Barcelona School of Economics

Master Program in Finance

"Hull-White Calibration for Swaptions using Neural Networks"

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June 2022

ABSTRACT IN ENGLISH:

The purpose of this project is to calibrate the Hull-White model using data on at-the-money payer swaptions and Artificial Neural Networks (ANN) for 100 different combinations of expiries (T_0) and tenors ($T_n - T_0$). The calibration of the model parameters a and σ is done in the following way: (i) first, we use market swap interest rates to generate 10,000 combinations of (T_0, T_n, a, σ); (ii) then, we calculate swaption prices for each of these combinations using a closed-form function derived from the Hull-White (one factor) model in the form of $F: (T_0, T_n, a, \sigma) \rightarrow Price$; (iii) we use ANN to learn the inverse mapping relationship ($F^{-1}: (T_0, T_n, Price) \rightarrow (a, \sigma)$); (iv) we use swaptions market prices as arguments for F^{-1} to find calibrated a and σ for each expiry and tenor. Finally, we use the calibrated parameters to calculate the theoretical Hull-White prices and compare these prices with the prices observed in the market. The results show that the prices generated are very close to the ones observed in the market, suggesting that the ANN is an effective method for calibrating the model parameters accurately.

ABSTRACT IN SPANISH:

El objetivo de este proyecto es calibrar el modelo de Hull-White utilizando datos sobre at-the-money payer swaptions y Redes Neuronales Artificiales (ANN por sus siglas en inglés) para 100 combinaciones diferentes de vencimientos (T_0) y “tenors” ($T_n - T_0$). La calibración de los parámetros a y σ del modelo se realiza de la siguiente manera: (i) en primer lugar, utilizamos los tipos de interés swap del mercado para generar 10,000 combinaciones de (T_0, T_n, a, σ); (ii) a continuación, calculamos los precios de las swaptions para cada una de estas combinaciones utilizando una función de forma cerrada derivada del modelo de Hull-White (un factor) en forma de $F: (T_0, T_n, a, \sigma) \rightarrow Precio$; (iii) utilizamos ANN para aprender la relación inversa ($F^{-1}: (T_0, T_n, Precio) \rightarrow (a, \sigma)$); (iv) utilizamos los precios de mercado de las swaptions como argumentos de F^{-1} para encontrar a y σ calibrados para cada vencimiento y tenor. Por último, utilizamos los parámetros calibrados para calcular los precios teóricos de Hull-White y los comparamos con los precios observados en el mercado. Los resultados muestran que los precios generados son muy próximos a los observados en el mercado, lo que sugiere que el ANN es un método eficaz para calibrar con precisión los parámetros del modelo.

KEYWORDS IN ENGLISH:

Hull-White; Swaptions; Artificial Neural Networks

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Abstract

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Acknowledgements

First and foremost, we would like to thank our advisor Prof. Eulalia Nualart, PhD, for her support and dedication throughout the project. Her help and guidance were key to make this project possible. We would also want to thank Makar Pravosud for his help with the implementation of the artificial neural networks. Last, but not least, we are grateful for the tools provided by the university and the support of our families.

Contents

1 Introduction

Short rate models are used to describe the dynamics of the interest rate. Under this class of models, interest rates, such as the Euro Interbank Offered Rate (EURIBOR), are defined as an Ito process with drift and volatility terms. Among the most common short rate models, the Hull-White (one factor) model stand out for its capacity to fit the data. It is an extension of the Vasicek model, in which the interest rate converges to a long-term mean $\theta(t)$ at a mean reversion speed defined by a constant parameter a and constant volatility σ. As can be seen in the plot below, the 12-Month EURIBOR appears to follow an Ito process.

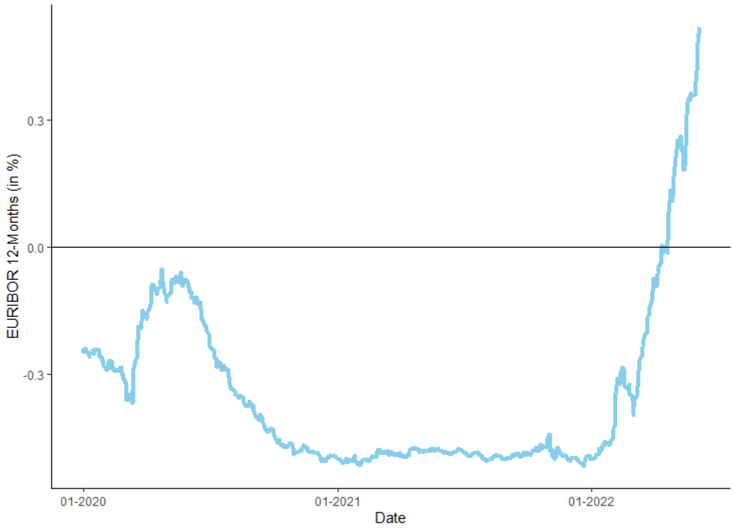

Figure 1: 12-Month EURIBOR

Practitioners use short rate models to price interest rate derivatives, such as swaptions. Model calibration is the process in which a pricing model parameters are adjusted to fit real market data. For the Hull-White (one factor) model used to price swaptions calibration implies tuning parameters a and σ to best describe quoted values in the market.

