

Designing relational sanctions in buyer–supplier relationships

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Abstract

This paper explores the role of supplier performance measures (scorecards and others) in the internal design of relational contracts. We analyze a simple supplier–buyer repeated interaction in which incentives arise by the threat of terminating the relationship—temporarily or permanently. As the periods with no-trade reduce the value of the relationship, the optimal relational contract minimizes the equilibrium punishment while preserving the supplier's incentives. We show that making the relational penalties conditional on additional supplier's performance measures increases total surplus. We also provide a rationale for “forgiveness” in relational contracting. The buyer may optimally forgive (decide not to sanction at all or to impose a lesser sanction) the supplier despite a bad outcome when some additional information is positive. We start with a binary performance measure, but we extend our analysis to more complex performance measures such as scorecards. Finally, we rank the scorecards in terms of their informativeness and we characterize the optimal investment in the design and improvement of these performance measures.

1 | INTRODUCTION

Since the landmark findings of Macaulay (1963) on commercial contracting practices, we know that many vertical relationships are characterized by relational contracting in which cooperation is not induced by external enforcement, but by the parties' concern about future dealings.^{1,2} One advantage of relational contracts³ is that they can rely also on subjective, nonverifiable information that cannot be used by external or third-party enforcers. The way in which this information is used for the internal design of relational contracts is a key element in the performance of supplier–buyer relationships.

In parallel, the concept of “balanced scorecard” developed by Kaplan and Norton (1992, 1993, 1996, 2001), has become a powerful instrument in formal incentive contracts, helping firms in their design of performance measurement systems by combining auditable information with nonfinancial and subjective information that may be linked to long-term firm goals. Gibbons and Kaplan (2015) also advocate the use of these balanced scorecards in “informal management,” where executive decisions involve discretion and judgment beyond the implementation of formal contracts.

This paper connects these two strands of thinking and combines the enforcement side of relational sanctions with the informative role of performance measures. We analyze on how nonenforceable performance metrics or assessments (in the figure of scorecards or in other forms) can be deployed in the design of relational contracts. In particular, the model concerns a simple supplier–buyer repeated relationship in which incentives arise only—at least in the base

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model- through the threat of termination by the client. Output, which is observable but not verifiable, depends on the supplier's effort but correlation is imperfect. Since output is a noisy signal of effort, even when both parties are honoring the relational contract, a bad realization of output may materialize and the provision of incentives requires that the observed low output may trigger the imposition of a relational punishment on the supplier. Following Green and Porter (1984), we focus on a sanctioning strategy whereby the buyer terminates the supplier for T periods upon observing low output and then resumes the relationship.

The no-trade periods reduce the value of the relationship for both parties. Thus, the sanctioning policy should be designed to minimize the equilibrium punishment while keeping the supplier under the right incentives for effort. We show that one way to increase the surplus from the relationship is to make the temporary termination decision also conditional on additional measures of the supplier's performance. Even when these measures are not verifiable by external adjudicators and cannot be part of an enforceable contract, they improve the functioning of the relational contract. They allow the buyer to reduce the length of the equilibrium sanction that is necessary to incentivize the supplier and thus decrease the loss of value associated with actual punishment. These measures allow the contract to display relational sanctions that are better targeted and less dependent on the noisy realization of output. For example, the buyer may forgive the supplier despite a bad outcome when the additional information is positive.⁴

We start by considering that the performance measure is related to the success or not in the execution of a single interim or ancillary task specified in a document known by the parties (it could be a long term agreement, or some other form of written schedule). We assume that the underlying effort undertaken by the supplier has an impact on the probability of success in this interim task (information channel) or on the cost of performing such a task (cost channel).

The information channel is intuitive since succeeding or not in the specified task becomes a signal of the unobservable supplier's effort. The cost channel affects the incentive compatibility constraint, increasing the relative cost for the supplier of not exerting proper effort in the first place. We show that both channels may increase the surplus of the relational contract. In our base model, the written specification of the ancillary tasks merely plays an informational role and is not part of an enforceable contract that could be brought to court and eventually give rise to penalties or additional payments to the supplier. However, we also show that our results hold when the terms of the document spelling out the performance of the tasks are weakly enforceable and we analyze how the degree of enforceability affects the shape of the relational contract.

We extend our baseline model by replacing the binary setting of a single task that can be performed or not by the supplier with a more complex scorecard that includes several nonverifiable dimensions of the supplier's contractual behavior (an overall "grade"). In this richer scenario we characterize the optimal relational sanctions and provide an useful definition of what is a more informative performance scoring. Finally, we characterize, in terms of the main parameters of the setting (value of the relationship, asymmetry of information, discount factor), the optimal investment in improving the performance metrics.

2 | RELATED LITERATURE

Our analysis provides support to the use of performance scorecards in relational collaborations. Additionally we provide a rationale for "forgiveness" in relational contracting. Buyers may be more willing to overlook observed failures in outcomes in their dealings with their suppliers as long as the latter evidence traits of good behavior in undertaking prespecified ancillary tasks or targets. Forgiveness of failures is a pattern in supplier-buyer relations that to mainstream views of relational contracting (Baker et al., 1994) would not explain. An exception is Vanneste and Frank (2014), who relate "forgiveness" to the stringency of performance thresholds in relational relationships. They show that more forgiving contracts (with a lower threshold) are optimal when the relationship value is high and outside options are low. While we also consider the impact of the value of the relationship on "forgiveness," we focus on the use of performance metrics, such as scorecards, and how they help parties use finer schemes of relational sanctions that enhance the value of the relationship, allowing for more complex responses than the simple alternative between terminate and forgive. We show that when the additional information arising from scorecards is positive, the supplier's failure is optimally fully excused or less harshly punished.

Empirical evidence shows the actual importance of "forgiveness" in relational contracting. Vanneste and Frank (2014) conduct a field study on dutch companies and report the case, among others, of a company rating suppliers using performance indicators that are largely unverifiable by outsiders (quality, delivery, responsiveness, etc.) and

aggregating these indicators in a simple “traffic light” score: green (performing), yellow (underperforming), or red (unacceptable). Suppliers with yellow lights as somehow forgiven and only the red light leads to the termination of the relationship.

The existence of nonverifiable performance measures in supplier agreements is also reported by Bernstein (2015). She documents the use of scorecards by large original equipment manufacturers in the US Midwest in their relationships with suppliers. In line with our analysis, these long term agreements (MSA) including the scorecards are not intended for legal enforcement, but rather for the improvement of the accuracy of the buyer's assessment concerning the overall contractual behavior of a given supplier and avoid a mistaken imposition of severe relational punishments on cases of a mishap that should be ignored and forgiven. Bernstein and Peterson (2021) provide evidence of “managerial provisions” (terms governing the administration of the contract, information flows, coordination and adaptation actions) contained in master agreements, handbooks, supplier scorecards, and similar documents and how they serve to improve the functioning of relational supplier–buyer contracts.

This evidence shows that written arrangements including performance measures play a role in relational contracting even though they may be unenforceable. This interpretation of contracts as informational devices for aligning parties' expectations and incentives goes in line with the empirical findings of Mayer and Argyres (2004), who show how firms learn to improve their ongoing relationships over time and use contracts as a repository of such learning. This is also consistent with Bozovic and Hadfield (2016), who show, in respect of business sectors that possess an important innovative dimension, that formalized contractual documents are widely used, despite explicit recognition by market participants of the fact that legal enforcement of the contract is often not a realistic outcome. The parties relying on written contracts are able to better align what is the future understanding of parties concerning what are the right actions to take in a future set of circumstances.⁵

Klein (2000) emphasizes that more detailed contract terms dealing explicitly with certain dimensions of the interaction between the parties actually help defining the self-enforcing range of the contractual relationship, extending the ability of self-enforcement mechanisms to ensure cooperation to a wider set of market conditions.⁶ However, Klein does not consider how contract terms may help to refine reputational sanctions and to make them more closely attuned to the underlying, unobservable behavior of the agent. The positive effect from explicit contract terms addressing the agent's performance comes through court enforcement⁷ and not through the informational and cost channels that we explore.

Several other papers analyze the effects of having an explicit enforceable contract written over some subset of verifiable actions in the relational contract: Baker et al. (1994, 2011) and Schmidt and Schnitzer (1995). In particular, Baker et al. (1994) show that when a—fully enforceable—explicit contract is sufficiently “good” to approximate the relevant action, the relational and the legal contract are substitutes.⁸ When this is not the case, only an appropriate combination of both can generate desirable outcomes, and they act as complements. Our paper extends their insights to a setting in which formal contracts are replaced by performance measures that are not enforceable and are used only to improve the internal design of the relational contract. In an extension, we also explore the case in which performance measures are used both for improving the relational contract, as well as for introducing explicit and partially enforceable incentives. We show that our results hold in this case and, as predicted by the previous literature, the relational punishment decreases (forgiveness increases) when the performance measures (and the explicit incentive mechanism linked to them) are sufficiently “good.” This result has a similar flavor than Kvaloy and Olsen (2009), who analyze a setting in which the level of enforcement of the formal contract is determined by a principal's investment. They show that relational contracting and formal contracting are substitutes in the sense that completeness of the formal contract is optimally reduced when the functioning of the relational contract is better.⁹ While we do not consider initially the contracting cost of the formal contract, in our framework relational incentives (punishment) are costly and we identify an analogous tradeoff.

