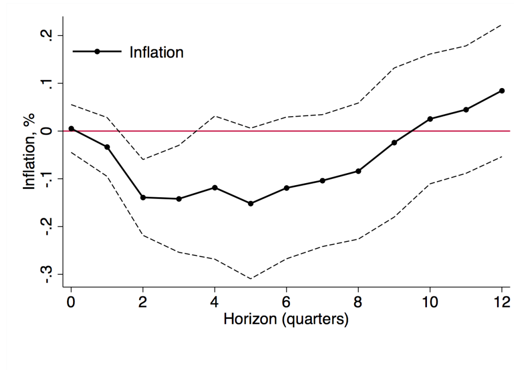
In Table I.11 we again define a recession as an episode where real GDP drops for than 3% below trend, but this time estimate our specification for tax shocks. Further, panels (b) and (c) of Figure I.21 exhibit estimation results for a broader set of horizons. At horizons beyond 8 quarters we find results consistent with our theory: tax cut multipliers are larger in demand-side expansions and supply-side recessions.

Figure I.12: Impulse response functions to a +1% productivity shock (utilization-adjusted shocks from Fernald (2014))

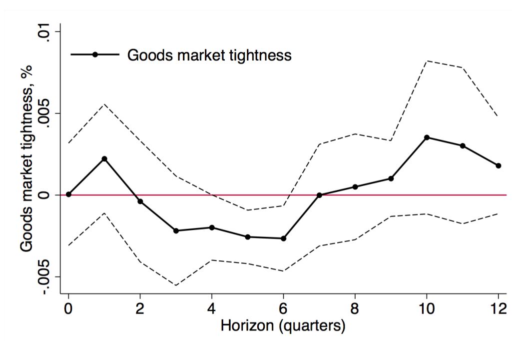
(a) Response of cyclical component of GDP



(b) Response of inflation (based on GDP deflator)



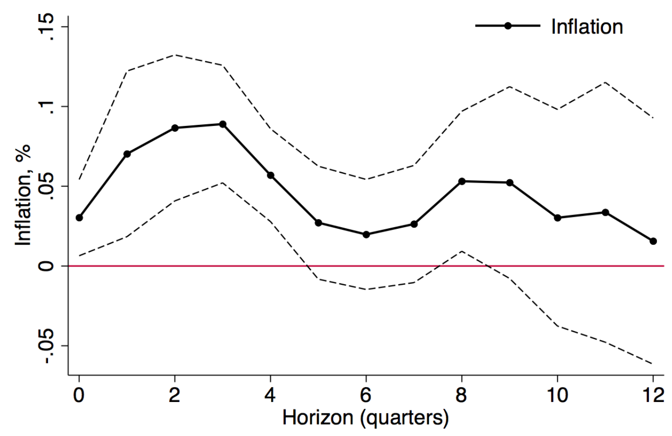
(c) Response of goods market tightness measure of Michaillat and Saez (2015)



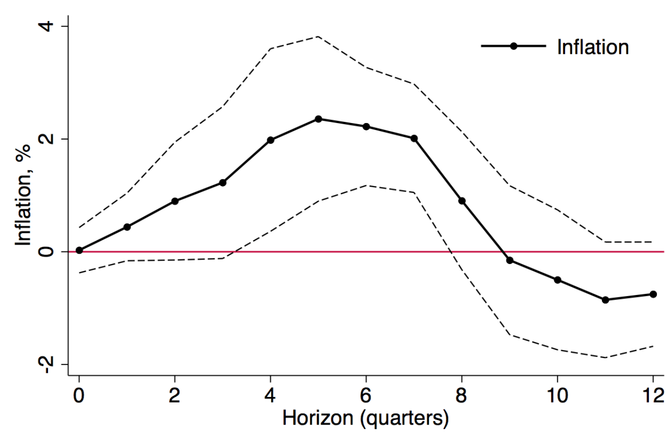
Notes: *the figure presents estimation using local projections based on econometric specification outlined in Appendix I.2 over the sample period 1973:Q4-2013:Q2; the set of controls includes one lag of cyclical GDP, inflation and goods market tightness. The dotted lines represent 90% confidence bands based on standard errors robust to autocorrelation and heteroskedasticity.*

Figure I.13: Impulse response functions of inflation to government spending and tax shocks

(a) Response to +1% military spending shock of Ramey and Zubairy (2018)



(b) Response to +1% tax rate shock of Romer and Romer (2010)



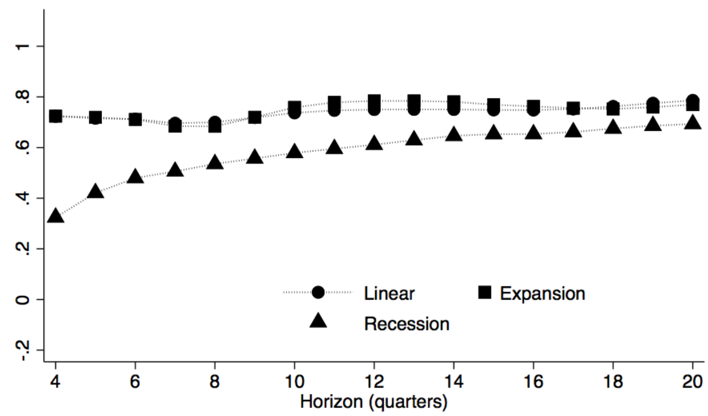
***Notes**: the figure presents estimation using local projections based on econometric specification outlined in Appendix I.3 over the sample period 1909:Q1-2015:Q4 (panel (a)) and 1947:Q1-2007:Q4 (panel (b)) ; the set of controls includes four lags of cyclical GDP, inflation and spending shock (panel (a)) and four lags of cyclical GDP, inflation and average tax rate (panel (b)). The dotted lines represent 90% confidence bands based on standard errors robust to autocorrelation and heteroskedasticity.*

Table I.4: Conditional state-dependent spending multipliers ($\bar{U} = 6.5\%$; US military spending news shocks)

