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Price competition with a stake in your rival☆

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ABSTRACT

We examine how revenue-sharing and profit-sharing stakes affect price competition intensity under duopoly. Our analysis builds on the price competition framework introduced by Varian (1980) and accounts for fundamental asymmetries in terms of cost and consumer loyalty. A stake exists when a firm appropriates a share of its rival's revenues or profits. For example, a marketplace owner that charges a third-party seller an ad valorem fee on its sales has a revenue-sharing stake, and a firm holding a minority ownership participation in another has a profit-sharing stake. We show that a revenue-sharing stake always has a stronger competition-dampening effect (leads to higher prices) than a profit-sharing stake, and explain how the introduction of a stake affects the intensity of competition between firms. Our analysis generates new insight into how stakes affect competitive interaction in the marketplace.

1. Introduction

Traditional price competition analysis is built on the benchmark case where firms are indifferent to the revenues and profits generated by their competitors. There are several instances of competition, however, where revenue-sharing or profit-sharing mechanisms imply that firms should internalize to some degree competitor payoffs. For example, a firm that earns revenue-based fees from a competitor will consider the impact of its own pricing choices on its fee revenues, and will thereby incorporate the competitor's revenues into its decision making. Similarly, a firm holding a minority stake (with no decision-making power) in a rival operates both as a competitor and a shareholder. The stake entitles the firm to a share of the competitor's profits, so it will incorporate these profits into its decision making. In both cases, due to the existence of a stake in the rival, we should expect price competition outcomes to differ from the benchmark case of pure price competition.

Mechanisms that enable firms to appropriate revenues or profits from rivals can arise in many settings. Revenue-sharing mechanisms are common in online platforms. Marketplace owners such as Amazon or WalMart charge third-party sellers operating in their online marketplaces an *ad valorem* fee based on their sales revenues. So do Apple, Google, or Valve on their software storefronts, as well as the owners of many popular application and video game ecosystems (Unity Asset Store, Zoom App Marketplace, Snapchat Lenses, Minecraft Marketplace). When these firms supply first-party products or content on their platform for consumers, they often compete with the same sellers they collect fees from.

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Another instance of revenue-sharing can arise from licensing agreements with competitors. Patent licensing agreements can include royalties levied as a percentage of sales (or renegotiated on a rolling basis) and give place to competition between the licensor and the licensee. For example, Du Pont licensed and also competed with several of its licensees in the production of polyester, cellophane, and nylon in the mid-20th century. Ford licensed many of its automotive innovations to competing car manufacturers over the last two decades, and has recently opened up its electric vehicle patent portfolio for licensing. Sony develops high-performance image sensors which it employs in its own digital cameras and also licenses to other camera manufacturers it competes against. Amazon has started to license its checkout-free technology to street supermarkets, such as Sainsbury's in the UK, while operating Amazon Fresh stores in close proximity to its licensees.

Profit-sharing mechanisms with competitors are implemented through ownership participation. Minority shareholding of less than 50% of outstanding shares (sometimes referred to as a minority stake or passive stake) ensures that the acquirer partially appropriates the target's profits without conferring control over its competitive responses. Minority shareholding among competitors has been reported to be common practice in industries with high R&D intensity, see Nain and Wang (2016). Moreover, minority acquisitions have been subject to less regulatory scrutiny than mergers, with fewer than 1% of acquisitions being challenged by antitrust authorities.

In this paper we study the properties of price competition when a firm has a stake in its competitor. There are two key elements to our modeling approach. First, we build on the price competition model introduced by Varian (1980) where firms benefit from consumer loyalty. This moderates the incentives of firms to undercut each other when pricing, allowing us to examine the impact of stakes on pricing outcomes. We capture demand and supply-side asymmetries by letting firms differ in the size of their loyal customer base and marginal cost. Second, we let one of the firms benefit from a stake in its competitor, and decompose the stake into a revenue-sharing component and a cost-sharing component (parameters τ_p and τ_c in our model). A revenue-sharing stake incorporates the first component and a profit-sharing stake incorporates both components. This enables us to analyze the separate mechanisms at play and explain how the type and the size of the stake affects firm pricing strategies and market outcomes.