The incentive engine in our model is the threat of termination. Asymmetric information alone is not enough to justify the welfare loss associated with termination. Levin (2003) studies self-enforced relational contracts under private information about the supplier (agent)'s effort when output is observable and without any payment constraints between the buyer and supplier. He shows that a simple stationary contract with a bonus is optimal and termination is not needed. We implicitly assume some additional “frictions” in the buyer-supplier relationship, which make termination a necessary device to give incentives. Such “frictions” may be related, among other reasons, to additional dimensions of asymmetric information (output may not be fully observable), limited liability or uncertain opportunity cost of paying bonuses. Levin (2003) and Fuchs (2007) show that if the buyer may manipulate to some extent the performance measures (subjective evaluation), termination may be part of the optimal contract. In a spirit close to our

approach, Li and Matouschek (2013) analyze relational incentives in a labor relationship in which a principal compensates a worker with a bonus after observing high effort. The principal has private information on his cost of paying the agent and sometimes he cannot honor the relational contract. Keeping incentives in their imperfect information setting requires punishment by the worker when the principal does not pay the bonus despite observing high effort. Similarly to our model, the equilibrium path of the optimal relational contract is characterized by combining cooperation and punishment. In the same vein, Yared (2010), Englmaier and Segal (2014), and Troya (2017) provide models of self-enforcing agreements with “frictions” where cycles of cooperation and punishment emerge on the equilibrium path. For example, Troya (2017) analyzes trade credit transactions between buyers and suppliers (buyer pays with delay goods already delivered) where the ability to repay is unknown to the supplier and the threat of trade suspension is used to discipline the buyer. As in our model, on the equilibrium path the supplier stops trading for T periods if the buyer fails to repay. The paper provides anecdotal evidence consistent with these cycles of cooperation and punishment in the trade credit market. We complement the approach of these papers focused on relational contract settings with “frictions,” by showing that the use of scorecards can reduce the length of the punishment periods and their associated inefficiencies.¹⁰

In a related spirit, Li et al. (2021) explore how rulebooks and standardized processes may be used in the design of relational contracts between the firm and its workers. When the actions available to the workers are more discretionary and are not constrained by rules of proper performance, even if not enforceable, surplus is higher, but incentives are costlier for the firm to provide. Rulebooks may be introduced by the firm in scenarios of higher costs of honoring the relational contract, even if surplus is lower, since implementing the contract is cheaper for the firm. There is a dynamic problem with more formalized and structured action guidelines for workers, since they are tempted to stick to the cheaper actions following the book and not to choose costlier but more productive discretionary moves, thus creating a long-term cost to the use of rigid playbooks in relational contracts. Contrary to the role that scorecards play in our model, rulebooks have not positive informative effect over the functioning on the relational contracts in theirs.

Close to our paper is also Iossa and Spagnolo (2011), who analyze a setting in which firms use formal contracts to improve relational contracting. The driving forces of their results are, however, very different from ours. Their main idea is that the formal contract may specify some irrelevant or inefficient tasks, used as a “threat” to discipline informal agreements over efficient and non-contractible tasks. Thus, in their model, auxiliary tasks are fully enforceable but are not undertaken in equilibrium.

3 | THE MODEL

A supplier (S) undertakes a project for a buyer (B). The outcome of the project, $y \in \{0, V\}$, is uncertain and the probability of the project being successfully completed depends on the effort exerted by S . In particular, we assume that S decides between two possible levels of effort, $e \in \{\underline{e}, \bar{e}\}$. The choice of effort is private information (not observable by B) and not directly contractible. Exerting effort is costly, $c_{\underline{e}} < c_{\bar{e}}$, and determines the probability of completion of the project, $p_{\underline{e}} < p_{\bar{e}}$. For simplicity and without loss of generality, we take $c_{\underline{e}} = 0$, $c_{\bar{e}} = c$, $p_{\underline{e}} = 0$, and $p_{\bar{e}} = \pi$.

If the project is successful, it delivers profits $y = V > 0$. High effort is efficient, $\pi V - c > 0$. S is financially constrained and cannot finance the project. B pays upfront an exogenous price P^{11} to S for undertaking the project, as $\pi V > P > c$.¹² Given these assumptions, B would be willing to hire S if effort is high ($\pi V > P$), but not otherwise. In a static framework, B correctly anticipates that given that the effort is not observable and contractible, S will exert low effort and therefore there will be no trade. Parties can overcome this market failure when the interaction is repeated by using a relational contract.

Now we consider an infinite horizon framework with an infinitely lived S and an infinitely lived B , in which the basic game above is repeated over and over again. As in the static game, we still assume that contracts cannot be verified by a third party who could enforce the explicit provisions of agreement.

This repeated game has multiple equilibria, including the repetition of the solution to the static game. We will focus on equilibria supporting cooperation between S and B . In particular, we consider the following grim strategy subgame perfect equilibrium inspired by Green and Porter (1984):¹³ When the project fails, the no-trade equilibrium in which the S chooses no effort and B does not buy, takes place for T periods. After expiration of these T periods, the cooperation phase is reinstated.

- B starts trusting S in Period 1 and financing the project by paying price P , initiating a cooperation phase.

- Cooperation phase. There is trade, S chooses high effort and B trusts S by financing the project by paying price P until a project failure occurs, starting a punishment phase.
- Punishment phase. When B observes project failure, she reacts by discontinuing to finance projects with S for T periods. After expiration of the T periods, B is willing to trade with S again. The cooperation phase may be reinstated.

We are in a setting of ex-post imperfect information: the fact that the project has failed is an imperfect signal of S 's level of effort. If the signal were perfect, then T could be infinite and the cost of punishment would be 0, since punishment would never be imposed in equilibrium. In our setting, the imperfect information leads agents to incur punishment costs. Both parties would be better off if they did not stop trading during the punishment phase (T periods). However, punishment is necessary to preserve incentives. We denote as the "optimal" contract in this setting, the one maximizing the number of periods in which trade occurs, or, equivalently, minimizes the number of periods in which the costly reputational sanction is imposed subject to providing incentives to the agent to exert high effort.

This relational contract is optimal within the set of "Green and Porter" grim strategies described above, but it is not globally optimal. In the Green and Porter model (and also in our setting) alternative and more complex strategies exist that generate equilibria in which parties get a higher surplus.¹⁴ However, this "Green and Porter" grim strategy is appealing, since it is simple and easy to implement and, more importantly for the present paper, it summarizes in a single parameter T the inefficiencies of the relational contract due to the imperfect monitoring of effort.

We assume that both parties face the same discount factor, $\delta \in (0, 1)$. When B and S play the strategy described above, let V^+ and V^- be the present discounted value of S 's profits in the cooperation and punishment phase, respectively. We have:

$$\begin{aligned} V^+ &= P - c + \pi\delta V^+ + (1 - \pi)\delta V^-, \\ V^- &= \delta^T V^+. \end{aligned}$$

Solving the equation system we obtain both present values in terms of the parameters of the model

$$V^+ = \frac{P - c}{1 - \pi\delta - (1 - \pi)\delta^{T+1}}, \quad (1)$$

$$V^- = \delta^T V^+ = \frac{\delta^T (P - c)}{1 - \pi\delta - (1 - \pi)\delta^{T+1}}. \quad (2)$$

Finally, to achieve this equilibrium we must add an incentive compatibility constraint. The following inequality captures the lack of incentives of S to choose low effort:

$$V^+ \geq P + \delta V^-$$

Using the definition of $V^+ = P - c + \pi\delta V^+ + (1 - \pi)\delta V^-$, the incentive compatibility constraint can also be written as:

$$\pi\delta(V^+ - V^-) \geq c. \quad (3)$$

We are interested in another equivalent expression for the inequality above, which can be found using the solution to the equation system V^+ and V^- (we plug Equations 1 and 2 into 3):

$$\pi\delta \frac{(1 - \delta^T)(P - c)}{1 - \pi\delta - (1 - \pi)\delta^{T+1}} \geq c.$$

Let $\Phi(T)$ be the left-hand side of the incentive compatibility constraint above. For our purposes, this function has a useful property:

Lemma 1. $\Phi(T)$ is increasing in T .

Hence, to solve optimally the infinitely repeated game, we want to choose T to maximize V^+ :

$$\max_T V^+ = \max_T \frac{P - c}{1 - \pi\delta - (1 - \pi)\delta^{T+1}}$$

subject to the following constraint:

$$\Phi(T) \geq c.$$

Given that our function satisfies $\frac{\partial V^+}{\partial T} < 0$, then the optimal T^* for our problem will be the minimum T that satisfies the identity $\Phi(T^*) = c$. But this equation has a unique solution, by Lemma 1.¹⁵

The optimal punishment T^* has been characterized for a given value of the discount factor δ , probability of success of the project under high effort π , and marginal profit $P - c$. Next Lemma establishes how the optimal punishment T^* depends on this set of parameters.

Lemma 2. *The optimal punishment T^* is decreasing in π , $P - c$, and δ .*

The intuition of Lemma 2 is as follows. The optimal punishment decreases with π , since the difference between the supplier's output in the cooperation and punishment phases increases with π and hence higher π provides more incentives to the supplier to exert effort for a given punishment.¹⁶ The optimal punishment T^* also decreases with $P - c$ and δ , since they increase the cost for S of the missing trade following project failure.

4 | SPECIFYING NONENFORCEABLE PERFORMANCE MEASURES

We now introduce in our previous setup a performance measure scheme $\Omega = (y, a)$ agreed by B and S . This specifies, in addition to the outcome of the project y , also whether or not S has succeeded in some ancillary task $a \in A$. For the sake of expositional simplicity, we also assume that B does not obtain any direct benefits from the tasks specified in Ω other than giving incentives to S . We start by considering that Ω is a performance measure that may affect the internal design of the relational relationship between B and S but without a “direct” impact over S payoffs¹⁷.

The way in which Ω may provide incentives can be approached in various ways. One possibility (that we denote as the cost channel) is that the cost of undertaking the stipulated tasks in Ω is smaller if S has exerted effort. Another path (the probability channel) is that the probability of success in discharging the specified tasks is larger when S has taken effort. Obviously, both channels may be at work at the same time.¹⁸ In this section, we will focus on the probability channel, and in a later extension we will look into the cost channel.

The probability channel would work as follows. At the end of the project (or the relevant project phase) the parties learn the realization of $\Omega = (y, a) \in \{(V, S), (V, NS), (0, S), (0, NS)\}$. If the project succeeds, B learns that high effort has been exerted and the outcome of the ancillary task plays no role. If the execution of the project fails, the outcome of the ancillary task provides imperfect information on effort exerted by S . We may regard the task a as an informative signal of the effort exerted by S , since we are assuming that the probability of performing this task is higher when S has exerted effort: $\Pr(S|\bar{e}) = \alpha > \beta = \Pr(S|e)$.

The next step is to analyze the interaction between Ω and the relational/informal interaction. The main idea is that parties can use the information of this performance measure for improving the functioning of the informal contract by tailoring the reputational or relational punishment more tightly to the ex post probability that no effort has been exerted.