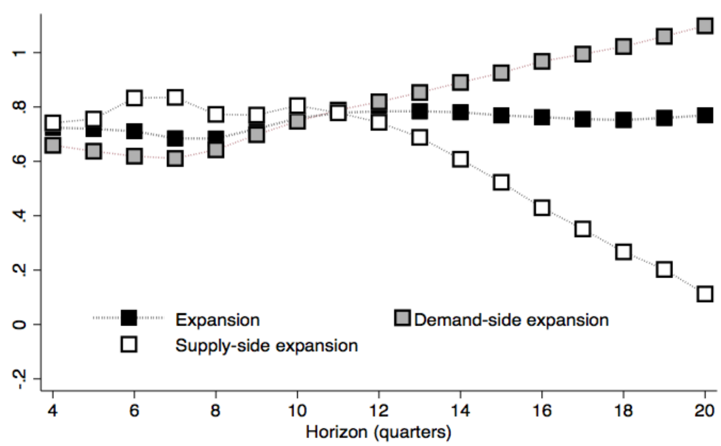
US data: 1909-2015		2y horizon				4y horizon		
State	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
β_H: Linear	0.70***				0.75***			
	(0.06)				*(0.06)*			
β_H^E : $\mathbf{1}\{U_t < \bar{U}\}$		0.68***	0.68***			0.76***	0.76***	
		(0.10)	*(0.09)*			*(0.13)*	*(0.12)*	
β_H^R : $\mathbf{1}\{U_t \geq \bar{U}\}$		0.54***				0.65***		
		(0.13)				*(0.08)*		
β_H^{DE} : $\mathbf{1}\{U_t < \bar{U}; \pi_t \geq \bar{\pi}_t\}$				0.64***				0.97***
				(0.06)				*(0.20)*
β_H^{SE} : $\mathbf{1}\{U_t < \bar{U}; \pi_t < \bar{\pi}_t\}$				0.77***				0.43
				(0.29)				*(0.47)*
β_H^{DR} : $\mathbf{1}\{U_t \geq \bar{U}; \pi_t < \bar{\pi}_t\}$			0.86***	0.86***			0.72***	0.72***
			(0.33)	*(0.33)*			*(0.12)*	*(0.12)*
β_H^{SR} : $\mathbf{1}\{U_t \geq \bar{U}; \pi_t \geq \bar{\pi}_t\}$			0.32***	0.32***			0.63***	0.63***
			(0.11)	*(0.11)*			*(0.09)*	*(0.09)*
$\beta_H^E = \beta_H^R$ (p-value)		0.37				0.44		
$\beta_H^E = \beta_H^{DR}$ (p-value)			0.62				0.81	
$\beta_H^E = \beta_H^{SR}$ (p-value)			0.01				0.40	
$\beta_H^{DR} = \beta_H^{SR}$ (p-value)			0.14	0.14			0.54	0.54
$\beta_H^{DE} = \beta_H^{SE}$ (p-value)				0.63				0.29
Paap-Kleibergen LM-test								
T	416	416	416	416	408	408	408	408

Figure I.14: Government Spending Multipliers across Horizons (US military spending news shocks, 1909-2015)

(a) Government spending multipliers in recessions and expansions across horizons ($\bar{U} = 6.5\%$)



(b) Government spending multipliers in demand-side and supply-side expansions across horizons



(c) Government spending multipliers in demand-side and supply-side recessions across horizons

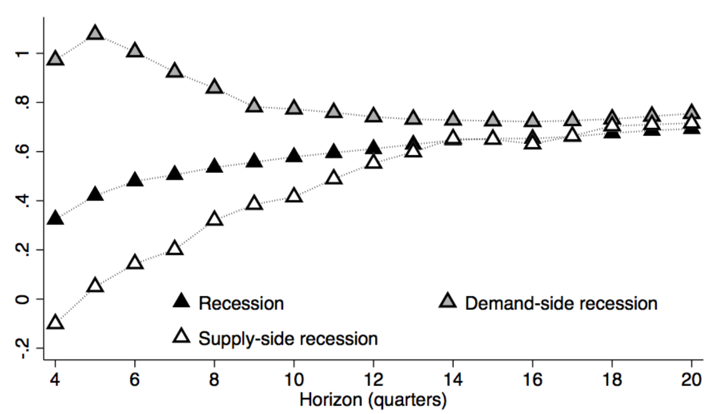
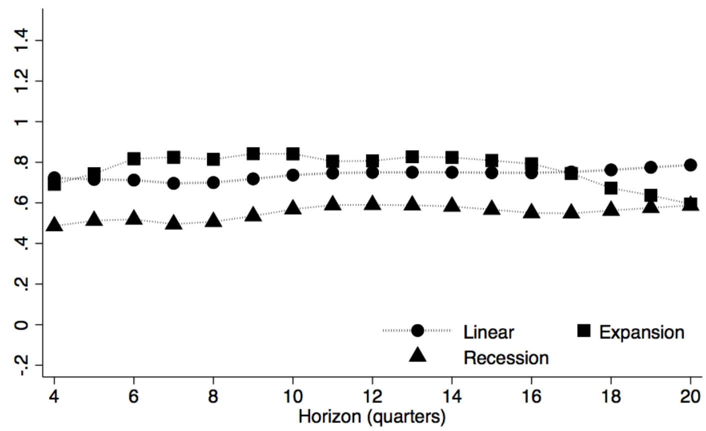


Table I.5: Conditional state-dependent spending multipliers ($\bar{U} = 4.5\%$; US military spending news shocks)