Artificial Neural Networks (ANN) have been commonly used in the financial industry for option pricing. An ANN can learn the mapping of an option price as a function of certain set of parameters, identifying properties and exogenous factors, particular to a certain pricing model. Learning the inverse of the mapping of the price function can be used to adjust the model parameter to the observed market option prices.

In this project we calibrate Hull-White model and compare the theoretical prices with the ones observed in the market. We begin by generating 10,000 prices for different parameters and use an ANN to learn the inverse mapping of the prices and parameters.

Once we have the model estimated, we fit the market prices to get what would be the corresponding parameters. Finally, we use the corresponding parameters to calculate what would be the theoretical prices they would yield and we compare with the ones observed in the market.

2 Concepts review

2.1 Interest Rate Swap Agreements

Interest rate swap contracts - that for simplicity will be referred to as simply "swaps" - are agreements in between two counterparties to exchange payments or cash flows. Whereas one counterpart pays a fixed rate, the other pays a floating rate. In the Eurozone, one of the most common floating rates is the Euro Interbank Offered Rate (EURIBOR). In this framework, it is important to define that the "payer" of a swap contract is the one that pays the fixed rate, while the "receiver" is the one that receives it and pays the floating rate.

This way, a swap contract is defined by the dates in which it makes the payments. Throughout this project we will refer to these payment dates by T_0, T_1, T_2,...,T_{i-1}, T_i, T_{i+1},..., T_{n-1}, T_n, where n is the Tenor of the swaption contract in case payments are made yearly, explained in more detail below.

At every payment date T_i, the cash flow of a payer swap is then given by

$$(f_{T_{i-1}}^{T_{i-1},T_i} - K)\delta N$$

Where $f_{T_{i-1}}^{T_{i-1},T_i}$ is the forward (floating) rate that refers to the period in between T_{i-1} and T_i, K is the fixed rate, N is the nominal value, and $\delta = T_i - T_{i-1}$. Now, it is straightforward to write the value at time t of a payer swap as zero coupon bonds P as

$$\Pi_p(t) = N\left(P_t^{T_0} - P_t^{T_n} - K\delta\sum_{i=1}^{n} P_t^{T_i}\right)$$

As a swap agreement is a "symmetrical" contract, the value of a receiver swap contract ($\Pi_r(t)$) is the negative of the payer swap value. That is:

$$\Pi_r(t) = -\Pi_p(t).$$

2.2 Swaptions

Swap contracts, as explained above, are agreements between two entities to exchange cash flows over a period of time. An option on a swap contract is known as swaption. These are non-standardized contracts that are traded over-the-counter (OTC). According to the report by Skantzos and Garston [2019], the monthly trading volume of swaptions in 2018 was around 1 trillion USD. A European swaption gives the holder the right to enter into a certain interest rate swap at a expiry. The length of the underlying swap contract is known as tenor. That is, the swaption is an option that expires when the underlying swap contract starts. This moment is called expiry and is referred to T_0. The tenor of the swap contract is given by $T_n - T_0$.

A European payer swaption gives the holder right to enter a swap contract where he/she will pay a fixed interest rate and will receive a floating one. Therefore, a payer swaption will only be exercised at maturity if the floating rate is greater than the fixed one. A European receiver swaption gives the holder right to enter a swap contract where he/she will pay a floating interest rate and receive a fixed one. It will only be exercised at maturity if the fixed rate is greater than the floating one. For at-the-money (ATM) swaptions, the holder is indifferent whether to exercise or not.

Finally, the price of a payer swaption at time $t < T_0$ can be derived from the value of swap contracts and Black's formula, which yield the following relation:

$$Swaption(t) = N\delta\left(\sum_{i=1}^{n} p_t^{T_i}\right)(R_{swap}(t)\Phi(d_1(t)) - \Phi(d_2(t)))$$

Where $\Phi(x)$ is the cumulative distribution function (CDF) of the normal distribution,

$$d_1(t) = \frac{ln\frac{R_{swap}(t)}{K} + 0.5\overline{\sigma}_t^2(T_0 - t)}{\overline{\sigma}_t\sqrt{T_0 - t}},$$

$$d_2(t) = d_1(t) - \overline{\sigma}_t\sqrt{T_0 - t},$$

and $\overline{\sigma}_t$ is the implied volatility.