Formally, we define a new infinite horizon game in which B makes the relational sanction dependent on the outcome of Ω . We proceed as in the previous case by computing the Present Discounted Value of S 's profits given the punishment by B , now based on the observed performance summarized in Ω .

$$\begin{aligned} V^{\Omega+} &= P - c + \pi\delta V^{\Omega+} + (1 - \pi)\delta\alpha V_S^{\Omega-} + (1 - \pi)\delta(1 - \alpha)V_{NS}^{\Omega-}, \\ V_S^{\Omega-} &= \delta^{T_S} V^{\Omega+} \\ V_{NS}^{\Omega-} &= \delta^{T_{NS}} V^{\Omega+} \end{aligned}$$

If the project succeeds, $\Omega_+ = \{(V, S), (V, NS)\}$, B learns that high effort has been exerted and no punishment takes place. In case of project failure $\Omega_- = \{(0, S), (0, NS)\}$, a relational sanction is triggered, but the length of the punishment may depend on the outcome of a . Then, we differentiate the case of task completion, $V_S^{\Omega-}$ ($\Omega = (0, S)$) from the case of task failure $V_{NS}^{\Omega-}$ ($\Omega = (0, NS)$). Solving the equation system, we obtain:

$$V^{\Omega+} = \frac{P - c}{1 - \pi\delta - (1 - \pi)(\alpha\delta^{T_S+1} + (1 - \alpha)\delta^{T_{NS}+1})}$$

Introducing Ω also affects the incentive compatibility constraint, so to express that the firm has no incentive to exert low effort, now we have:

$$V^{\Omega+} \geq P + \delta [\beta V_S^{\Omega-} + (1 - \beta)V_{NS}^{\Omega-}]$$

Following similar computations than in the previous section, we obtain the incentive compatibility constraint with the performance measure Ω as the inequality given by:

$$\Psi^{\Omega}(T_S, T_{NS}, \alpha, \beta) \geq c$$

where this new function is:

$$\Psi^{\Omega}(T_S, T_{NS}, \alpha, \beta) = \frac{\delta [\pi + (1 - \pi)(\alpha\delta^{T_S} + (1 - \alpha)\delta^{T_{NS}}) - (\beta\delta^{T_S} + (1 - \beta)\delta^{T_{NS}})](P - c)}{1 - \pi\delta - (1 - \pi)(\alpha\delta^{T_S+1} + (1 - \alpha)\delta^{T_{NS}+1})}$$

Notice that $\Psi^{\Omega}(T, T, \alpha, \beta) = \Phi(T)$, if we impose that penalties are independent of the performance measure, $T_S = T_{NS} = T$.

We are interested in characterizing the optimal relational sanctions under Ω which will be the solution to the following problem:

$$\max_{T_S, T_{NS}} V^{\Omega+} = \frac{P - c}{1 - \pi\delta - (1 - \pi)(\alpha\delta^{T_S+1} + (1 - \alpha)\delta^{T_{NS}+1})}$$

subject to the incentive constraint:

$$\Psi^{\Omega}(T_S, T_{NS}, \alpha, \beta) \geq c.$$

Then, we need to determine the optimal relational punishment when S succeeds in the additional task, T_S , and when the latter is not performed, T_{NS} . To compare the solution to this problem (with two punishment variables (T_S, T_{NS})) with the optimal relational punishment in the previous framework with only one instrument T , we focus on the impact of the punishment on the objective function. We say that (T_S, T_{NS}) generates lower expected relational punishment costs than T if $\alpha\delta^{T_S+1} + (1 - \alpha)\delta^{T_{NS}+1} > \delta^T$. In fact, the solution to the problem is the pair (T_S, T_{NS}) that satisfies the incentive compatibility constraint and maximizes $\alpha\delta^{T_S+1} + (1 - \alpha)\delta^{T_{NS}+1}$ (minimizes the expected relational punishment costs).

First, we characterize what is the optimal punishment policy when the information provided by Ω is used.

Proposition 1. *The optimal relational contract (i) uses feedback from performance measures and generates lower expected relational punishment costs than without feedback, that is, $\alpha\delta^{T_S^*} + (1 - \alpha)\delta^{T_{NS}^*} > \delta^{T^*}$. (ii) maximizes the relational punishment in case of negative performance (minimizes the punishment in case of positive performance).*

This implies that the optimal relational contract (T_S, T_{NS}) may have two formats: Never again ($T_S = T, T_{NS} = \infty$) and Full forgiveness ($T_S = 0, T_{NS} = T$).

In the current setting, we can provide incentives to exert effort by penalizing either when $\Omega_- = (0, S)$ or when $\Omega_- = (0, NS)$. However, penalizing when the additional task is not performed is more effective since it provides additional incentives given that $\Pr(S|\bar{e}) = \alpha > \beta = \Pr(S|e)$. Thus, we would optimally want to focus the costly punishment on those cases where signals reveal the “worst news” concerning the underlying effort that Ω is interested in inducing. Our core idea is that given the costly nature of the relational punishment, once incentives have been ensured, minimizing the punishment enhances contract surplus. The buyer starts by penalizing only when the supplier fails both in the project and the additional task (full forgiveness ($T_S = 0, T_{NS} = T$)). When the maximum penalty imposed in the worst case—when at the same time the project has failed and the task is not satisfied—does not provide enough incentives for high effort, the buyer needs to punish also if the supplier fails in the project but succeeds in the additional task (never again ($T_S = T, T_{NS} = \infty$)).¹⁹

One could think that a possible solution would be to make the punishment dependent only on the outcome of the project and disregard the additional information arising from Ω . But by using feedback from the performance measures (T_S, T_{NS}) , the supplier can provide incentives with lower expected punishment costs by penalizing more in the case in which the task is not performed, something that is relatively more likely to happen if the supplier has chosen low effort.²⁰

All previous results depend on the assumption that the performance of the task a embodied in Ω is informative as to the effort decision by S , $\alpha > \beta$. Proposition 2 below provides an intuitive comparative static result regarding these two parameters.

Proposition 2. *The expected equilibrium punishment of the optimal relational contract with feedback is decreasing in the informativeness of Ω , decreasing in α , and increasing in β .*

Remember that the expected equilibrium punishment of the optimal relational contract with feedback (T_S, T_{NS}) is $\alpha\delta^{T_S+1} + (1 - \alpha)\delta^{T_{NS}+1}$. The intuition of Proposition 2 is that the more informative over the exerted effort the performance measures are, the better tailored the punishment will be, and consequently the more efficient the relational contract. Higher informativeness (higher α or lower β) makes the optimal relational contract more effective, since minimizing the punishment in case of performance has larger impact over incentives, the higher is the informativeness of the task performance, as clearly captured by the term $(\alpha - \beta)(\delta^{T_S} - \delta^{T_{NS}})$.

5 | EXTENSIONS

5.1 | Scoring

Up to now, we have considered a performance measure Ω containing the outcome of the project and a single task with a binary outcome. However, typically supplier performance metrics, as scorecards, may have several dimensions that are aggregated and result in a grade or score.²¹ In this line, we now extend the model by allowing Ω to include, in addition to the outcome of the project, several performance measures that are summarized in a nonbinary outcome. In particular, we assume that at the end of the project or relevant phase thereof, there is a score signal s that is observable by S and B , $\Omega = (y, s)$. The rest is identical to the baseline model, including the binary level of effort. To ensure that taking high effort translates into more evidence (a higher score) that the agent took high effort, we assume that this score signal is monotone, that is, $f(s|e)$ satisfies the monotone-likelihood ratio property (MLRP):

$$\frac{f(s|\bar{e})}{f(s|e)} \text{ is increasing in } s.$$

This condition ensures that more evidence is “good news” about effort (Milgrom, 1981), that is, $\Pr(\bar{e}|s)$ is increasing in s . To prove the results, it is convenient to take the score s as a discrete variable, $s_1 < s_2 < \dots < s_N$. The MLRP implies in that case:

$$\frac{\Pr(s_j|\bar{e})}{\Pr(s_j|\underline{e})} > \frac{\Pr(s_i|\bar{e})}{\Pr(s_i|\underline{e})} \text{ if } j > i,$$

equivalently,

$$\frac{\Pr(s_j|\bar{e})}{\Pr(s_i|\bar{e})} > \frac{\Pr(s_j|\underline{e})}{\Pr(s_i|\underline{e})} \text{ if } j > i$$

Following similar computations from the previous sections, we can rewrite the problem as follows:

$$\max_{T(s)} V^{\Omega+} = \frac{P - c}{1 - \pi\delta - (1 - \pi)\left(\sum_{i=1}^N \Pr(s_i|\bar{e})\delta^{T(s_i)}\right)}$$

subject to the incentive constraint:

$$\Psi^{\Omega}(T(s), \Pr(s|\underline{e}), \Pr(s|\bar{e})) \geq c.$$

where $\Psi^{\Omega}(T(s), \Pr(s|\underline{e}), \Pr(s|\bar{e}))$ is equal to

$$\frac{\delta \left[\pi + (1 - \pi)\left(\sum_{i=1}^N \Pr(s_i|\bar{e})\delta^{T(s_i)}\right) - \left(\sum_{i=1}^N \Pr(s_i|\underline{e})\delta^{T(s_i)}\right) \right] (P - c)}{1 - \pi\delta - (1 - \pi)\left(\sum_{i=1}^N \Pr(s_i|\bar{e})\delta^{T(s_i)}\right)}$$

Now, $T(s)$ is a punishment function that depends on the score obtained by S . $\Pr(s|\underline{e})$ and $\Pr(s|\bar{e})$ are the distributions of the score that depend on whether or not S has exerted effort. Notice that our previous setting with a single task and binary outcome is just a particular case of the present formulation.