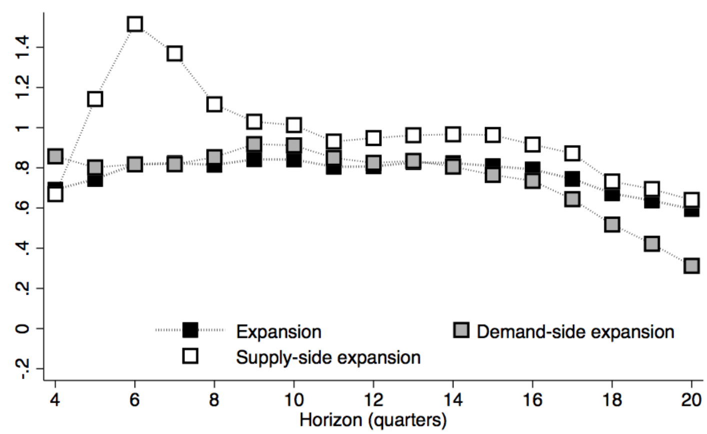
US data: 1909-2015			2y horizon			4y horizon		
State	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
β_H: Linear	0.70***				0.75***			
	(0.06)				*(0.06)*			
β_H^E : $\mathbf{1}\{U_t < \bar{U}\}$		0.81***	0.81***			0.79***	0.79***	
		(0.20)	*(0.20)*			*(0.28)*	*(0.30)*	
β_H^R : $\mathbf{1}\{U_t \geq \bar{U}\}$		0.51***				0.55***		
		(0.12)				*(0.12)*		
β_H^{DE} : $\mathbf{1}\{U_t < \bar{U}; \pi_t \geq \bar{\pi}_t\}$				0.85**				0.74
				(0.34)				*(0.66)*
β_H^{SE} : $\mathbf{1}\{U_t < \bar{U}; \pi_t < \bar{\pi}_t\}$				1.12*				0.92*
				(0.61)				*(0.55)*
β_H^{DR} : $\mathbf{1}\{U_t \geq \bar{U}; \pi_t < \bar{\pi}_t\}$			0.69**	0.69**			0.51**	0.51**
			(0.32)	*(0.32)*			*(0.23)*	*(0.25)*
β_H^{SR} : $\mathbf{1}\{U_t \geq \bar{U}; \pi_t \geq \bar{\pi}_t\}$			0.31***	0.31***			0.55***	0.55***
			(0.09)	*(0.09)*			*(0.10)*	*(0.10)*
$\beta_H^E = \beta_H^R$ (p-value)		0.11				0.41		
$\beta_H^E = \beta_H^{DR}$ (p-value)			0.73				0.43	
$\beta_H^E = \beta_H^{SR}$ (p-value)			0.02				0.48	
$\beta_H^{DR} = \beta_H^{SR}$ (p-value)			0.22	0.22			0.88	0.88
$\beta_H^{DE} = \beta_H^{SE}$ (p-value)				0.72				0.83
Paap-Kleibergen LM-test								
T	416	416	416	416	408	408	408	408

Figure I.15: Government Spending Multipliers across Horizons (US military spending news shocks, 1909-2015)

(a) Government spending multipliers in recessions and expansions across horizons ($\bar{U} = 4.5\%$)



(b) Government spending multipliers in demand-side and supply-side expansions across horizons



(c) Government spending multipliers in demand-side and supply-side recessions across horizons

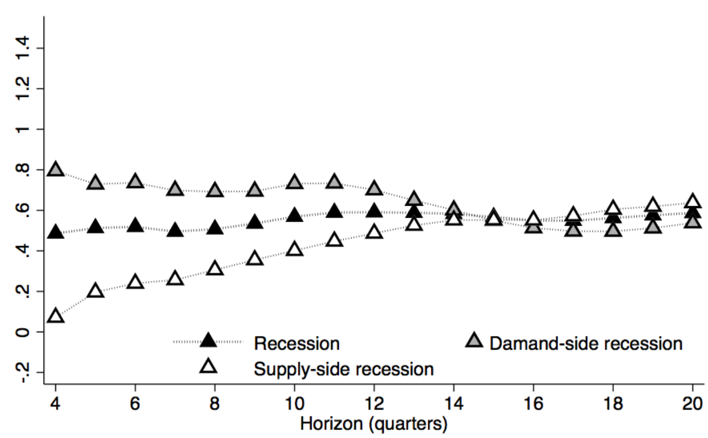
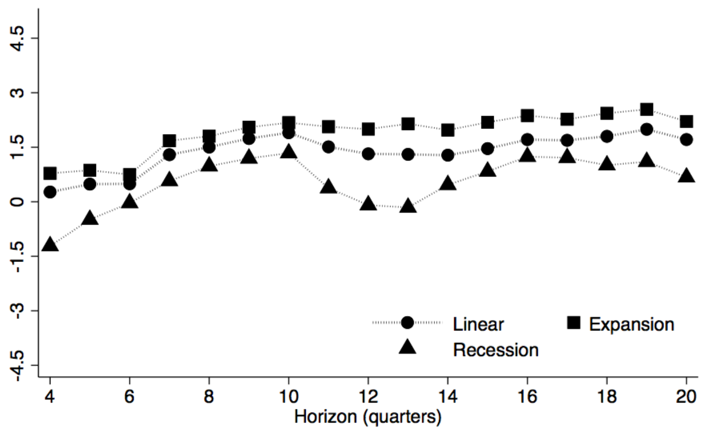


Table I.6: Conditional state-dependent tax cut multipliers ($\bar{U} = 6.5\%$; US Romer-Romer narrative tax shocks)

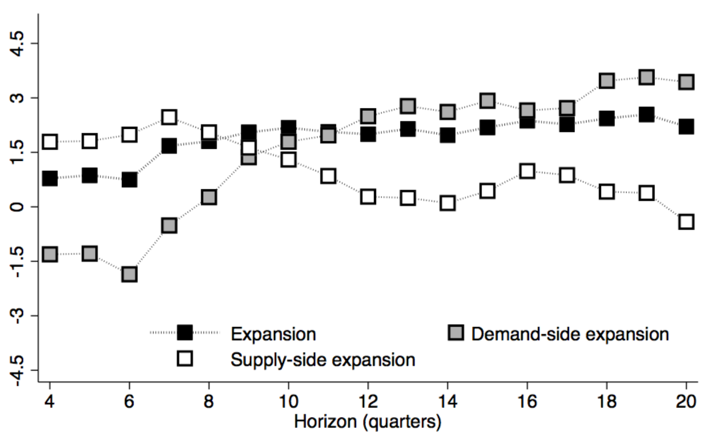
US data: 1947-2097			2y horizon			4y horizon		
State	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
β_H: Linear	1.50				1.71**			
	(1.14)				*(0.82)*			
β_H^E : $\mathbf{1}\{U_t < \bar{U}\}$		1.81	1.81			2.37**	2.37**	
		(1.17)	*(1.16)*			*(0.99)*	*(0.99)*	
β_H^R : $\mathbf{1}\{U_t \geq \bar{U}\}$		0.98				1.24		
		(1.07)				*(0.87)*		
β_H^{DE} : $\mathbf{1}\{U_t < \bar{U}; \pi_t \geq \bar{\pi}_t\}$				0.27				2.65*
				(1.08)				*(1.50)*
β_H^{SE} : $\mathbf{1}\{U_t < \bar{U}; \pi_t < \bar{\pi}_t\}$				2.05				0.99
				(1.34)				*(1.89)*
β_H^{DR} : $\mathbf{1}\{U_t \geq \bar{U}; \pi_t < \bar{\pi}_t\}$			1.49	1.49			-1.98	-1.98
			(1.04)	*(1.04)*			*(2.75)*	*(2.77)*
β_H^{SR} : $\mathbf{1}\{U_t \geq \bar{U}; \pi_t \geq \bar{\pi}_t\}$			4.29*	4.29*			1.80*	1.80*
			(2.18)	*(2.18)*			*(1.00)*	*(1.00)*
$\beta_H^E = \beta_H^R$ (p-value)		0.48				0.39		
$\beta_H^E = \beta_H^{DR}$ (p-value)			0.84				0.12	
$\beta_H^E = \beta_H^{SR}$ (p-value)			0.28				0.70	
$\beta_H^{DR} = \beta_H^{SR}$ (p-value)			0.25	0.25			0.20	0.20
$\beta_H^{DE} = \beta_H^{SE}$ (p-value)				0.32				0.51
Paap-Kleibergen LM-test								
T	240	240	240	240	240	240	240	240