2.3 Hull-White (one factor) Model

In their seminal paper, Hull and White [1990] provide an extension of the Vasicek Model, the Hull-White model (HW), in which the interest rate follows the stochastic differential equation:

$$dr = a(\frac{\theta(t)}{a} - r)dt + \sigma dW_t \tag{1}$$

Where $\theta(t)$ is the time dependent reversion level, a is a the speed of reversion parameter and σ the volatility. Both a and σ are constant and for a given time (t) the parameter theta θ is calculated from today's term structure:

$$\theta(t) = \frac{\delta F(0,t)}{\delta t} + aF(0,T_0) + \frac{\sigma^2}{2a}(1 - e^{-2at}) \tag{2}$$

Where $F(0, T_0)$ is the forward rate computed at today's term, T_0 is the expiry of the option, and the δ is the size of the steps, which in our case represent one year.

2.4 Model calibration

The process by which a model's parameters are adjusted to best describe the real market prices is called calibration. This process is of central importance when considering the usability of a model. While in practice, the speed with which a model can be calibrated is of central importance, this aspect is beyond the scope of this work. In terms of the HW, the calibration process implies adjusting the parameters a and σ to best describe market prices, since the mean reverting parameter $\theta(t)$ can be characterized with available market information as described in equation (2).

2.5 Artificial Neural Networks (ANN)

ANN models consist on a supervised learning method where the algorithm does not assume any kind of distribution in advance. The ANN tries to adjust its behavior to the data used in the training phase using a large number of parameters, being totally focused on the prediction results.

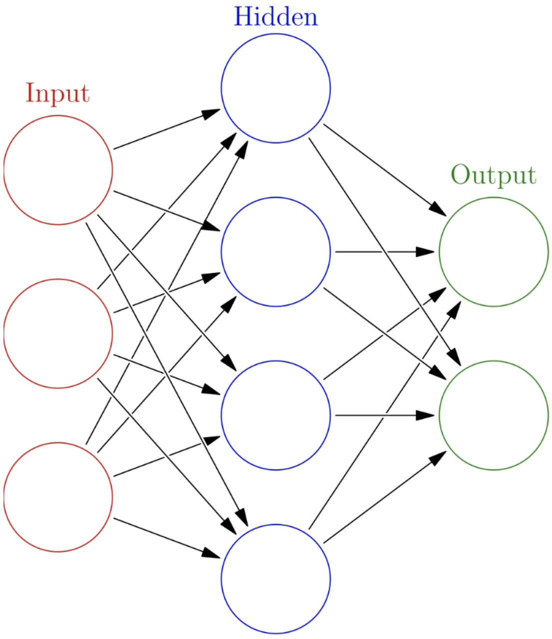

Figure 2: ANN

The architecture of the ANN references the human brain functionality having neurons that are connected between each other sending "signals" in order to process information received (inputs) and generate an action/prediction (output) having in the middle hidden layers that correspond to how the inputs are translated into outputs, but these don't have a specific meaning.

In the case of the ANN algorithm, each input is the argument of an activation function and is attributed a weight. These weights will be chosen in order to minimize the loss function. For instance, trying to minimize the mean squared error (MSE).

This search of the "best" weights is called learning and is computed by starting with a random weight for each neuron that is adjusted using a methodology (in our case is back-propagation) which consists of the calculation of the gradient descent of the cost function for each instance of the weights and re-assigning the weights.

3 Literature Review

One of the first papers to introduce the concept of Artificial Feedforward Neural Networks (AFNN) and how useful this method can be was Hornik et al. [1989]. In this seminal work, the authors show what is now known as the Universal Approximation theorem. The theorem states that this method (AFNN) can approximate any function, as long as it is uniformly continuous in compact set and the number of nodes can increase arbitrarily.

Due to the fact that these methods are computationally demanding, they gained much more relevance recently, with the development of computers with a significantly increase in processing power. The method is applied in a wide range of areas, including finance. According to Hamid and Iqbal [2004], the most common uses of ANN in finance are mostly concerning pricing and financial distress prediction, which together would account to roughly 45% of all the studies until their paper's publication in 2004. More recent examples of works that use ANN for pricing financial derivatives are Liu et al. [2019], who also uses the method for computing implied volatilities, and Ferguson and Green [2018], that also makes a guide about how to use deep neural networks in finance. Ruf and Wang [2019] also provide a comprehensive review of the literature on the applications of ANN in option pricing, hedging and model calibration. Starting from the 1990s, practitioners have used ANN to price options and several papers in the academic literature have been wrote on the subject.

If on one hand, using ANN for pricing financial assets and its derivatives is not necessarily new, on the other hand the literature on the use of ANN for model calibration is more recent. For instance, in a pioneering paper, Abu-Mostafa [2001] uses ANN to calibrate the Vasicek model. Andreou et al. [2010] propose an ANN that calibrates parametric models.