Our findings contribute to our understanding of the determinants of price competition, specifically on how stakes affect competitive interaction. To the best of our knowledge, our framework is the first to integrate both revenue-sharing and profit-sharing into a single model and provide a direct comparison of their effect. This depends on the type of the stake and the competitive standing of both firms. We show that a stake can increase or reduce the intensity of competition, and the direction of the effect depends on how the competitive standing of both firms compares before and after the stake is introduced. Furthermore, a revenue-sharing stake always has a stronger competition-dampening effect than a profit-sharing stake. Our results are particularly relevant to platforms and gatekeepers that charge revenue-sharing fees or royalties and suggests that these practices deserve heightened antitrust scrutiny.

The paper is organized as follows. In the next section we review the related literature on price competition, revenue-sharing, and profit-sharing mechanisms. Section 3 introduces the model and building blocks for our analysis. We characterize the equilibria of the game and comparative statics in Section 4, and examine the equilibrium impact of stakes on market outcomes in Section 5. We conclude in Section 6.

2. Literature

This paper builds on our work on retailer-led marketplaces in Hervas-Drane and Shelegia (2022). Retailers operating marketplaces typically charge third-party sellers an ad valorem fee based on their sales revenues while simultaneously competing against them. In that paper we study the retailer's problem with a price competition model based on a sequential game with a revenue-sharing stake (the ad valorem fee) and a demand allocation mechanism inspired by the marketplace setting. In this paper we expand and generalize that model. We study a simultaneous game that generates mixed strategy equilibria, generalize the demand allocation mechanism with a loyal customer base for each firm, expand the specification to include both profit-sharing as well as revenue-sharing stakes, and solve the game over the full parameter space encompassing loyalty and cost asymmetries between firms.

Our model builds on the extensive literature on price competition. We use the model of Bertrand price competition with consumer loyalty introduced by Varian (1980) as the building block for our analysis. Narasimhan (1988) extends Varian's model with asymmetric loyalty across firms, which we also incorporate into our model together with asymmetric marginal costs. As in the preceding papers, our model generates a mixed strategy equilibrium where firms quote high and low prices with different probability. Chen (2008) and Villas-Boas (1999) present duopoly models that generate similar pricing strategies in the absence of stakes, and are driven instead by price discrimination across consumers executing repeated purchases. We are aware of few instances in the price competition literature that have examined the impact of stakes. Two contributions that have studied the impact of cross-ownership (profit-sharing stakes) under oligopoly are Gilo et al. (2006), who focus on pricing dynamics and tacit collusion, and Shelegia and Spiegel (2012) who characterize pricing outcomes with cost asymmetries.

Our analysis of revenue-sharing stakes relates to the literature on platforms and gatekeepers. Our model is most relevant to the case where the platform or gatekeeper charges other firms a revenue-sharing fee and also competes against them. The literature has only recently started to examine the implications of this market configuration. De Cornière and Taylor (2019) analyze the incentives of a gatekeeper that competes with a rival and can direct consumers to his own offering or that of the rival, though fees do not play an important role in the analysis. Hagiu et al. (2020) and Hagiu et al. (2021) explore cases where platforms may charge a unit fee to host a rival and simultaneously compete against it. Anderson and Bedre-Defolie (2021) study competition in online marketplaces where the marketplace owner charges an ad valorem fee and competes against sellers. Their model specification incorporates revenue-sharing, though their analysis of competition differs from the one presented here in several ways given that firms supply differentiated products.

Another strand of the literature has examined the implications of revenue-sharing stakes in the context of fee collection and taxation. Johnson (2017) provides a review of this literature, which has focused mostly on fee-setting strategies and regulatory outcomes in the context of vertical relations rather than competitive interactions between firms.