Figure I.16: Tax Cut Multipliers across Horizons (US Romer-Romer narrative tax shocks, 1947-2007)

(a) Tax cut multipliers in recessions and expansions across horizons ($\bar{U} = 6.5\%$)



(b) Tax cut multipliers in demand-side and supply-side expansions across horizons



(c) Tax cut multipliers in demand-side and supply-side recessions across horizons

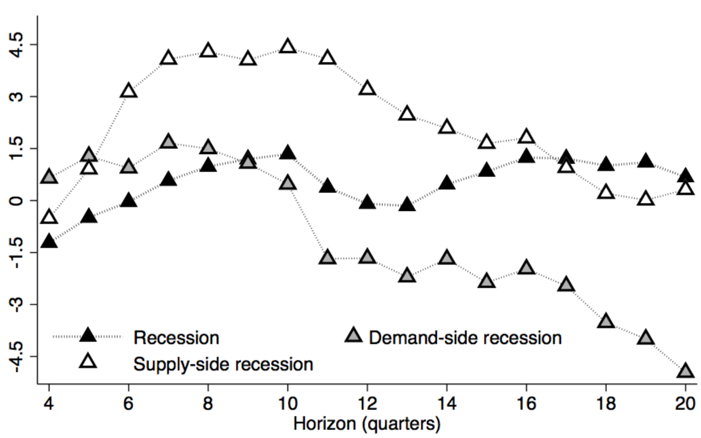
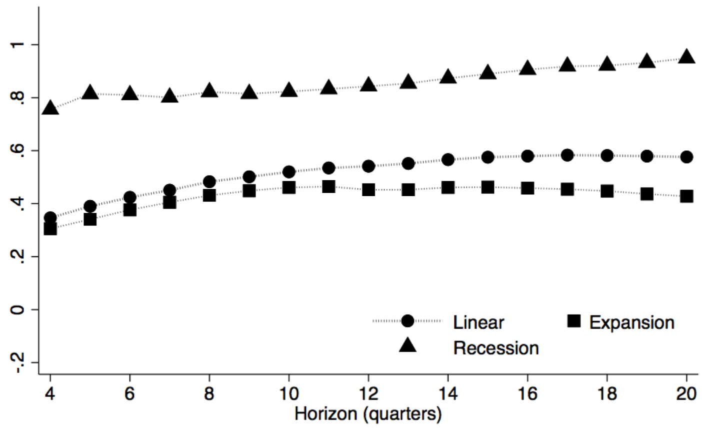


Table I.7: Conditional state-dependent spending multipliers ($\bar{U} = 6.5\%$; US Blanchard-Perotti spending shocks)

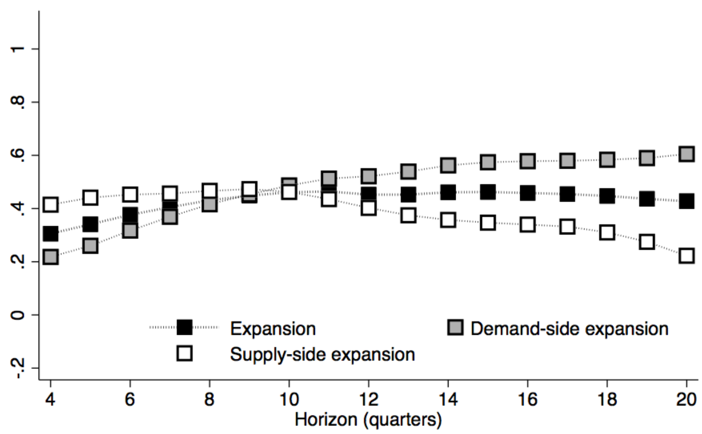
US data: 1909-2015			2y horizon			4y horizon		
State	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
β_H: Linear	0.48***				0.58***			
	(0.09)				*(0.10)*			
β_H^E : $\mathbf{1}\{U_t < \bar{U}\}$		0.43***	0.42***			0.46***	0.44***	
		(0.08)	*(0.09)*			*(0.12)*	*(0.13)*	
β_H^R : $\mathbf{1}\{U_t \geq \bar{U}\}$		0.82***				0.91***		
		(0.11)				*(0.07)*		
β_H^{DE} : $\mathbf{1}\{U_t < \bar{U}; \pi_t \geq \bar{\pi}_t\}$				0.42***				0.58***
				(0.08)				*(0.14)*
β_H^{SE} : $\mathbf{1}\{U_t < \bar{U}; \pi_t < \bar{\pi}_t\}$				0.47**				0.34
				(0.19)				*(0.25)*
β_H^{DR} : $\mathbf{1}\{U_t \geq \bar{U}; \pi_t < \bar{\pi}_t\}$			0.54***	0.56***			0.62***	0.65***
			(0.16)	*(0.15)*			*(0.13)*	*(0.14)*
β_H^{SR} : $\mathbf{1}\{U_t \geq \bar{U}; \pi_t \geq \bar{\pi}_t\}$			0.84***	0.84***			0.92***	0.91***
			(0.14)	*(0.14)*			*(0.21)*	*(0.21)*
$\beta_H^E = \beta_H^R$ (p-value)		0.00				0.00		
$\beta_H^E = \beta_H^{DR}$ (p-value)			0.50				0.36	
$\beta_H^E = \beta_H^{SR}$ (p-value)			0.01				0.05	
$\beta_H^{DR} = \beta_H^{SR}$ (p-value)			0.19	0.22			0.34	0.37
$\beta_H^{DE} = \beta_H^{SE}$ (p-value)				0.81				0.35
Paap-Kleibergen LM-test								
T	416	416	416	416	408	408	408	408