While these papers introduced techniques based on ANN to calibrate the option pricing models, the work done by Hernandez [2016] helped renew the interest in model calibration with ANN. Hernandez presents a method to calibrate models using ANN. He tests his approach with the Hull-White model for over three years of daily quotes of swaption contracts of 12 expiries and 13 tenors. The main argument presented by the author is the gain in velocity in the calibration process by using ANN. Several papers have followed Hernandez pioneering work. McGhee [2020] use ANN to calibrate the SABR model, by pointing out that this method can be about 10,000 faster than the commonly SABR Approximation. Liu et al. [2019] use ANN to calibrate stochastic volatility under Heston Bates models, proposed by Bates [1996]. Stone [2020] used ANN to calibrate rough volatility models. Bayer et al. [2019] provide an ANN method for calibrating rough volatility models. Thorin

[2021] uses ANN to obtain calibrated SABR parameters.

Gurrieri et al. [2009] describe several strategies for the calibration of one factor model Hull-White. They give the closed-forms for exact pricing using explicit integrals of the model parameters and propose parametric forms for the mean reversion a and volatility σ. Moreover, they provide closed-form formulas for payer swaption prices that we use in this paper and we introduce in the Methodology section. The approach is to express the price of a payer swaption as a weighted-average of zero-coupon bonds options. This idea, in turn, was first proposed by Jamshidian [1989] and is known as the Jamshidian's decomposition.

4 Methodology

In this section we describe the methodology we followed to calibrate Hull-White model.

4.1 Swaption Pricing

We used the Black's Formula to obtain the market prices of swaptions. Black's formula is a derivation of the Black Scholes formula, in which the underlying asset is a swap. It is a model used to price swaptions with information available from the market: implied volatility σ and yield curve $P_0^{T_0}...P_{T_0}^{T1}...P_{T_0}^{T_n}$. Interest swap rates and implied volatilities are from EUR ATM payer swaptions from April 29, 2022 and were downloaded from financial data provider Refinitiv.

4.1.1 Yield curve and T-bond prices

We start by obtaining annual interest swap rates for 10 years.

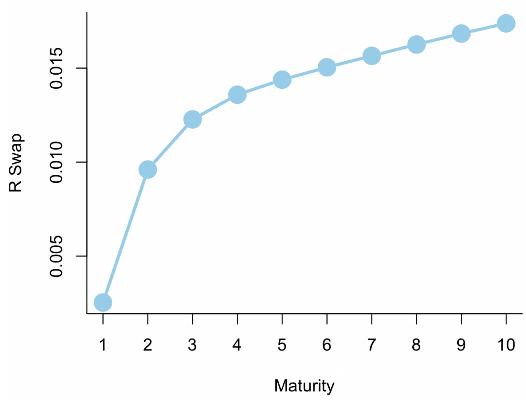

Figure 3: R Swaps against Maturity

As it is possible to see in Figure 3, the yield curve is upward sloping, in which the yield of the bonds increases as a function of their maturity.

The next step is to calculate the T-bond prices. Knowing that size of the steps $\delta = \delta(T_i, T_{i+1})$ is 1, we start by defining the T-bond prices as follows:

$$P_{T_0}^{T_n} = \frac{1 - R(U_n)\delta \sum_{i=1}^{n} P_{T_0}^{T_i}}{1 + R(U_n)\delta} = \frac{1 - R(U_n) \sum_{i=1}^{n} P_{T_0}^{T_i}}{1 + R(U_n)}$$

Where $R(U_n)$ correspond to the swap interest rate at tenor n. As proposed by Wilmott [2007], we also assume that $P_0^{T_0} = e^{-rT_0}$, where r is 0.118% (EURIBOR 12 months from April 29, 2022). The resulting prices of the T-bond are shown in Figure 4.

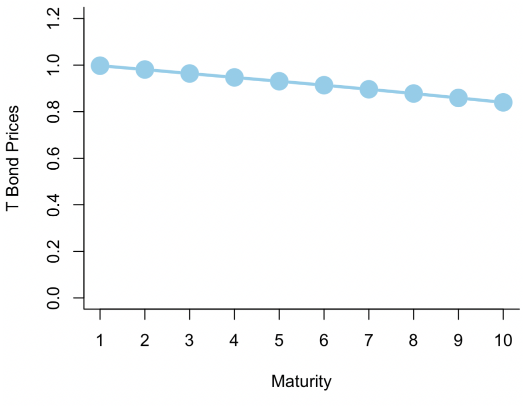

Figure 4: T Bond Prices against Maturity

4.1.2 Implied volatility

Next, we obtain the implied volatilities for ATM payer EUR swaptions for 10 expiries and 10 tenors. The implied volatility is the volatility implied by the option prices observed in the market. Usually, swaptions are priced in terms of implied volatilities instead of in dollar or euro amounts. As it possible to see in Table 1 and Figure 5, implied volatility is greater for short tenors and expiries.