Proposition 3. *Let $U^* = \sum_{i=1}^N \Pr(s_i|\bar{e})\delta^{T^*(s_i)}$ be the optimal punishment, then there exists a score $s^* \in \{s_1, s_2, \dots, s_N\}$ such that if $s_i < s^*$ then $T(s) = \infty$ (never again), and if $s_i > s^*$ then $T(s) = 0$ (total forgiveness).*

In other words, there is an optimal standard or minimum score, s^* , such that if the outcome of the performance measure is higher than s^* , S is forgiven if the project fails. Otherwise, when the score is lower than s^* and the project fails, the relationship is terminated by B forever.²² The ground of this result lies in the point that between two scores, s_i and a larger one, s_{i+1} , one wants to maximize the punishment in s_i (if at all needed), because by doing so the MLRP $\left(\frac{\Pr(s_{i+1}|\bar{e})}{\Pr(s_i|\bar{e})} > \frac{\Pr(s_{i+1}|\underline{e})}{\Pr(s_i|\underline{e})}\right)$ implies that the punishment in s_i increases more, in relative terms, the punishment of the firm when it has exerted low effort. As a consequence the punishment increases the incentives to exert high effort. Finally, it is important to point out that there are no restrictions over the number of elements and structure of $s^* \in \{s_1, s_2, \dots, s_N\}$. Thus, in the limit, the scoring set could be continuous. Although the proof is now technically more challenging, the intuition at work here is similar to that behind Proposition 1: since punishment is costly, we desire to calibrate it in such a way that focuses the relational sanctions—and thus saves costs that otherwise would be incurred—on the cases revealing the worst signals concerning the variable of interest as to incentives, that is, agent's effort.

Proposition 3 generalizes Proposition 1 given that the binary signal was a particular case of the set of signals that we consider in this section. As in the previous section, Proposition 3 implies that disregarding the information provided by the score is not optimal.

Proposition 2 established that the optimal relational punishment with feedback from Ω is decreasing in the informativeness of the performance measure. To generalize it, we need a criterion of informativeness that we can apply to scores resulting from general performance measures.

Definition 1. Ω_1 is more informative than Ω_2 , if $F_1(s|\bar{e}) \leq F_2(s|\bar{e}) \nabla s \left(\sum_{i=1}^x \Pr(s_i|\bar{e})_1 \leq \sum_{i=1}^x \Pr(s_i|\bar{e})_2 \nabla x \right)$ and $F_1(s|\underline{e}) \geq F_2(s|\underline{e}) \nabla s \left(\sum_{i=1}^x \Pr(s_i|\underline{e})_1 \geq \sum_{i=1}^x \Pr(s_i|\underline{e})_2 \nabla x \right)$

Next proposition states that the informativeness order of the scores implies all common informativeness criteria based in the value of information for a decision maker (Blackwell sufficiency and Lehmann efficiency).²³ Those informativeness criteria are built in terms of the value of information in decision making problems: a signal X is more informative than some other signal Y if every decision-maker with preferences in a particular class prefers X to Y . Thus, a signal is more informative if it allows decision-makers to make better decisions and to reduce Type I and II decision errors.

Proposition 4. *If the scoring from Ω_1 is more informative than the scoring from Ω_2 , according to Definition 1, then Ω_1 is more informative than Ω_2 according to Blackwell sufficiency and Lehmann efficiency, and it generates less decision errors.*

Finally, using our concept of performance measure's informativeness, we can state that more informative performance measures translate into more productive relationships and lower reputational sanctions.

Proposition 5. *If contract Ω_1 is more informative than contract Ω_2 , according to Definition 1, then optimal relational punishment under Ω_1 , $\sum_{i=1}^N \Pr(s_i|\bar{e})_1 \delta^{T_1^*(s_i)}$ is lower than under Ω_2 , $\sum_{i=1}^N \Pr(s_i|\bar{e})_2 \delta^{T_2^*(s_i)}$.*

This result generalizes Proposition 2 and states that a more informative performance measures, by reducing decision errors, allows to decrease the equilibrium punishment while preserving incentives.

5.2 | Performance measures as weakly enforceable contracts

In the previous sections, we have assumed that Ω is known by both parties as being a performance measure that will not ultimately be part of an enforceable contract. In this section, we reconsider our binary framework when the performance measure Ω plays also the role of weakly (or imperfectly) enforceable contract. We mean by this that in case of bad “performance” $\Omega = (0, NS)$ (the project and the ancillary task fail), B will go to Court and receive some monetary compensation D (perhaps related to the price paid by B at the start of the project, P) by S with probability γ . This γD is the expected compensation to B in case of project failure and the supplier also does not succeed in the tasks embodied in Ω . Then, Ω , plays the role of performance measure but also of enforceable—albeit imperfectly—contract over the interim task. We introduce two conditions over the “quality” of Ω : (i) $(1 - \beta)D - (1 - \pi)(1 - \alpha)D > c$; and (ii) $P - c > (1 - \pi)(1 - \alpha)D$. These two conditions are easier to interpret under full enforceability ($\gamma = 1$). In such a case, condition (i) guarantees that Ω provides enough incentives to S to exert effort; condition (ii) ensures that S gets some surplus and is willing to trade. Notice that these conditions are related to the “quality” of Ω , since both conditions are easier to meet if Ω is more informative (higher α or lower β) over the behavior of S .

Now we turn to characterize the optimal relational contract in this richer environment. The design of the optimal relational contract with a weakly enforceable performance measure Ω requires to recompute the Present Discounted Value of S 's profits to include the expected legal monetary sanction, $(1 - \pi)(1 - \alpha)\gamma D$.

$$\begin{aligned} V^{WE+} &= P - \Omega - (1 - \pi)(1 - \alpha)\gamma D + \pi \delta V^{WE+} + (1 - \pi)\delta \alpha V_S^{WE-} + (1 - \pi)\delta(1 - \alpha)V_{NS}^{WE-}, \\ V_S^{WE-} &= \delta^{T_S} V^{\Omega+} \\ V_{NS}^{WE-} &= \delta^{T_{NS}} V^{\Omega+} \end{aligned}$$

Solving the equation system, we obtain:

$$V^{WE+} = \frac{P - \Omega - (1 - \pi)(1 - \alpha)\gamma D}{1 - \pi\delta - (1 - \pi)(\alpha\delta^{T_S+1} + (1 - \alpha)\delta^{T_{NS}+1})},$$

Most importantly, weak enforceability of Ω also affects the incentive compatibility constraint:

$$V^{WE+} \geq P - (1 - \beta)\gamma D + \delta [\beta V_S^{WE-} + (1 - \beta)V_{NS}^{WE-}],$$

We can rewrite the IC as:

$$\Psi^{WE}(T_S, T_{NS}, \alpha, \beta, \gamma) \geq c$$

where $\Psi^{WE}(T_S, T_{NS}, \alpha, \beta, \gamma)$ is:

$$\frac{\delta [\pi + (1 - \pi)(\alpha\delta^{T_S} + (1 - \alpha)\delta^{T_{NS}}) - (\beta\delta^{T_S} + (1 - \beta)\delta^{T_{NS}})](P - c - (1 - \pi)(1 - \alpha)\gamma D)}{1 - \pi\delta - (1 - \pi)(\alpha\delta^{T_S+1} + (1 - \alpha)\delta^{T_{NS}+1})} + (\beta - \alpha - (1 - \pi)\alpha)\gamma D$$

By construction, if $\gamma = 0$ then $\Psi^\Omega = \Psi^{WE}$. Thus, the optimal relational sanction with weakly enforceable contracting will be the solution to the following problem:

$$\max_{T_S, T_{NS}} V^{WE+} = \frac{P - c - (1 - \pi)(1 - \alpha)\gamma D}{1 - \pi\delta - (1 - \pi)(\alpha\delta^{T_S+1} + (1 - \alpha)\delta^{T_{NS}+1})}$$

subject to the incentive constraint:

$$\Psi^{WE}(T_S, T_{NS}, \alpha, \beta, \gamma) \geq c.$$

As before, the solution to the problem is the pair (T_S, T_{NS}) that satisfies the incentive compatibility constraint and maximizes $\alpha\delta^{T_S+1} + (1 - \alpha)\delta^{T_{NS}+1}$ (minimizes the expected punishment costs). Thus, all our previous results hold when Ω is weakly enforceable: The optimal relational contract $(T_S^{WE*}, T_{NS}^{WE*})$ has the familiar double format: (i) never again, $(T_S^{WE*} = T, T_{NS}^{WE*} = \infty)$, or (ii) full forgiveness $(T_S^{WE*} = 0, T_{NS}^{WE*} = T)$. The optimal relational punishment $(T_S^{WE*}, T_{NS}^{WE*})$, is decreasing in the informativeness of Ω , decreasing in α and increasing in β .

In addition, we can state a new result regarding the impact of the degree of enforcement of Ω on the efficiency of the optimal relational contract.

Proposition 6. *The optimal relational punishment $(T_S^{WE*}, T_{NS}^{WE*})$ is decreasing in γ , the degree of enforcement of Ω .*

This result goes in line with Ganuza (2016) who show how, in settings of product markets with asymmetric information about product quality that the legal system may simultaneously reduce the cost of market sanctions and sustain cooperation between firms and consumers for a larger set of relevant parameter values. Here, we have shown that if the enforcement of Ω increases, the optimal reputational sanction decreases, and thus there is a substitution effect between the two dimensions. Moreover, as in Ganuza (2016), enforceability makes it possible for cooperation to emerge for a larger set of parameter values. Along such dimension, the legal (enforceable) elements and the relational elements are complements.²⁴

It is important to notice that the proof of Proposition 6 relies on the stated minimal conditions over the quality of Ω : (i) $(1 - \beta)D - (1 - \pi)(1 - \alpha)D > c$; and (ii) $P - c > (1 - \pi)(1 - \alpha)D$. For example, consider that the quality of Ω is low (high β and low α), and the above conditions are not satisfied. Then, it is possible that, contrary to Proposition 6, optimal relational punishment increases if the degree of enforceability of the written contract goes up. This is due to

the fact that in such a case, the legal consequences of the performance measures are severely misaligned with the underlying relevant effort, and increasing enforcement reduces the value of the relationship without substantially improving incentives, having an overall negative impact on the efficiency of the relational contract. Thus, if the “quality” of Ω and of the legal apparatus entrusted with its enforcement are sufficiently poor, doing away with the legal contract and relying solely on the relational sanctions is more advantageous.