Figure I.17: Government Spending Multipliers across Horizons (US Blanchard-Perotti spending shocks, 1909-2015)

(a) Government spending multipliers in recessions and expansions across horizons ($\bar{U} = 6.5\%$)



(b) Government spending multipliers in demand-side and supply-side expansions across horizons



(c) Government spending multipliers in demand-side and supply-side recessions across horizons

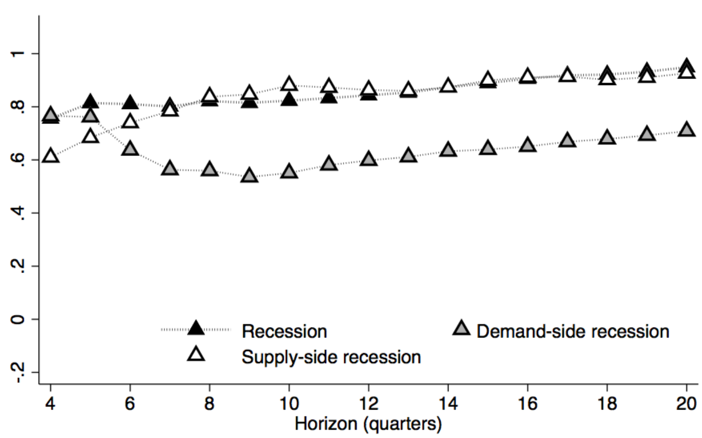
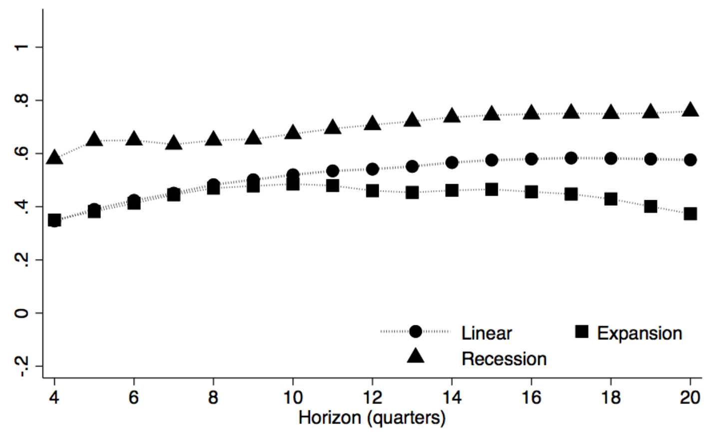


Table I.8: Conditional state-dependent spending multipliers ($\bar{U} = 4.5\%$; US Blanchard-Perotti spending shocks)

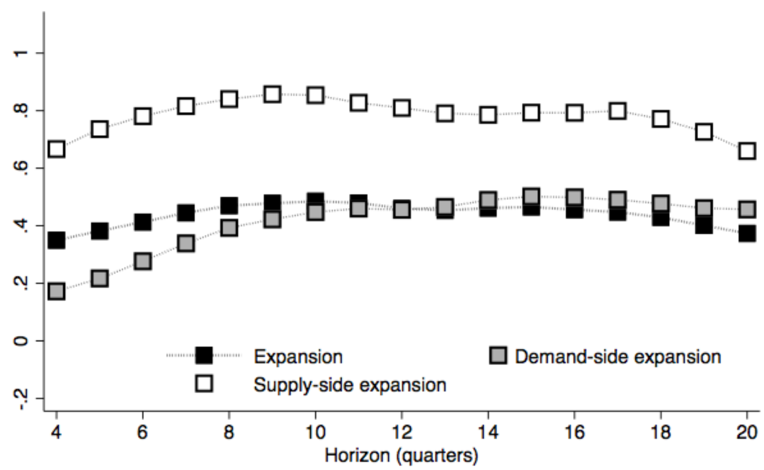
US data: 1889-2015				2y horizon			4y horizon		
State		(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
β_H:	Linear	0.48***				0.58***			
		(0.09)				*(0.10)*			
β_H^E :	$\mathbf{1}\{U_t < \bar{U}\}$		0.47***	0.47***			0.46**	0.46**	
			(0.15)	*(0.15)*			*(0.18)*	*(0.18)*	
β_H^R :	$\mathbf{1}\{U_t \geq \bar{U}\}$		0.65***				0.75***		
			(0.10)				*(0.10)*		
β_H^{DE} :	$\mathbf{1}\{U_t < \bar{U}; \pi_t \geq \bar{\pi}_t\}$				0.39***				0.50***
					(0.09)				*(0.14)*
β_H^{SE} :	$\mathbf{1}\{U_t < \bar{U}; \pi_t < \bar{\pi}_t\}$				0.84***				0.79***
					(0.11)				*(0.26)*
β_H^{DR} :	$\mathbf{1}\{U_t \geq \bar{U}; \pi_t < \bar{\pi}_t\}$			0.75***	0.76***			0.74***	0.76***
				(0.16)	*(0.16)*			*(0.10)*	*(0.09)*
β_H^{SR} :	$\mathbf{1}\{U_t \geq \bar{U}; \pi_t \geq \bar{\pi}_t\}$			0.54***	0.51***			0.73***	0.70***
				(0.15)	*(0.15)*			*(0.12)*	*(0.12)*
$\beta_H^E = \beta_H^R$ (p-value)			0.25				0.08		
$\beta_H^E = \beta_H^{DR}$ (p-value)				0.22				0.12	
$\beta_H^E = \beta_H^{SR}$ (p-value)				0.70				0.09	
$\beta_H^{DR} = \beta_H^{SR}$ (p-value)				0.38	0.33			0.99	0.68
$\beta_H^{DE} = \beta_H^{SE}$ (p-value)					0.00				0.31
Paap-Kleibergen LM-test									
T		416	416	416	416	408	408	408	408