	1 Yr	2 Yr	3 Yr	4 Yr	5 Yr	6 Yr	7 Yr	8 Yr	9 Yr	10 Yr
1 Yr	115.32	118.30	113.25	110.97	109.40	108.48	107.17	105.51	103.56	101.40
2 Yr	115.57	114.51	110.12	106.87	103.54	103.33	102.41	100.94	99.12	97.13
3 Yr	110.71	108.71	105.29	102.32	99.12	98.58	97.51	96.00	94.20	92.27
4 Yr	104.49	102.19	100.21	98.03	95.64	94.45	93.08	91.52	89.81	87.96
5 Yr	98.97	96.86	95.41	93.85	92.24	90.80	89.39	87.91	86.37	84.68
6 Yr	93.66	92.44	91.14	89.75	88.31	87.09	85.87	84.60	83.26	81.81
7 Yr	89.27	88.33	87.15	85.92	84.65	83.64	82.59	81.51	80.36	79.15
8 Yr	86.48	85.57	84.39	83.18	81.90	81.08	80.18	79.21	78.18	77.10
9 Yr	83.66	82.91	81.68	80.52	79.28	78.63	77.86	77.00	76.08	75.13
10 Yr	80.09	80.38	78.76	77.72	76.75	76.26	75.62	74.88	74.06	73.24

Table 1: Implied Volatilities by Expiry (rows) and Tenor (columns)

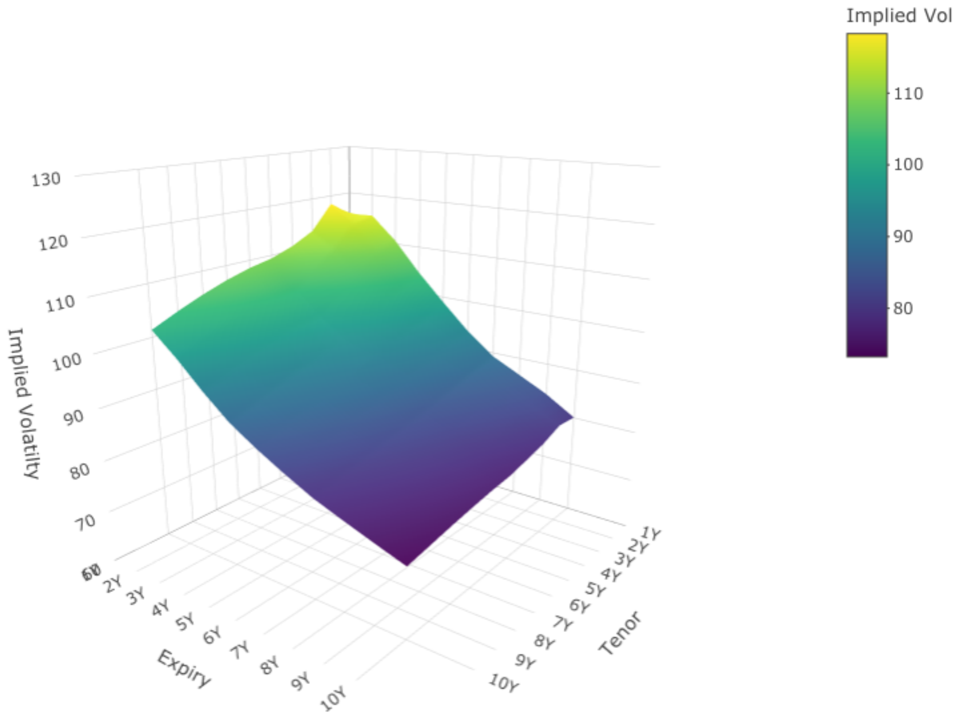

Figure 5: Implied Volatility Surface

4.1.3 Black's formula

Next, we use Black's formula introduced in *Concepts review* to derive market prices. The price of a payer swaption at time $t = 0$ is given by the following expression:

$$Swaption(0) = NP_0^{T_0}(1 - P_{T_0}^{T_n})(\Phi(d_1(0)) - \Phi(d_2(0))) \tag{3}$$

Where $d_{1,2}(0) = \pm\frac{1}{2}\sigma(0)\sqrt{T_0}$

In Figure 6 it is possible to observe that the swaption prices increases with respect to expiry, while remain relatively similar with different tenors.

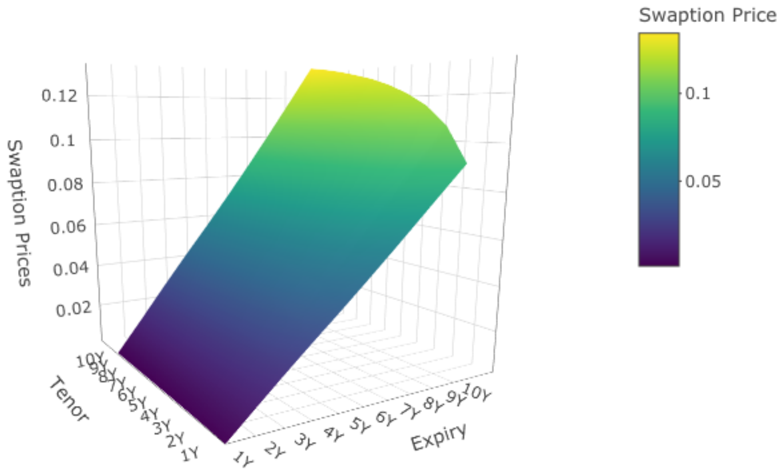

Figure 6: Market prices

4.2 Hull-White price simulation

Having obtained the market prices, we now generate 10,000 swaption prices with their respective pair of parameters a and σ under the Hull-White model for different expiries and tenors. In order to do so, we must get to a close-form equation for the swaption price, as a function of σ, a, expiry (T_0) and tenor ($T_n - T_0$).