Finally, we could consider that the contract, instead of leading to a penalty for bad performance, leads to a bonus for succeeding in the tasks specified in Ω . Consider that S would receive a bonus of size b in case of “good performance,” if $\Omega = \{(V, S), (V, NS), (0, S)\}$. The analysis and the effects of this pay for performance mechanism (“carrot”) in the optimal relational contract, is basically the same as the penalty (“stick”) case discussed above. Consider that there is a maximum bonus b_{\max} that satisfies two conditions in isolation (without considering the relational contract): (i') $\beta b_{\max} < (1 - (1 - \pi)(1 - \alpha))b_{\max} - c$ (b_{\max} provides enough incentives for S to exert effort); and (ii') $P + (1 - (1 - \pi)(1 - \alpha))b_{\max} > c$ (S gets some surplus from trade). Notice that these two conditions are qualitatively equivalent to conditions (i) and (ii) above if we, replace D by b_{\max} .²⁵ Then, when positive formal incentives are also in place ($b < b_{\max}$) and conditions (i') and (ii') are satisfied, following the same arguments as above, all our previous results hold. Thus, the larger the bonus b , the lower the optimal relational punishment. As in the punishment case, there is substitution between formal and informal incentives.

5.3 | Exploring the cost channel

The cost channel exists when the cost of undertaking the interim or ancillary tasks $a \in A$ specified in Ω varies in the level of underlying effort, $c(a|\bar{e}) < c(a|\underline{e})$. For isolating the impact of the cost channel, we shut down the probability channel by assuming that it is out of the equilibrium path that S does not undertake the ancillary tasks a and once S incurs in the cost $c(a)$, the task is always performed. In such a case, we will show that a performance measure Ω may improve the functioning of the relational contract through the cost channel.²⁶ The idea is simple: as S has to incur a higher cost to perform the tasks when it chooses low effort, introducing Ω softens the incentive compatibility constraint $\pi\delta(V^+ - V^-) \geq c + c(a|\bar{e}) - c(a|\underline{e})$ (a positive incentive effect). However, introducing Ω also has a downside, since it burdens S with an extra cost that does not provide benefits (remember, the tasks are not per se valuable to B) and thus reduces the value of the relationship, with a negative impact on incentives (loss of value effect). Then the problem becomes

$$\max_T V^+ = \max_T \frac{P - c - c(a|\bar{e})}{1 - \pi\delta - (1 - \pi)\delta^{T+1}}$$

subject to the following constraint:

$$\pi\delta \frac{(1 - \delta^T)}{1 - \pi\delta - (1 - \pi)\delta^{T+1}} \geq \frac{c - (c(a|\underline{e}) - c(a|\bar{e}))}{P - c - c(a|\bar{e})}.$$

As in our baseline model, the optimal relational contract T^* will be the minimum T that satisfies the incentive compatibility constraint. Lemma 1 and its proof in the appendix states that the left-hand side of the incentive compatibility constraint is increasing in T . Then, as discussed above, the optimal T^* will be decreasing in the cost difference between exerting effort and not exerting it, $c(a|\underline{e}) - c(a|\bar{e})$ (a positive incentive effect), and increasing in the performance costs of Ω under high effort $\Omega(a|\bar{e})$ (loss of value effect). These comparative statics can be summarized in the following proposition.

Proposition 7.

- (i) The optimal punishment T_a^* is decreasing in $c(a|\underline{e})$ and increasing in $c(a|\bar{e})$.
- (ii) Let Ω_1 and Ω_2 with two different sets of tasks a and a' and two optimal punishment T_a^* and $T_{a'}^*$, then $T_a^* \leq T_{a'}^*$ iff
- $$\frac{c - (c(a|\underline{e}) - c(a|\bar{e}))}{P - c - c(a|\bar{e})} \leq \frac{c - (c(a'|\underline{e}) - c(a'|\bar{e}))}{P - c - c(a'|\bar{e})}.$$

It is interesting to illustrate the result with the following example. Consider the number of tasks as a continuous variable, where $c(a|\underline{e}) = \kappa a$ and $c(a|\bar{e}) = a$. Then, simple computations show that the optimal punishment T^* is decreasing in the number of tasks a if and only if $\kappa \geq \frac{P}{P-c} \geq 1$.²⁷ When this condition is satisfied, the higher κ (the higher the cost difference in performing tasks with low effort and high effort), the higher the surplus of the buyer-supplier relationship will be, since it is easier to provide incentives without affecting the cost of the supplier on the equilibrium path. If this condition is not satisfied, the cost channel does not work and we should not rely on these ancillary tasks if they do not provide additional information on supplier behavior (if the information channel is not at work).

In words, including costly nonproductive tasks into our performance measure may increase the efficiency of the relational contract as long as the cost difference of undertaking these tasks between exerting and not exerting effort is large enough. Notice that the previous condition is only a necessary one, since optimally deciding the number of tasks to include in Ω requires to consider not only the reduction in punishment costs but also how $c(a|\bar{e}) = a$ reduces V^+ . This problem is analyzed in more detail in the next section.

6 | INVESTING IN IMPROVING PERFORMANCE MEASURES

In previous sections, we have taken the performance measure Ω as exogenous, and we have explored the probability and the cost channels independently. Now, we want to consider that the set of tasks that determines the performance measure Ω is chosen optimally to maximize the value of the relationship. In addition, the tasks, $a \in A$, are characterized by different costs of performance, $\{c(a|\bar{e}), c(a|\underline{e})\}$ and probabilities of performance, $\{\Pr(S|\underline{e}), \Pr(NS|\underline{e})\}$, and are likely to have an impact on the relational contract through both the probability and the cost channels simultaneously. The precise characterization of the optimal Ω should depend on the particular structure of the set of tasks. We take a more parsimonious approach and we define an investment parameter λ , in such a way that the inverse measure of the equilibrium punishment $IP(\lambda) = \alpha\delta^{Ts+1} + (1-\alpha)\delta^{Tns+1}$ increases with λ .²⁸ Tasks are included in Ω optimally, in a way such that the overall effect of the investment in improving Ω (the increase in the Ω performance costs due to the new task is compensated by the increase in the effectiveness of the relational contract through the reduction of the equilibrium punishment) enhances the value of the relationship.

Under such characterization, we can define the optimal level of investment λ^* as the solution to the following problem

$$\lambda^* \in \arg \max \frac{P - c - \lambda}{1 - \pi\delta - (1 - \pi)IP(\lambda)}$$

Consider the previous example in which we focus on the cost channel, $c(a|\underline{e}) = \kappa a$ and $c(a|\bar{e}) = a$. We showed that if $\kappa \geq \frac{P}{P-c} \geq 1$, the larger the number of tasks the lower the equilibrium punishment. Therefore, if only those sorts of tasks are available, the optimal investment in contracting is given by the optimal number of tasks, $\lambda = a$, and the optimal contracting is characterized by $a^* \in \arg \max \frac{P - c - a}{1 - \pi\delta - (1 - \pi)IP(a)}$.²⁹

A simple comparative statics analysis over λ^* provides interesting results.

Proposition 8. *The optimal investment in contracting λ^* is increasing in $P - c$ and may increase or decrease with π .*

The intuition of Proposition 8 is as follows. The optimal investment in improving the performance metric λ^* increases with $P - c$, since the larger the trade surplus, the costlier the relational punishment is, and consequently, the higher the investment in decreasing it should be. The effect of π over investment in contracting λ^* is ambiguous, because an increase in π reduces the asymmetric information and, with it, the need of improving the functioning the relational contract by investing in better performance measures. But a higher π also increases the value of the relationship and the cost of punishment, which enhances the productivity of investing in Ω . In the proof of Proposition 8 it is shown that if δ is low enough, and consequently, the positive impact of π on the value of the relationship becomes less relevant, the optimal investment λ^* decreases in π .

7 | CONCLUSIONS

This paper has shown that introducing additional performance measures in the design of relational contracts may improve their functioning. Making relational penalties depend on these performance measures reduces the cost of reputational punishments that firms may need to inflict upon suppliers to keep them under the right incentives to provide “core” effort. We have also provided a rationale about “forgiveness” in buyer-suppliers relationship and how it depends on performance measures. In particular, performance measures impact the way in which reputational punishments will be structured by the sanctioning party. This party will use a more eschewed pattern of sanctioning when using additional performance measures. A firm will be less forgiving with those counterparties for which the additional information conveyed in the performance measures is not positive, and more forgiving with those other ones who although may have failed in the main goal of the project, they have performed well in the dimensions captured by the performance measures.

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ENDNOTES

- ¹ The role of informal contract relationships, social norms in business networks, and reputation has also been identified as crucial for cooperation and has been explored in various historical and economic circumstances. See, Klein and Leffler (1981), Greif (1989, 2012), Bernstein (1992, 2001), Baker et al. (2002), Baker and Hubbard (2004), and Richman (2017).
- ² A large number of recent empirical papers provide relevant evidence of the important role of relational contracts in improving buyer-suppliers relationships: Macchiavello and Ameet (2015), Barron et al. (2020) and Gil et al. (2021).
- ³ See, MacLeod (2007) and Malcomson (2013) for useful surveys of the theoretical literature on relational contracts.
- ⁴ Bernstein (2015) and Bernstein and Peterson (2021) show how formal buyer-supplier contracts and scorecards used in these relationships provide for mechanisms (root cause analysis, oversight clauses, scorecards review meetings, cure periods) that allow buyers to identify cases of bad performance that should be largely overlooked and cooperatively resolved, while reserving termination for instances that deserve a harsh response in the face of opportunism or serious structural misperformance on the part of the supplier. Provisions entitling buyers to withhold payment for bad performance, which is kind of lesser sanction, are also reported as being commonly found in the buyer-supplier agreements.
- ⁵ Gilson et al. (2009, 2010, 2013) provide complementary analyses of the phenomenon they label as “braiding,” that is, the use of legal contracts to support informal contracting specially in technology-intensive industries.
- ⁶ The express contract terms that Klein has in mind, however, are enforceable in court, since the paper underscores how self-enforcement and court-enforcement are not alternative mechanisms, but complementary ones that the contracting parties combine (Klein, 2000, p. 68 and 75).
- ⁷ Klein (1996) already raised the point that court enforcement and private enforcement are not alternative instruments, but are jointly deployed by contractors to define the self-enforcing range of their contractual interaction. Kenney and Klein (2000) apply the combination of self-enforced and court-enforced instruments to explain block booking in the contracts in film exhibition contracts.
- ⁸ Bernheim and Whinston (1998) provide an alternative argument for this substitutability: it may be optimal to leave formal contracts incomplete to allow the principal to adapt payments (and incentives) to observable but nonverifiable information.
- ⁹ Contracting costs are also a driving force in Battigalli and Maggi (2008), where the optimal contract drafting strategy for long-term interactions is analyzed when relational contracting is also an option.