Figure I.18: Government Spending Multipliers across Horizons (US Blanchard-Perotti spending shocks, 1909-2015)

(a) Spending multipliers in recessions and expansions across horizons ($\bar{U} = 4.5\%$)



(b) Spending multipliers in demand-side and supply-side expansions across horizons



(c) Spending multipliers in demand-side and supply-side recessions across horizons

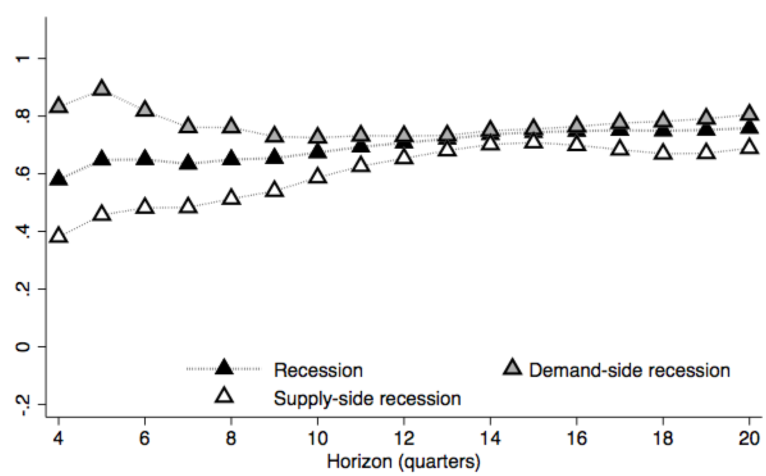
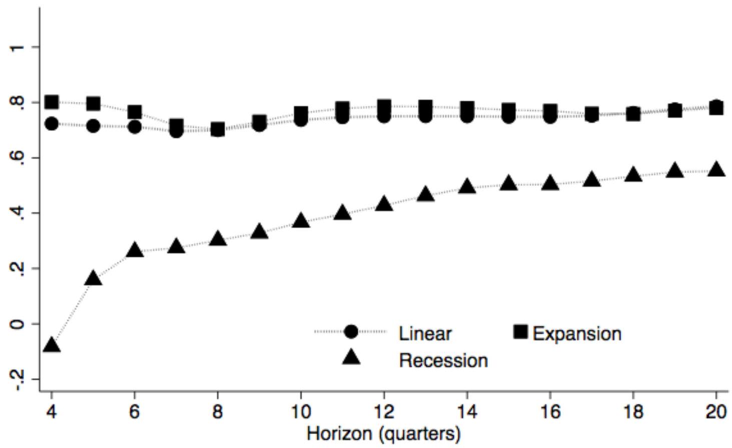


Table I.9: Conditional state-dependent spending multipliers (cyclical GDP-based threshold $\hat{GDP}_t \equiv (GDP_t - \overline{GDP}_t)/\overline{GDP}_t$ where $\overline{GDP}_t$ is trend GDP; US military spending news shocks)

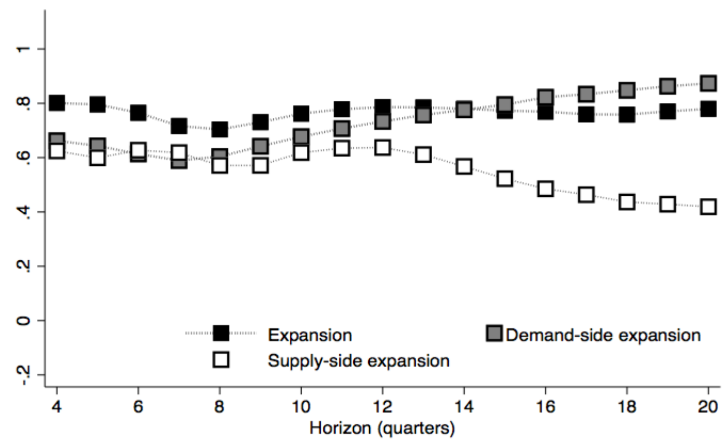
US data: 1909-2015			2y horizon				4y horizon		
State		(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
β_H:	Linear	0.70***				0.75***			
		(0.06)				*(0.06)*			
β_H^E :	$\mathbf{1}\{\hat{GDP}_t \geq -3\%\}$		0.70***	0.70***			0.77***	0.77***	
			(0.11)	*(0.11)*			*(0.08)*	*(0.08)*	
β_H^R :	$\mathbf{1}\{\hat{GDP}_t < -3\%\}$		0.30				0.50***		
			(0.24)				*(0.17)*		
β_H^{DE} :	$\mathbf{1}\{\hat{GDP}_t \geq -3\%; \pi_t \geq \bar{\pi}_t\}$				0.60***				0.82***
					(0.07)				*(0.11)*
β_H^{SE} :	$\mathbf{1}\{\hat{GDP}_t \geq -3\%; \pi_t < \bar{\pi}_t\}$				0.57***				0.49**
					(0.18)				*(0.19)*
β_H^{DR} :	$\mathbf{1}\{\hat{GDP}_t < -3\%; \pi_t < \bar{\pi}_t\}$			0.55**	0.55**			0.60***	0.60***
				(0.25)	*(0.24)*			*(0.12)*	*(0.12)*
β_H^{SR} :	$\mathbf{1}\{\hat{GDP}_t < -3\%; \pi_t \geq \bar{\pi}_t\}$			0.11	0.11			0.48***	0.48***
				(0.29)	*(0.31)*			*(0.15)*	*(0.15)*
$\beta_H^E = \beta_H^R$ (p-value)			0.13				0.20		
$\beta_H^E = \beta_H^{DR}$ (p-value)				0.56				0.28	
$\beta_H^E = \beta_H^{SR}$ (p-value)				0.06				0.13	
$\beta_H^{DR} = \beta_H^{SR}$ (p-value)				0.26	0.28			0.48	0.48
$\beta_H^{DE} = \beta_H^{SE}$ (p-value)					0.85				0.12
Paap-Kleibergen LM-test									
T		416	416	416	416	408	408	408	408

Figure I.19: Government Spending Multipliers across Horizons (US military spending news shocks, 1909-2015, cyclical GDP-based threshold $\hat{GDP}_t \equiv (GDP_t - \overline{GDP}_t)/\overline{GDP}_t$ where $\overline{GDP}_t$ is trend GDP, threshold of -3%)