The approach to price swaptions through HW is to construct a portfolio of options of zero coupon bonds that replicate the payoffs of a swaption contract. The price of the resulting portfolio must be equal to the price of the swaption, otherwise arbitrage opportunities would be created.

The price of the bond is given by:

$$X_i = e^{A(T_0,T_i)-B(T_0,T_i)r^*} \tag{4}$$

Where $B(T_0,T_i)$, $A(T_0,T_i)$ are given by:

$$B(T_0,T_i) = \frac{1-e^{-a(T_i-T_0)}}{a} \tag{5}$$

$$A(T_0,T_i) = ln(P_{T_0}^{T_i}) + B(T_0,T_i)r - \frac{\sigma^2}{4a}B(T_0,T_i)^2(1-e^{-2aT_0}) \tag{6}$$

and r^*, is the solution of:

$$\sum_{i=1}^{n} c_i e^{A(T_0,T_i)-B(T_0,T_i)r^*} = 1 \tag{7}$$

Where:

$$c_i = R(U_i) \text{ and } c_n = 1 + R(U_n) \tag{8}$$

Next, for calculating the prices of a European put option (ZBP) that matures at time T_0 on a zero-coupon bond maturing at time t, we have that:

$$ZBP(T_0,T_i,X_i) = X_i P_0^{T_0} N(d_1) - P_0^{T_i} N(d_2) \tag{9}$$

with:

$$d_1 = \frac{ln(\frac{X_i P_0^{T_0}}{P_0^{T_i}})}{\sigma_p} + \frac{\sigma_p}{2} \text{ and } d_2 = \frac{ln(\frac{X_i P_0^{T_0}}{P_0^{T_i}})}{\sigma_p} - \frac{\sigma_p}{2} \tag{10}$$

And σ_p is calculated in order to calculate d_1 and d_2 in the form of:

$$\sigma_p = \sigma\sqrt{\frac{1-e^{-2aT_0}}{2a}} \cdot \frac{1-e^{-a(T_i-T_0)}}{a} \tag{11}$$

Finally, the Hull-White price for the payer swaption is given by:

$$Swaption(T_0,T_n,a,\sigma) = \sum_{i=1}^{n} c_i ZBP(T_0,T_i,X_i)$$

$$Swaption(T_0,T_n,a,\sigma) = ZBP(T_0,T_n,X_n) + \sum_{i=1}^{n} R(U_i) ZBP(T_0,T_i,X_i) \tag{12}$$

Now that we have a closed-form equation for the payer swaptions under Hull-White, we can generate multiple prices for the different expiries and tenors by sampling different as and σs.

We generate 10,000 values of HW prices with a and σ ranging from 0.01 to 0.35. The densities of the prices the real market data and the randomly simulated prices can be seen in the figure below.

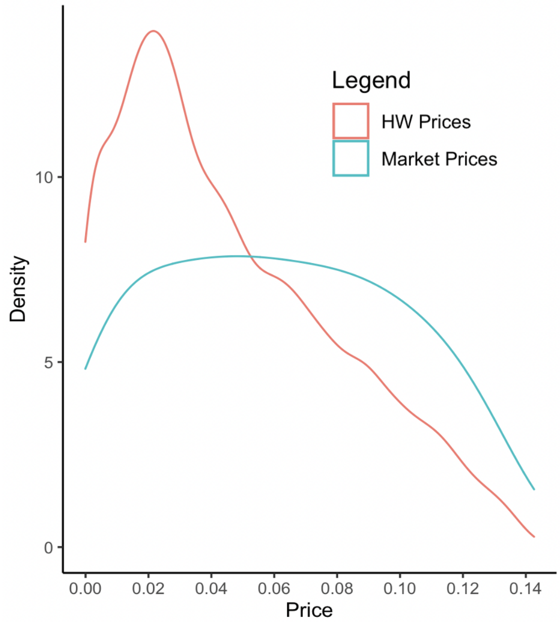

Figure 7: Densities of HW randomly generated prices vs. market prices

4.3 Neural Network model

Now that we have a function that maps payer swaption prices from the arguments (T_0, T_n, a, σ), that is $F : (T_0, T_n, a, \sigma) \longrightarrow Price$, we want to build an ANN that learns the inverse mapping of this function. That is, that learns the relationship: $F^{-1} : (T_0, T_n, Price) \longrightarrow (a, \sigma)$.