- ¹⁰ Gil and Zananone (2021) survey empirical papers that provide evidence on relational relationships characterized by these relational contract setting with “frictions” and also on indefinitely alternating cycles of cooperation and punishment.
- ¹¹ We take P as exogenous to simplify the presentation. We could add a bargaining mechanism to the game to endogenize P and obtain the same results. For example, we could endogenize the price by giving full bargaining power to S . Then, the latter would set the price at the level in which B is indifferent between buying or not, $P = \pi V$. Then, one could replace P by πV in all the subsequent expressions and verify that our results hold.
- ¹² We only consider payments from B to S . We are implicitly assuming that S is protected by limited liability and we cannot use the results of Levin (2003).
- ¹³ In fact, our way of modeling relational contracting is a simplified version of the collusion model by Green and Porter (1984) as presented in Tirole (1988) and Cabral (2005).
- ¹⁴ See, for example, Abreu et al. (1996) for a characterization of the optimal discrete strategies in the Green and Porter model.
- ¹⁵ For expositional convenience we treat T as a continuous variable. If T were a discrete variable (the number of no-trade periods), the optimal punishment T^* should be defined by the following conditions: $\Phi(T^* - 1) < c$ and $\Phi(T^*) \geq c$.
- ¹⁶ Additionally, π is a measure of the level of imperfect information, which explains why the optimal punishment decreases with it.
- ¹⁷ In Section 5.2, we will discuss the case that Ω is also part of a formal contract and is partially enforceable.
- ¹⁸ One could think of various examples of tasks assigned to the buyer whose probabilities of success or cost of performance will be differentially affected by the underlying uncontractible effort invested in the project. Building a model or prototype, preparing and making presentations on interim results, setting meetings with buyer’s representatives to explain ongoing developments, allowing “in house” visits or audits by buyer’s personnel or advisors may provide little direct benefit to the client outside the value of the project itself, but may be affected differently by the seller’s choice of effort and thus play a useful informative role.
- ¹⁹ Besides this intuition, we can also provide a sketch of the technical proof in the appendix. There is a set of pairs (T_S, T_{NS}) that generate the same expected punishment when S exerts effort, $\alpha\delta^{T_S} + (1 - \alpha)\delta^{T_{NS}} = U$. The optimal solution is characterized by finding the maximum U^* that satisfies the incentive compatibility condition $\Psi^\Omega(T_S, T_{NS}, \alpha, \beta) \geq c$. Using a change of variable we can rewrite Ψ^Ω as a function of U and δ^{T_S} . We show in the proof that Ψ^Ω is decreasing in U and increasing in δ^{T_S} . The higher the relational punishment (the lower U) the higher the incentives to exert effort, since by doing so S reduces the probability of that punishment. This result implies that the incentive compatibility constraint must be binding $\Psi^\Omega(U^*, \delta^{T_S}, \alpha, \beta) = c$. Among all the pairs that generate the same expected punishment U^* , choosing the one with higher δ^{T_S} maximizes the incentives of S to exert effort. Conditional on being punished, the difference between exerting effort and not exerting it is $(\alpha - \beta)(\delta^{T_S} - \delta^{T_{NS}})$, which is maximized with the highest δ^{T_S} . Then, the optimal punishment requires to maximize δ^{T_S} , implying that if conditions do not require a tough punishment, S is forgiven if there is project failure but the specific tasks have been performed. Otherwise, S is punished in case of performance of the task, but the relationship with B is completely severed forever in case of nonperformance (“never again”).
- ²⁰ In a way, this result rewrites the Holmstrom (1979) Informativeness Principle for relational contracting. The Informativeness Principle reads: “any measure of performance (*although may not be verifiable by third parties* in our relational set up) that reveals information about the effort level chosen by the agent should be included (*taken into account* in our relational setup) in the compensation (*relational*) contract”.
- ²¹ In a related mood, Gibbons and Kaplan (2015) emphasize the importance of appropriately balanced scoring for formal and informal decisions in agency settings and discuss both formal and informal weights on such scores.
- ²² The threshold result resembles that of Kvaloy and Olsen (2020) in the use of scorecards for designing a bonus incentive scheme in a multitask setting.
- ²³ Ganuza (2010) provide alternative criteria of informativeness based on the dispersion of posterior conditional expectations. These criteria have the advantage that the dispersion of conditional expectations is easily verified. Ganuza (2010) show that the weakest of these criteria, integral precision (based on the convex order) is equivalent to Lehmann efficiency in dichotomous settings and then it is also implied by the defined contract informativeness order.
- ²⁴ Similar results are shown by Baker and Choi (2015), who analyze a setting of a relational contract in which court enforcement is possible. Parties may resort to reputational and to legal sanctions, both of them costly, and characterize certain advantages of adding legal sanctions to a pure relational contract.
- ²⁵ In particular, (i) and (i’) are fully equivalent, and (ii) and (ii’) are also identical if we redefine $P + b_{\max}$ as a new price P' .
- ²⁶ We can interpret this assumption as implicitly assuming that a failure to undertake the task would be understood as cheating (exerting \underline{e}) and leading to the automatic termination of the relationship.
- ²⁷ See the proof of Proposition 7 for the details of the computations.
- ²⁸ The curvature and specific functional form of $IP(\lambda)$ depends on the set of tasks. We only know that $IP(\lambda)$ is increasing (punishment is decreasing in λ) since it is the outcome of an optimal design of Ω . If we include a new task into Ω , this translates into an additional cost

on the supplier that reduces the value of the relationship. For this to be optimal, it is required that this task leads to a reduction of the relational punishment.

²⁹ In the proof of Proposition 7 in the appendix, we characterize the optimal punishment T^* when $c(al\underline{g}) = \alpha a$ and $c(al\bar{e}) = a$. T^* is decreasing in a , and then $IP(a) = \delta^{T^*}$ is increasing in a as assumed.

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APPENDIX A

Proof of Lemma 1. From the main text, $\Phi(T) = \pi\delta \frac{(1-\delta^T)(P-c)}{1-\pi\delta-(1-\pi)\delta^{T+1}}$. Let $\varphi(x) = \frac{1-x}{1-\pi\delta-(1-\pi)x\delta}$. Then, we have $\Phi(T) = \pi\delta(P-c)\varphi(x(T))$, for $x(T) = \delta^T$. As $x(T)$ is decreasing, to show that Φ is increasing in T , we have to show that $\varphi(x)$ is decreasing in x .

$$\begin{aligned}\varphi'(x) &= \frac{-(1-\pi\delta-(1-\pi)x\delta)+(1-x)(1-\pi)\delta}{(1-\pi\delta-(1-\pi)x\delta)^2} \\ &= \frac{-(1-\pi\delta)+(1-\pi)\delta}{(1-\pi\delta-(1-\pi)x\delta)^2} \\ &= \frac{-1+\delta}{(1-\pi\delta-(1-\pi)x\delta)^2} < 0\end{aligned}$$

this concludes the proof. □

Proof of Lemma 2. We write the binding incentive compatibility condition that characterizes the optimal punishments as follows, $\Phi(T^*(a), a) - c = 0$, where $a \in \{\pi, \delta, P - c\}$. By the implicit function theorem we obtain $T^{*'}(a) = -\frac{\frac{\partial\Phi(T^*, a)}{\partial a}}{\frac{\partial\Phi(T^*, a)}{\partial T^*}}$. Given that for Lemma 1 $\frac{\partial\Phi(T^*, a)}{\partial T^*} > 0$, the $\text{sign}\{T^{*'}(a)\} = -\text{sign}\{\frac{\partial\Phi(T^*, a)}{\partial a}\}$. Given that, (i) $\frac{\partial\Phi(T^*, P-c)}{\partial P-c} = \pi\delta \frac{(1-\delta^T)}{1-\pi\delta-(1-\pi)\delta^{T+1}} > 0$ and $\frac{\partial T^*}{\partial P-c} < 0$. (ii)

$$\begin{aligned}\frac{\partial\Phi(T^*, \pi)}{\partial \pi} &= (P-c)(1-\delta^T)\delta \left[\frac{1-\pi\delta-(1-\pi)\delta^{T+1}+\pi(\delta-\delta^{T+1})}{(1-\pi\delta-(1-\pi)\delta^{T+1})^2} \right] \\ &= (P-c)(1-\delta^T)\delta \left[\frac{1-\delta^{T+1}}{(1-\pi\delta-(1-\pi)\delta^{T+1})^2} \right] > 0\end{aligned}$$

and $\frac{\partial T^*}{\partial \pi} < 0$. Finally,

$$\begin{aligned} \frac{\partial \Phi(T^*, \delta)}{\partial \delta} &= (P - c)\pi \left[\frac{(1 - (T + 1)\delta^T)(1 - \pi\delta - (1 - \pi)\delta^{T+1}) + (\delta - \delta^{T+1})(\pi + (1 - \pi)(T + 1)\delta^T)}{(1 - \pi\delta - (1 - \pi)\delta^{T+1})^2} \right] \\ &= (P - c)\pi \left[\frac{(1 - (T + 1)\delta^T)(1 - \delta^{T+1}) + (\delta - \delta^{T+1})(T + 1)\delta^T}{(1 - \pi\delta - (1 - \pi)\delta^{T+1})^2} \right] \\ &= (P - c)\pi \left[\frac{(1 - \delta^{T+1} - (T + 1)\delta^T + (T + 1)\delta^{T+1})}{(1 - \pi\delta - (1 - \pi)\delta^{T+1})^2} \right] \\ &= (P - c)\pi \left[\frac{(1 - (T + 1)\delta^T + T\delta^{T+1})}{(1 - \pi\delta - (1 - \pi)\delta^{T+1})^2} \right] > 0 \end{aligned}$$

where the positive sign comes from the fact that $1 - (T + 1)\delta^T + T\delta^{T+1}$ is strictly decreasing, and 0 when $\delta = 1$, therefore for all $\delta < 1$, the expression is positive. Then $\frac{\partial \Phi(T^*, \delta)}{\partial \delta} > 0$ and $\frac{\partial T^*}{\partial \delta} < 0$. \square