(a) Spending multipliers in recessions and expansions (recession if GDP more than 3% below trend)



(b) Spending multipliers in demand-side and supply-side expansions



(c) Spending multipliers in demand-side and supply-side recessions

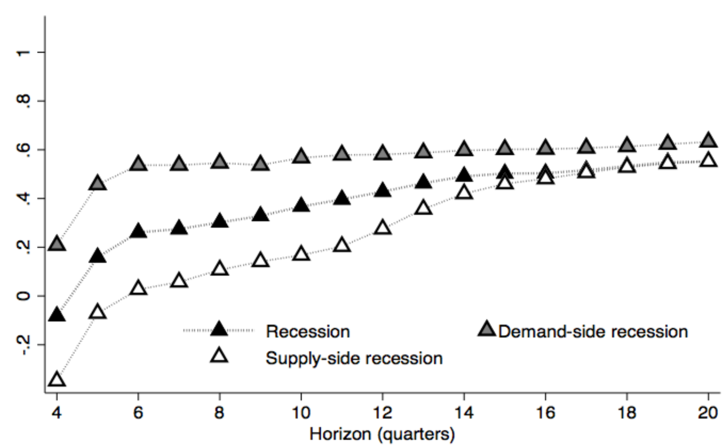
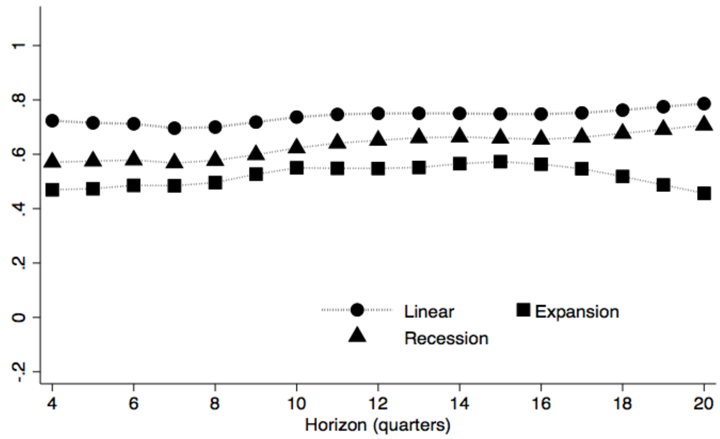


Table I.10: Conditional state-dependent spending multipliers (cyclical GDP-based threshold $\hat{GDP}_t \equiv (GDP_t - \overline{GDP}_t)/\overline{GDP}_t$ where $\overline{GDP}_t$ is trend GDP; US military spending news shocks)

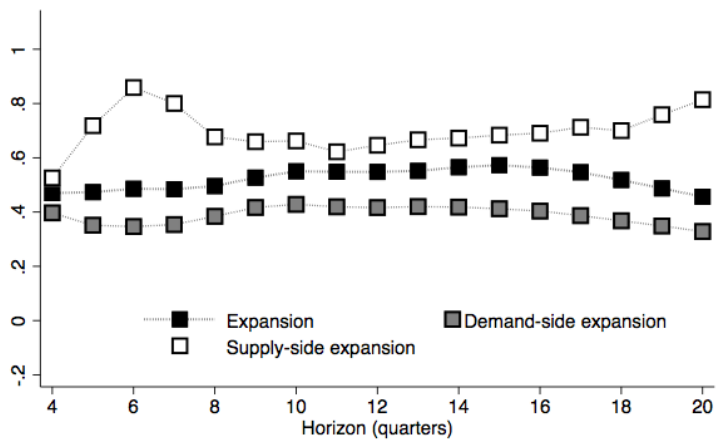
US data: 1909-2015			2y horizon			4y horizon		
State	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
β_H: Linear	0.70***				0.75***			
	(0.06)				*(0.06)*			
β_H^E : $\mathbf{1}\{\hat{GDP}_t \geq 3\%\}$		0.50***	0.50***			0.56***	0.56***	
		(0.08)	*(0.08)*			*(0.09)*	*(0.10)*	
β_H^R : $\mathbf{1}\{\hat{GDP}_t < 3\%\}$		0.58***				0.66***		
		(0.07)				*(0.07)*		
β_H^{DE} : $\mathbf{1}\{\hat{GDP}_t \geq 3\%; \pi_t \geq \bar{\pi}_t\}$				0.38***				0.40***
				(0.08)				*(0.12)*
β_H^{SE} : $\mathbf{1}\{\hat{GDP}_t \geq 3\%; \pi_t < \bar{\pi}_t\}$				0.68**				0.69**
				(0.30)				*(0.32)*
β_H^{DR} : $\mathbf{1}\{\hat{GDP}_t < 3\%; \pi_t < \bar{\pi}_t\}$			0.33	0.33			0.32	0.32
			(0.28)	*(0.27)*			*(0.26)*	*(0.40)*
β_H^{SR} : $\mathbf{1}\{\hat{GDP}_t < 3\%; \pi_t \geq \bar{\pi}_t\}$			0.57***	0.57***			0.76***	0.76***
			(0.07)	*(0.06)*			*(0.12)*	*(0.10)*
$\beta_H^E = \beta_H^R$ (p-value)		0.42				0.48		
$\beta_H^E = \beta_H^{DR}$ (p-value)			0.54				0.33	
$\beta_H^E = \beta_H^{SR}$ (p-value)			0.50				0.28	
$\beta_H^{DR} = \beta_H^{SR}$ (p-value)			0.43	0.39			0.11	0.28
$\beta_H^{DE} = \beta_H^{SE}$ (p-value)				0.36				0.41
Paap-Kleibergen LM-test								
T	416	416	416	416	408	408	408	408

Figure I.20: Government Spending Multipliers across Horizons (US military spending news shocks, 1909-2015, cyclical GDP-based threshold $\hat{GDP}_t \equiv (GDP_t - \overline{GDP}_t)/\overline{GDP}_t$ where $\overline{GDP}_t$ is trend GDP, threshold of 3%)

(a) Spending multipliers in recessions and expansions (expansion if GDP more than 3% above trend)