To do so, we implement the framework provided by Ferguson and Green [2018] for the calibration problem. This way, we construct an ANN model consisting of three inputs (Price, Expiry and Tenor), going through one hidden layer containing 4 nodes and having as an output two nodes (a and σ).

These parameters provided by the neural network correspond to the ones that minimize the MSE according to the algorithm.

The learning method chosen was back-propagation, the optimizer was Adaptive Moment Estimation (Adam), the number of epochs where the results converge was 100, and the size of the hidden layer was 4 nodes plus a constant. The figure below shows the final ANN estimated.

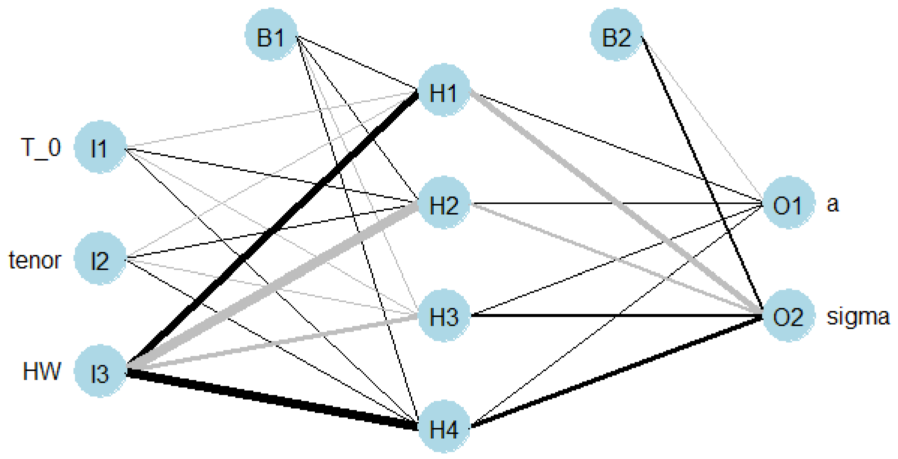

Figure 8: Architecture of ANN model

In this figure, it is possible to observe the different weights of the ANN by looking at the thickness of each arrow connecting the neurons. Being the thicker the more weight assigned by the neural network, it is possible to affirm that the price its the most important input for the model.

Once the neural network is trained using the data simulated by the Hull-White model, it is possible to estimate the a and σ corresponding to each expiry and tenor of the market prices.

With these parameters calculated, the Hull-White price is once again calculated using the calibrated parameters a and σ, in order to verify the accuracy of the results. The closer the market prices are from the Hull-White prices generated by the a and σ estimated, the better the calibration.

5 Results

After estimating the inverse mapping function with the ANN, we predict what would be the pairs of (a, σ) for each tenor and expiry in the sample. Having this, we calculate the theoretical HW prices from the calibrated model and we plot its surface together with the initial market price surface. The closer these two surfaces are from each other, the better the calibrating method. The results can be seen below:

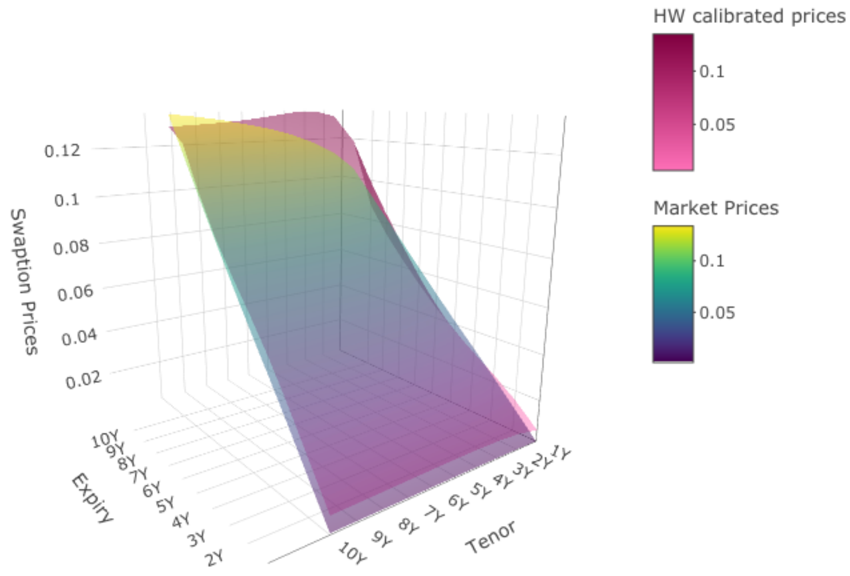

Figure 9: Market surface vs. HW calibrated prices

With a simple visual inspection, one can see that the results are very close to the ones observed in the market. However, one might question whether this result is simply due to a similarity in the generated prices and the observed. That is, maybe for the range of a and σ we used, the closed-form equation for the HW prices generate most of the prices around the same ones observed. In this case, the ANN would not be very useful. We address this question by plotting below the histograms of the (i) market prices, (ii) the randomly generated HW prices, and (iii) the ANN-calibrated prices:

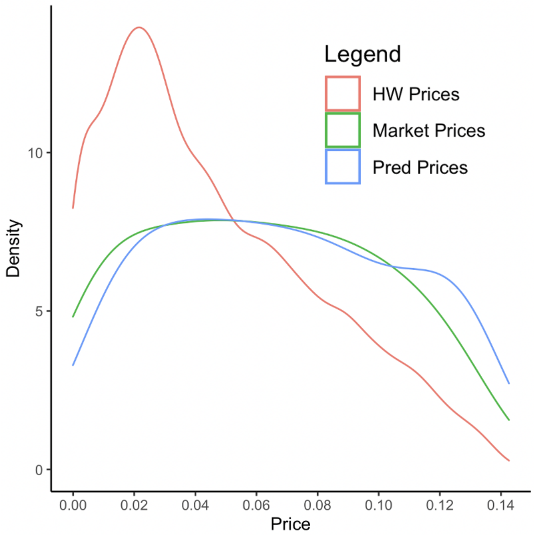

Figure 10: Histograms of the (i) market prices, (ii) the randomly generated HW prices, and (iii) the ANN-calibrated prices

It is evident that the ANN plays an important role in approximating the generated prices to the market prices. The similarity in between the histogram of the market data and the calibrated prices is a good indication of the power of this method.

Finally, the difference in between the two surfaces is shown in the heat map of the MSEs as a function of tenor and expiry below.

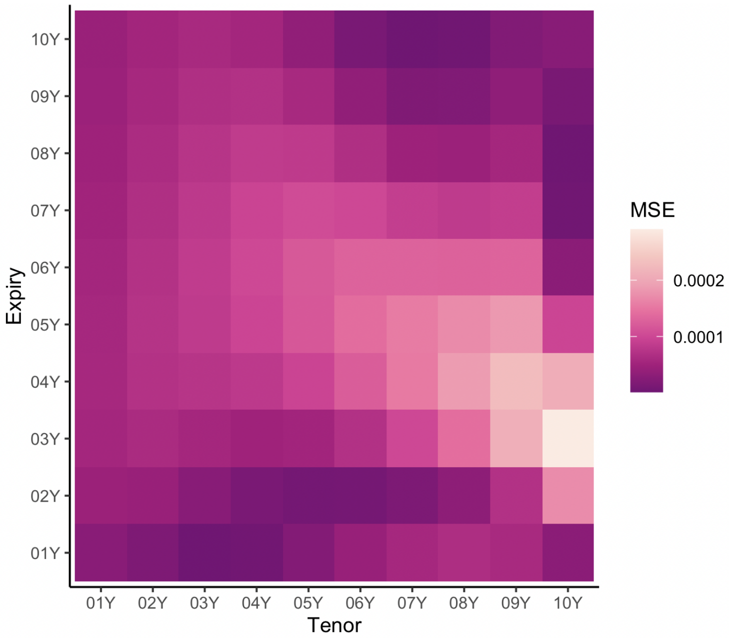

Figure 11: Heat map of the MSEs

It is possible to observe how the error is relatively small across all the combinations of expiry and tenor. The worst fit is for tenor equal 10 and expiry equal 3.

6 Discussion and Future research

In this project, we used Artificial Neural Networks (ANN) to learn the inverse mapping of the Hull-White (one factor) model for pricing swaptions. This result allowed us to calibrate the model's parameters for each pair of expiry and tenor, using data from EUR payer swaptions. With the newly calibrated model, we were able to calculate what would be theoretical prices for each of these contracts and compare it with the ones observed in the market.

Despite the fact that we used data that referred to a single day, we managed to derive a good approximation of the market prices. This result shows the power of the Universal Approximation theorem of ANNs and of the method itself. It also implicitly validates the usefulness and the widespread use of the Hull-White (one factor) model.

Naturally, this presents some limitations. For instance, the data availability and its usage limits our parameters calibration. This leads to a good fitting of the model for the specific date we used, which does not necessarily translate into a model that can be used to predict prices in the future. Therefore, future work may want to address this matter by expanding this analysis to use implied volatility from a range of dates, instead of a single day.

Additionally, further research could assess the relative performance of the ANN with numerical methods. Such method comparison should focus on the precision and speed of the calibration process. The Newton-Rapshon is a numerical method commonly used in the industry and could be used as a benchmark. It is a calibration method used to find the root of a function with only one variable. Therefore it is commonly used to calibrate the Black-Scholes model, since the model has only one parameter: the implied volatility. In this setting, the calibration problem is to find the implied volatility that makes the theoretical price of an option equal to the one observed in the market. It is however possible to use this method to calibrate models of more than one parameter. Nevertheless, the calibration of the parameters cannot occur simultaneously. In terms of the HW model, the implementation of the Newton-Rapshon method would imply calibrating one parameter while having the other fixed.

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