Proof of Proposition 1. We start by characterizing the optimal relational contract, point (ii) of the Proposition. For doing so, we rewrite the incentive compatibility constraint.

$$\begin{aligned} \frac{\delta [\pi + (1 - \pi)(\alpha\delta^{T_s} + (1 - \alpha)\delta^{T_{NS}}) - (\beta\delta^{T_s} + (1 - \beta)\delta^{T_{NS}})](P - c)}{1 - \pi\delta - (1 - \pi)(\alpha\delta^{T_s+1} + (1 - \alpha)\delta^{T_{NS}+1})} &\geq c \\ \frac{\delta [\pi(1 - (\alpha\delta^{T_s} + (1 - \alpha)\delta^{T_{NS}})) + (\alpha - \beta)(\delta^{T_s} - \delta^{T_{NS}})](P - c)}{1 - \pi\delta - (1 - \pi)(\alpha\delta^{T_s+1} + (1 - \alpha)\delta^{T_{NS}+1})} &\geq c \end{aligned}$$

Consider the following change of variable $U = \alpha\delta^{T_s} + (1 - \alpha)\delta^{T_{NS}}$, which implies $\delta^{T_{NS}} = \frac{U}{(1 - \alpha)} - \frac{\alpha}{(1 - \alpha)}\delta^{T_s}$, and then $\delta^{T_s} - \delta^{T_{NS}} = \frac{\delta^{T_s}}{(1 - \alpha)} - \frac{U}{(1 - \alpha)}$.

$$\frac{\delta \left[\pi(1 - U) + (\alpha - \beta) \left(\frac{\delta^{T_s}}{1 - \alpha} - \frac{U}{1 - \alpha} \right) \right] (P - c)}{1 - \pi\delta - (1 - \pi)\delta U} \geq c$$

Let $\chi(x) = \frac{\left[\pi(1 - x) + (\alpha - \beta) \left(\frac{\delta^{T_s}}{1 - \alpha} - \frac{x}{1 - \alpha} \right) \right]}{1 - \pi\delta - (1 - \pi)\delta x}$. Now, we want to show that $\chi(x)$ is decreasing in x .

$$\begin{aligned} \chi'(x) &= \frac{\left(-\left(\frac{\alpha - \beta}{1 - \alpha} + \pi \right) (1 - \pi\delta - (1 - \pi)\delta x) + (1 - \pi)\delta \left[\pi(1 - x) + (\alpha - \beta) \left(\frac{\delta^{T_s}}{1 - \alpha} - \frac{x}{1 - \alpha} \right) \right] \right)}{(1 - \pi\delta - (1 - \pi)\delta x)^2} \\ &= \frac{-\pi(1 - \delta) - \frac{(\alpha - \beta)}{1 - \alpha} (1 - \pi\delta - (1 - \pi)\delta^{T_{NS}+1})}{(1 - \pi\delta - (1 - \pi)\delta x)^2} \leq 0 \end{aligned}$$

As the optimal relational punishment policy is characterized by the maximum $U = \alpha\delta^{T_L} + (1 - \alpha)\delta^{T_{NL}}$ that satisfied the incentive compatibility constraint, and $\chi(x)$ is decreasing, this implies that the incentive compatibility constraint must be binding.

Then

$$\frac{\delta \left[\pi(1 - U^*) + (\alpha - \beta) \left(\frac{\delta^{T_s}}{1 - \alpha} - \frac{U^*}{1 - \alpha} \right) \right] (P - c)}{1 - \pi\delta - (1 - \pi)\delta U^*} = c$$

As the left-hand side of the equality is decreasing in U^* and increasing in δ^{T_S} , this implies that $\frac{\partial U^*}{\partial \delta^{T_S}} > 0$. Then, the optimal policy requires to maximize δ^{T_S} (minimize T_S). This implies that in the optimal solution, $T_{NS} \neq \infty \rightarrow T_S = 0$, or alternatively $T_S \neq 0 \rightarrow T_{TP} = \infty$.

Now, we move to prove that the optimal punishment with contracting feedback from the written contract, (T_S, T_{NS}) , generates lower expected punishment cost than without it, T^* , that is, $U^* = \alpha\delta^{T_S} + (1 - \alpha)\delta^{T_{NS}} > \delta^{T^*}$. This just requires to notice that $T_S = T_{NS} = T^*$ was feasible and it is not optimal. We can also verify this by comparing the two binding incentive compatibility constraints.

$$\frac{\delta\pi(1 - U^*)(P - c)}{1 - \pi\delta - (1 - \pi)\delta U^*} = c - \frac{\delta\left[(\alpha - \beta)\left(\frac{\delta^{T_S}}{1 - \alpha} - \frac{U^*}{1 - \alpha}\right)\right](P - c)}{1 - \pi\delta - (1 - \pi)\delta U^*} \tag{A1}$$

$$\frac{\delta\pi(1 - \delta^{T^*})(P - c)}{1 - \pi\delta - (1 - \pi)\delta\delta^{T^*}} = c \tag{A2}$$

Notice that the left-hand side of both equalities is the same decreasing function of U^* and δ^{T^*} , respectively. The right-hand side of the first equality (A1) is lower (the second term is negative) than the right-hand side of (A2) and this implies that $U^* = \alpha\delta^{T_S} + (1 - \alpha)\delta^{T_{NS}} > \delta^{T^*}$. □

Proof of Proposition 2. By the implicit function theorem and $\Psi^\Omega(U^*, \delta^{T_S}, \alpha, \beta) = c$, we obtain $\frac{\partial U^*}{\partial \alpha} = -\frac{\frac{\partial \Psi^\Omega}{\partial \alpha}}{\frac{\partial \Psi^\Omega}{\partial U^*}} = -\frac{>0}{<0} > 0$. Similarly, $\frac{\partial U^*}{\partial \beta} = -\frac{\frac{\partial \Psi^\Omega}{\partial \beta}}{\frac{\partial \Psi^\Omega}{\partial U^*}} = -\frac{<0}{<0} < 0$. Finally, notice that higher $U^* = \alpha\delta^{T_S} + (1 - \alpha)\delta^{T_{NS}}$ means a lower expected relational punishment. □

Proof of Proposition 3. As in the previous section, the value of the relationship between S and B is captured by $V^{\Omega+}$

$$\max_{T(s)} V^{\Omega+} = \frac{P - c}{1 - \pi\delta - (1 - \pi)\left(\sum \Pr(s_i|\bar{e})\delta^{T(s_i)}\right)}$$

that is increasing in $\sum \Pr(s_i|\bar{e})\delta^{T(s_i)}$. Then, similarly to previous results, the optimal relational punishment $T^*(s_i)$ maximizes $U^* = \sum_{i=1}^N \Pr(s_i|\bar{e})\delta^{T^*(s_i)}$ subject to satisfying the incentive compatibility constraint:

$$\begin{aligned} \Psi^\Omega(T(s), \Pr(s|\underline{e}), \Pr(s_i|\bar{e})) &\geq c \\ \frac{\delta\left[\pi + (1 - \pi)\left(\sum_{i=1}^N \Pr(s_i|\bar{e})\delta^{T(s_i)}\right) - \left(\sum_{i=1}^N \Pr(s_i|\underline{e})\delta^{T(s_i)}\right)\right](P - c)}{1 - \pi\delta - (1 - \pi)\left(\sum_{i=1}^N \Pr(s_i|\bar{e})\delta^{T(s_i)}\right)} &\geq c \end{aligned}$$

To prove the result, take as given the punishment of all scores but the two first ones: $\sum_{i=1}^N \Pr(s_i|\bar{e})\delta^{T(s_i)} = \alpha_1\delta^{T_1} + \alpha_2\delta^{T_2} + A$ and $\sum_{i=1}^N \Pr(s|\underline{e})\delta^{T(s_i)} = \beta_1\delta^{T_1} + \beta_2\delta^{T_2} + B$. Then, the function $\Psi^\Omega(T(s), \Pr(s|\underline{e}), \Pr(s_i|\bar{e}))$ becomes:

$$\frac{\delta\left[\pi + (1 - \pi)(\alpha_1\delta^{T_1} + \alpha_2\delta^{T_2} + A) - (\beta_1\delta^{T_1} + \beta_2\delta^{T_2} + B)\right](P - c)}{1 - \pi\delta - (1 - \pi)(\alpha_1\delta^{T_1} + \alpha_2\delta^{T_2} + A)}$$

We make the following change of variable $\bar{U} = \alpha_1\delta^{T_1} + \alpha_2\delta^{T_2}$ and $\delta^{T_2} = \frac{\bar{U}}{\alpha_2} - \frac{\alpha_1\delta^{T_1}}{\alpha_2}$, and we rewrite $\Psi^\Omega(T(s), \Pr(s|\underline{e}), \Pr(s_i|\bar{e}))$ as

$$\begin{aligned} & \frac{\delta \left[\pi + (1 - \pi)(\bar{U} + A) - \left(\beta_1 \delta^{T_1} + \beta_2 \left(\frac{\bar{U}}{\alpha_2} - \frac{\alpha_1 \delta^{T_1}}{\alpha_2} \right) + B \right) \right] (P - c)}{1 - \pi \delta - (1 - \pi)(\bar{U} + A)} \\ = & \frac{\delta \left[\pi + (1 - \pi)(\bar{U} + A) - \left(\beta_2 \frac{\bar{U}}{\alpha_2} + B \right) - \beta_2 \delta^{T_1} \left(\frac{\beta_1}{\beta_2} - \frac{\alpha_1}{\alpha_2} \right) \right] (P - c)}{1 - \pi \delta - (1 - \pi)(\bar{U} + A)} \end{aligned}$$

For the MLRP, $\frac{\beta_1}{\beta_2} - \frac{\alpha_1}{\alpha_2} > 0$, which implies that for a given \bar{U} , Ψ^Ω is decreasing in δ^{T_1} . In other words, we want to maximize T_1 punishment with respect to T_2 . This implies $\delta^{T_1} = \min\{0, \frac{\bar{U} - \alpha_2}{\alpha_1}\}$. We can repeat this proof for all pairs, T_i and T_{i+1} and obtain the same result. Then, the global solution has to be $\delta^{T_i} = 0$ ($T_i = \infty$) for all initial scores until we can guaranty that $\Psi^\Omega > 0$. \square