Our examination of profit-sharing stakes is directly related to the extensive literature on cross-ownership, and to minority shareholding in particular. Minority shareholding implies that the shareholder has an ownership participation in the target that does not exceed 50%, and therefore does not confer control or preclude product market competition between the shareholder and the target. Ouimet (2012) provides a detailed review of the factors explaining minority acquisitions. The literature has examined the theoretical implications of minority shareholding for competition outcomes, tacit collusion, mergers, and cross-ownership networks of many firms. Our focus in this paper is narrowly on product market competition. Two relevant contributions in the literature, Reynolds and Snapp (1986) and Farrell and Shapiro (1990), examine the impact of a profit-sharing stake under Cournot competition and find that it raises price and reduces output (i.e, softens competition). We discuss this result in the context of our findings in Section 5.

Other contributions have studied the empirical evidence on minority shareholding. Nain and Wang (2016) use manufacturing industry sector data covering three decades to examine the impact of minority stake acquisitions on product market competition. They find that both prices and profits increase as a result. Other studies have reported similar findings in specific industries. Parker and Röller (1997) report that cross-ownership contributes to explain non-competitive prices in the US cellular telephone industry, and Dietzenbacher et al. (2000) find that minority shareholding drives higher price-cost margins in the Dutch financial sector.

3. The model

Consider a duopoly market where one firm has a stake in its rival. We identify Firm 1 as the stakeholder and Firm 2 as the firm in which the stake is held, and formalize the properties of the stake further below. There is a unit mass of consumers in this market. All consumers are identical in their willingness to pay v and demand one unit each of the product sold by both firms. This unit demand specification captures the essential market forces under price competition and simplifies the analysis.

In addition to the stake, there are two other possible sources of asymmetry between firms. First, firms may differ in their marginal cost. We denote the marginal cost of Firm 1 by $c_1 \geq 0$ and the marginal cost of Firm 2 by $c_2 \geq 0$. To avoid uninteresting cases where there is no effective competition, we assume $\max(c_1, c_2) \leq v$ so that both firms are able to profitably serve consumers. For simplicity, we assume away any fixed costs.

A second source of asymmetry originates from the demand side. Some consumers exhibit loyalty to one of the firms and refuse to purchase from the rival. These loyal consumers generate captive demand for each firm, and their loyalty could be driven by (unmodeled) switching costs or differentiation between firms. We let fraction $L_1 \in (0, 1)$ of consumers be loyal to Firm 1 and fraction $L_2 \in (0, 1 - L_1)$ be loyal to Firm 2.[1] The remaining fraction $1 - (L_1 + L_2) > 0$ of consumers are not loyal to any firm and will purchase from the one quoting the lowest price as long as it does not exceed their valuation v. We refer to this fraction of consumers as *shoppers*.

We can now formalize Firm 1's stake in Firm 2. We let Firm 1 appropriate fraction τ_p of Firm 2's revenues and incur fraction τ_c of Firm 2's costs. Our goal is to analyze both revenue-sharing and profit-sharing stakes, so there are two cases of interest to focus on. If $\tau_p > \tau_c = 0$, Firm 1 has a revenue-sharing stake in Firm 2 and will only internalize Firm 2's revenues (not its costs). If $\tau_p = \tau_c > 0$, Firm 1 has a profit-sharing stake in Firm 2 and will internalize Firm 2's profits, which includes both its revenues and costs. We will refer to the preceding stakes as a *revenue stake* and a *profit stake*, respectively. In the analysis that follows we assume that the stake is exogenous, but we examine the implications for firms implementing a stake in Section 5.

Stakes in our model cover Firm 2's sales to both loyals and shoppers. That is, we assume that both loyals and shoppers purchase from Firm 2 on the same terms and through the same sales channel (i.e., in the case of a platform, all consumers purchasing Firm 2's product do so via Firm 1's platform).[2] The assumption is not critical to our results; if Firm 2's loyals are exempted from the stake, it can be shown that equilibrium prices differ but the qualitative properties of the results reported below continue to hold. Furthermore, our analysis is unaffected by the existence of additional sales channels or markets in which Firm 2 operates independently of Firm 1 (without incurring the stake).