(b) Spending multipliers in demand-side and supply-side expansions



(c) Spending multipliers in demand-side and supply-side recessions

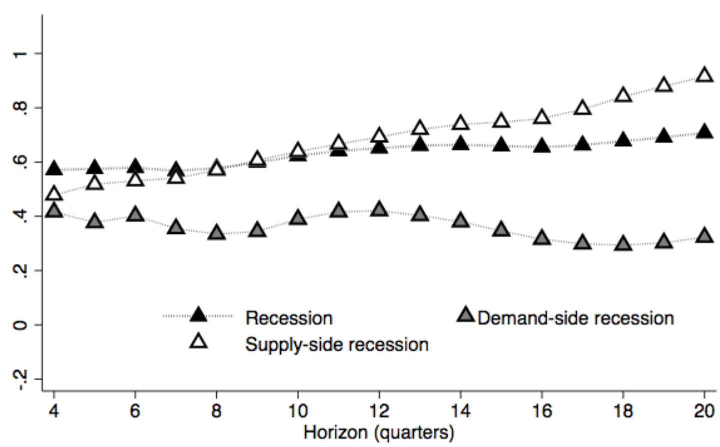
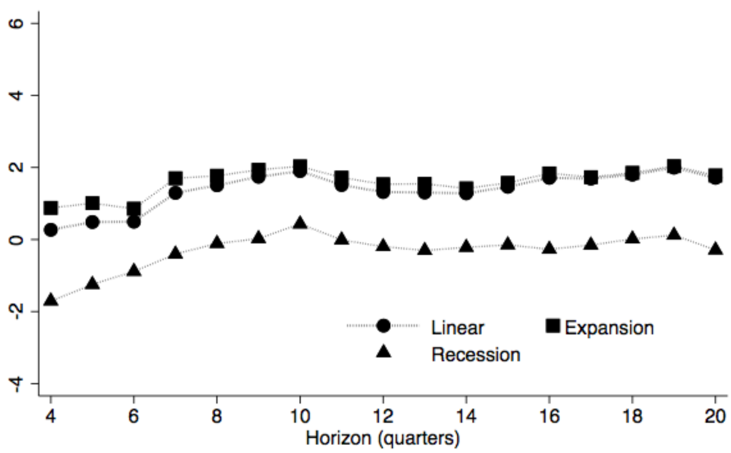


Table I.11: Conditional state-dependent tax cut multipliers (cyclical GDP-based threshold $\hat{GDP}_t \equiv (GDP_t - \overline{GDP}_t)/\overline{GDP}_t$ where $\overline{GDP}_t$ is trend GDP; US Romer-Romer narrative tax shocks)

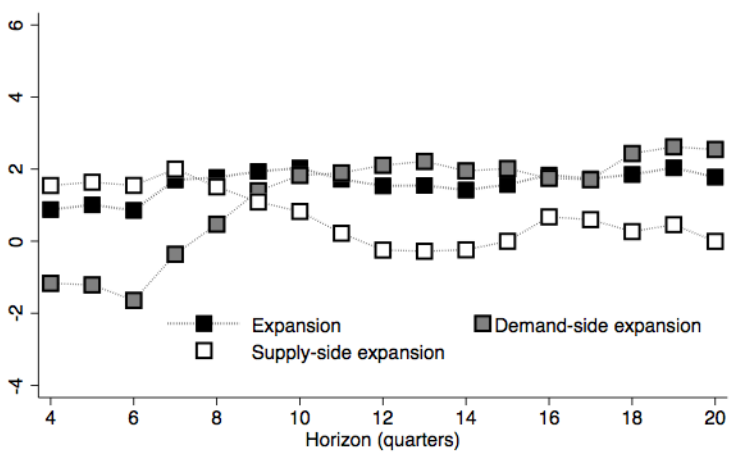
US data: 1909-2015		2y horizon				4y horizon			
State		(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
β_H:	Linear	1.50				1.71**			
		(1.14)				*(0.82)*			
β_H^E :	$\mathbf{1}\{\hat{GDP}_t \geq -3\%\}$		1.77*	1.77*			1.84*	1.84*	
			(1.01)	*(1.01)*			*(0.95)*	*(1.01)*	
β_H^R :	$\mathbf{1}\{\hat{GDP}_t < -3\%\}$		-0.11				-0.27		
			(0.60)				*(0.45)*		
β_H^{DE} :	$\mathbf{1}\{\hat{GDP}_t \geq -3\%; \pi_t \geq \bar{\pi}_t\}$				0.47				1.75
					(1.56)				*(1.25)*
β_H^{SE} :	$\mathbf{1}\{\hat{GDP}_t \geq -3\%; \pi_t < \bar{\pi}_t\}$				1.51				0.67
					(1.16)				*(2.13)*
β_H^{DR} :	$\mathbf{1}\{\hat{GDP}_t < -3\%; \pi_t < \bar{\pi}_t\}$			1.05	1.05			1.51	1.51
				(1.45)	*(1.45)*			*(1.35)*	*(1.38)*
β_H^{SR} :	$\mathbf{1}\{\hat{GDP}_t < -3\%; \pi_t \geq \bar{\pi}_t\}$			0.81	0.81			2.27*	2.27*
				(0.89)	*(0.89)*			*(1.30)*	*(1.30)*
$\beta_H^E = \beta_H^R$ (p-value)			0.04				0.04		
$\beta_H^E = \beta_H^{DR}$ (p-value)				0.70				0.85	
$\beta_H^E = \beta_H^{SR}$ (p-value)				0.45				0.78	
$\beta_H^{DR} = \beta_H^{SR}$ (p-value)				0.88	0.88			0.69	0.69
$\beta_H^{DE} = \beta_H^{SE}$ (p-value)					0.61				0.66
Paap-Kleibergen LM-test									
T		416	416	416	416	408	408	408	408

Figure I.21: Tax Cut Multipliers across Horizons (US Romer-Romer narrative tax shocks, 1947-2007, cyclical GDP-based threshold $\hat{GDP}_t \equiv (GDP_t - \overline{GDP}_t)/\overline{GDP}_t$ where $\overline{GDP}_t$ is trend GDP, threshold of -3%)

(a) Tax cut multipliers in recessions and expansions (recession if GDP more than 3% below trend)



(b) Tax cut multipliers in demand-side and supply-side expansions



(c) Tax cut multipliers in demand-side and supply-side recessions