Proof of Proposition 4. To simplify the notation we will prove the results using a continuous distribution of signals. Then consider two signals $F_1(s|e)$ and $F_2(s|e)$ that we want to rank according to their informativeness. Jewitt (2007) shows the equivalence of Lehmann efficiency and Blackwell sufficiency in a dichotomous setting as ours, $e \in \{\underline{e}, \bar{e}\}$ in which signals satisfy MLRP. Lehmann criterion establishes that a signal F_1 is more informative than another F_2 if the following condition over quantiles holds:

$$\forall p \in [0, 1], F_1\left(F_1^{-1}(p|\underline{e})|\bar{e}\right) \leq F_2\left(F_2^{-1}(p|\underline{e})|\bar{e}\right). \quad (A3)$$

By definition, the c.d.f.s $F_1(x|e)$ and $F_2(x|e)$ are nondecreasing functions, so that

$$\forall x, F_1(x|e) \geq F_2(x|e) \Leftrightarrow \forall p, F_1^{-1}(p|e) \leq F_2^{-1}(p|e).$$

By Definition 1, $F_1(x|\underline{e}) \geq F_2(x|\underline{e})$ and hence, for any p ,

$$\begin{aligned} F_1^{-1}(p|\underline{e}) &\leq F_2^{-1}(p|\underline{e}) \Rightarrow \\ F_2\left(F_1^{-1}(p|\underline{e})|\bar{e}\right) &\leq F_2\left(F_2^{-1}(p|\underline{e})|\bar{e}\right) \end{aligned}$$

By Definition 1, $F_1(x|\bar{e}) \leq F_2(x|\bar{e})$, then replacing $F_2\left(F_1^{-1}(p|\underline{e})|\bar{e}\right)$ by $F_1\left(F_1^{-1}(p|\underline{e})|\bar{e}\right)$ then, we obtain

$$F_1\left(F_1^{-1}(p|\underline{e})|\bar{e}\right) \leq F_2\left(F_2^{-1}(p|\underline{e})|\bar{e}\right).$$

Then, our criterion of informativeness captured by Definition 1 implies Lehmann efficiency and using Jewitt's result, also Blackwell sufficiency. \square

Proof of Proposition 5. Let $\sum_{i=1}^N \Pr(s_i|\bar{e}) \delta^{T_1^*(s_i)}$ and $\sum_{i=1}^N \Pr(s_i|\bar{e}) \delta^{T_2^*(s_i)}$ be the optimal punishment under Ω_1 and Ω_2 . First, we show that $T_2^*(s_i)$ is feasible under Ω_1 .

$$\Psi^{\Omega_1}\left(T_2^*(s_i), \Pr(s|\underline{e})_1, \Pr(s|\bar{e})_1\right) \geq \Psi^{\Omega_2}\left(T_2^*(s_i), \Pr(s|\underline{e})_2, \Pr(s|\bar{e})_2\right)$$

This is due to the following reasons: (i) Ψ^Ω increases with $\sum \Pr(s_i|\bar{e}) \delta^{T(s_i)}$ and decreases with $\sum \Pr(s_i|\underline{e}) \delta^{T(s_i)}$; (ii) $\delta^{T(s_i)}$ is an increasing function of s_i ; (iii) Scoring distributions are ordered according to the first-order stochastic dominance, $(\sum_{i=1}^x \Pr(s_i|\bar{e}))_1 \leq \sum_{i=1}^x \Pr(s_i|\bar{e})_2 \forall x$ and $(\sum_{i=1}^x \Pr(s_i|\underline{e}))_1 \geq \sum_{i=1}^x \Pr(s_i|\underline{e})_2 \forall x$. Then, by (ii) and (iii) $\sum_{i=1}^N \Pr(s_i|\bar{e})_1 \delta^{T_2^*(s_i)} > \sum_{i=1}^N \Pr(s_i|\bar{e})_2 \delta^{T_2^*(s_i)}$ and $\sum_{i=1}^N \Pr(s_i|\underline{e})_1 \delta^{T_2^*(s_i)} < \sum_{i=1}^N \Pr(s_i|\underline{e})_2 \delta^{T_2^*(s_i)}$, which jointly with (i) implies the inequality above. Finally, as $T_2^*(s_i)$ is feasible under Ω_1 , we can state that

$$V^{\Omega_1+} \left(\sum_{i=1}^N \Pr(s_i|\bar{e})_1 \delta^{T_1^*(s_i)} \right) > V^{\Omega_1+} \left(\sum_{i=1}^N \Pr(s_i|\bar{e})_1 \delta^{T_2^*(s_i)} \right) > V^{\Omega_2+} \left(\sum_{i=1}^N \Pr(s_i|\bar{e})_2 \delta^{T_2^*(s_i)} \right)$$

This is because, V^{Ω_1+} , the value of the relationship between S and B , is increasing in $\sum \Pr(s_i|\bar{e}) \delta^{T(s_i)}$, and $\sum_{i=1}^N \Pr(s_i|\bar{e})_1 \delta^{T_2^*(s_i)} > \sum_{i=1}^N \Pr(s_i|\bar{e})_2 \delta^{T_2^*(s_i)}$, which implies the last two inequalities. The first inequality is implied by the fact that for Ω_1 , the optimal punishment is $T_1^*(s_i)$. \square

Proof of Proposition 6. The optimal relational punishment $(T_S^{WE*}, T_{NS}^{WE*})$ is given by the following equality

$$\Psi^{WE}(T_S^{WE*}, T_{NS}^{WE*}, \alpha, \beta, \gamma) = c$$

Following the arguments of the proof of Proposition 1 we can rewrite this equality as follows:

$$\frac{\delta \left[\pi(1 - U^*) + (\alpha - \beta) \left(\frac{\delta^{T_S}}{1 - \alpha} - \frac{U^*}{1 - \alpha} \right) \right]}{1 - \pi\delta - (1 - \pi)\delta U^*} = \frac{c - d\gamma}{P - c - a\gamma}$$

Where, as in Proposition 1, $U^* = \alpha\delta^{T_S} + (1 - \alpha)\delta^{T_{NS}}$ refers to the optimal relational punishment, and $d = (1 - \beta)D - (1 - \pi)(1 - \alpha)D$ and $a = (1 - \pi)(1 - \alpha)D$ are two constants. From Proposition 1 we know that the left-hand side of the equality is decreasing in U^* , then the lower is the right-hand side, the higher is $U^* = \alpha\delta^{T_S} + (1 - \alpha)\delta^{T_{NS}}$, and the lower is the optimal relational punishment.

If we derive $\frac{c - d\gamma}{P - c - a\gamma}$ with respect to γ

$$\frac{d}{d\gamma} \left(\frac{c - d\gamma}{P - c - a\gamma} \right) = \frac{-d(P - c - a\gamma) + a(c - d\gamma)}{(P - c - a\gamma)^2}$$

This derivative is negative:

$$-d(P - c) + ac < 0 \Leftrightarrow \frac{c}{d} < \frac{(P - c)}{a}$$

This inequality is satisfied since we are assuming that (i) $d = (1 - \beta)D - (1 - \pi)(1 - \alpha)D > c \Rightarrow \frac{c}{d} < 1$ and (ii) $a = (1 - \pi)(1 - \alpha)D < P - c \Rightarrow \frac{(P - c)}{a} > 1$. Then, the right-hand side is decreasing in γ , and we can conclude that the optimal relational punishment $(T_S^{WE*}, T_{NS}^{WE*})$ is decreasing in the degree of enforceability of the written contract γ . \square

Proof of Proposition 7. As in the baseline model, the optimal relational contract T^* will be such that the incentive compatibility constraint is binding.

$$\pi\delta \frac{(1 - \delta^{T^*})}{1 - \pi\delta - (1 - \pi)\delta^{T^*+1}} = \frac{c - (c(al\underline{e}) - c(al\bar{e}))}{P - c - c(al\bar{e})}$$

The proof of Lemma 1 above shows that the left-hand side of the incentive compatibility constraint is increasing in T^* . Then, as part (i) of the proposition follows from the right-hand side being decreasing in $c(al\underline{e})$ and increasing in $c(al\bar{e})$. Part (ii) of the proposition states that the higher is the right-hand side, the higher is T^* . Following the example of the main text, we plug $c(al\underline{e}) = \kappa a$ and $c(al\bar{e}) = a$ into the right-hand side of the equality and we deriving, we obtain

$$\frac{d}{da} \left(\frac{c - (\kappa - 1)a}{P - c - a} \right) = \frac{-(\kappa - 1)(P - c) + c}{(P - c - a)^2}$$

This is lower than 0 if $\kappa > \frac{P}{P-c}$. In such a case, the RHS is decreasing in a , as the LHS is increasing in T^* , we can conclude that the optimal punishment T^* is decreasing in a . \square

Proof of Proposition 8. The contracting problem is defined as follows:

$$\lambda^* \in \arg \max V(\lambda) = \frac{P - c - \lambda}{1 - \pi\delta - (1 - \pi)IP(\lambda)}$$

We focus on increasing (decreasing) differences that are a sufficient condition for supermodularity (submodularity) and then for comparative statics. Then

$$\frac{\partial V(\lambda, P - c)}{\partial \lambda \partial P - c} = \frac{(1 - \pi)IP(\lambda)'}{(1 - \pi\delta - (1 - \pi)IP(\lambda))^2} \geq 0$$

As the cross derivative is positive, the value function $V(\lambda, P - c)$ is supermodular in λ and $P - c$, and the investment in contracting λ^* is increasing in $P - c$.

$$\frac{\partial V(\lambda, \pi)}{\partial \lambda \partial \pi} = \frac{(-1 + \delta + (\delta - IP(\lambda))(1 - \pi))}{(1 - \pi\delta - (1 - \pi)IP(\lambda))^3} IP(\lambda)'$$

The sign of the cross derivative depends on the expression $(-1 + \delta + (\delta - IP(\lambda))(1 - \pi))$ that may be positive or negative for some parameter values (since $\delta \in [0, 1]$ and $\delta > IP(\lambda)$). Notice, however, that if δ is low enough, the whole expression and the cross derivative are negative and consequently, the value function $V(\lambda, \pi)$ is submodular in λ and π , and the investment in contracting λ^* is decreasing in π . \square