Our analysis retains the timing of one-shot price competition. Firm 1 and Firm 2 simultaneously quote their prices p_1 and p_2. Consumers then decide whether to purchase or not, and in the case of shoppers (non-loyal consumers) from which firm

[1] We rule out the case $L_i = 0$ to simplify the exposition, as it can be shown that the analysis below holds provided that $\max(L_1, L_2) > 0$. When $L_1 = L_2 = 0$ a pure strategy equilibrium obtains, which for the case of a profit-sharing stake is described in Shelegia and Spiegel (2012).

[2] Consider for example the case of Anker, an electronics manufacturer that sells on Amazon's storefront and competes with Amazon's own branded electronics products. In our model, Anker's loyals are consumers who shop on Amazon's storefront and prefer Anker's products over those of Amazon. Anker can only serve these consumers by selling on Amazon's marketplace and paying fees to Amazon.

to purchase. We apply the following equilibrium selection criteria: if both firms quote the same price, shoppers purchase from the firm with the lowest marginal cost. If both firms have the same marginal cost, shopper demand is split between both firms. This demand allocation rule ensures surplus is maximized and is only relevant to the corner case where a pure strategy equilibrium holds (in the mixed strategy equilibrium the probability of a price tie $p_1 = p_2$ is zero).

4. Equilibrium pricing with a stake

We proceed to characterize the equilibrium of the game. We start our analysis by identifying the lowest price each firm is willing to quote. If Firm 1 serves a consumer at price p_1 it derives profit $p_1 - c_1$. If the consumer is served instead by Firm 2 at price p_2, Firm 1 derives profit $\tau_p\, p_2 - \tau_c\, c_2$ as stakeholder. Thus the price p that renders Firm 1 indifferent between selling to shoppers or allowing them to be served by Firm 2 is given by

$$p - c_1 = \tau_p\, p - \tau_c\, c_2.$$

Substituting $p = \tilde{c}_1$ and rearranging obtains

$$\tilde{c}_1 = \frac{c_1 - \tau_c\, c_2}{1 - \tau_p}. \tag{1}$$

We refer to $\tilde{c}_1$ as Firm's 1 stake-adjusted marginal cost. Firm 1 is indifferent between selling at price $p_1 = \tilde{c}_1$ or letting Firm 2 supply at the same price, given that the stake it earns from Firm 2 is equivalent to its own markup at this price. Therefore, Firm 1 is unwilling to undercut below $\tilde{c}_1$.

Firm 2 is burdened by the stake and indifferent between selling or not at price p_2 such that

$$(1 - \tau_p)p_2 - (1 - \tau_c)c_2 = 0.$$

Substituting $p_2 = \hat{c}_2$ and rearranging delivers

$$\hat{c}_2 = \frac{1 - \tau_c}{1 - \tau_p} c_2. \tag{2}$$

We refer $\hat{c}_2$ as Firm 2's stake-adjusted marginal cost. Firm 2 is indifferent between selling or not at price $p_2 = \hat{c}_2$ given that it has to pay the stake. This implies Firm 2 will never undercut below $\hat{c}_2$ in order to serve shoppers.

The stake-adjusted marginal costs derived above describe the opportunity cost of each firm when supplying a unit in the presence of the stake. Firm 1 incurs opportunity cost $\tilde{c}_1$ when supplying a unit to a shopper, and Firm 2 incurs an opportunity cost $\hat{c}_2$ when supplying a unit to a shopper or a loyal. The existence of the stake has an asymmetric impact across both firms. The stakeholder, Firm 1, has less incentives to undercut when selling to shoppers but the markup it derives from its own sales is unaffected. Firm 2 is burdened by the stake and suffers a markup squeeze on all its sales (recall that the stake is levied on sales to both loyals and shoppers). Note that the ordering of (c_1, c_2) is preserved in $(\tilde{c}_1, \hat{c}_2)$; $\tilde{c}_1$ is higher (lower) than $\hat{c}_2$ when c_1 is higher (lower) than c_2 because $\tilde{c}_1 - \hat{c}_2 = \frac{1}{1-\tau_p}(c_1 - c_2)$.

Our model converges to standard Bertrand price competition only when there is no stake $\tau_c = \tau_p = 0$ and no consumer loyalty $L_1 \to 0$ and $L_2 \to 0$. In that scenario, the Bertrand outcome implies that the most efficient firm with the lowest marginal cost undercuts the competitor and serves all demand. In the scenario we study, where a stake exists and some consumers are loyal, a similar outcome becomes a corner solution for certain parameter ranges. We next characterize the pure strategy equilibria in this corner solution.

Proposition 1. *If $\max(\tilde{c}_1, \hat{c}_2) \geq v$ a unique (pure strategy) equilibrium exists with the following properties:*

(a) *If $\hat{c}_2 > v$ firms set prices $p_1^* = v$ and $p_2^* \geq \hat{c}_2$, Firm 1 serves both its loyals and shoppers, and Firm 2 is not viable and sells to no one.*
(b) *Otherwise, firms set prices $p_1^* = p_2^* = v$, each firm serves its loyal consumers, and*
  (i) *all shoppers purchase from Firm 1 if $c_1 < c_2$.*
  (ii) *shoppers are equally split among both firms if $c_1 = c_2$.*
  (iii) *all shoppers purchase from Firm 2 if $c_1 > c_2$.*

Proof. Consider first the case $\hat{c}_2 > v$. Firm 2 is not viable, and will set price $p_2 \geq \hat{c}_2$ and sell to no one. Firm 1 will then maximize profit by setting $p_1 = v$ and sell to both its loyals and shoppers.

In the remaining cases Firm 2 is viable. Consider the case $\tilde{c}_1 \geq v > \hat{c}_2$, which implies that $c_1 > c_2$. By definition of $\tilde{c}_1$, Firm 1 cannot profitably undercut Firm 2 by setting a price $p_1 \leq \tilde{c}_1$ to serve shoppers. Given that Firm 1 is unwilling to undercut, Firm 2 earns maximum profit at $p_2 = v$. Thus $p_1 = p_2 = v$ in equilibrium. Each firm serves its loyal consumers and shoppers purchase from Firm 2 given efficient tie breaking.

Finally, consider the case $\hat{c}_2 = v \geq \tilde{c}_1$. Clearly, $p_2 = v$. We next argue that Firm 1 maximizes profit by also setting $p_1 = v$ and matching Firm 2's price. If $\tilde{c}_1 = v$ this follows directly, and shoppers will split between both firms given that $c_1 = c_2$ and efficient tie breaking. If $\tilde{c}_1 < v$, which implies $c_1 < c_2$, efficient tie breaking ensures Firm 1 can serve all shoppers without undercutting Firm 2. □

This pure strategy equilibrium holds when there is no effective competition between both firms. This outcome hinges on the stake (τ_p, τ_c) given that we have assumed both firms are viable without it. The lack of effective competition can arise because Firm 2 is rendered unviable with the stake (first part of the proposition) or because firms remain viable but one is unwilling to undercut the other (second part of the proposition).

Firm 2 becomes an unviable competitor when $\hat{c}_2 > v$ because it is unable to serve consumers at the monopoly price without incurring a loss. Inspection of $\hat{c}_2$ reveals that this is the case when Firm 1 appropriates a large share of Firm 2's revenues relative to its costs, $\tau_p >> \tau_c$. I.e., when there is high revenue stake. This follows from the fact that a high revenue-sharing stake can be larger than Firm 2's markup, taking it from positive to negative profit, which is never the case with a profit-sharing stake.

The second part of the proposition relates to the cases where one or both firms are unwilling to undercut the other. When the stake-adjusted marginal cost of a firm matches the monopoly price, $\tilde{c}_1 = v$ or $\hat{c}_2 = v$, the firm ceases to be an effective competitor. It is enough for one of the two firms to no longer wish to undercut for the market to become uncompetitive and for consumers to pay the monopoly price v.

We next turn to the general case where both firms are able and willing to undercut the rival so the above result does not apply.

Lemma 1. *If* $\max(\tilde{c}_1, \hat{c}_2) < v$ *no pure strategy equilibrium exists.*

Proof. Assume the opposite, there is some pure strategy equilibrium (p_1^*, p_2^*). No firm will price above consumer's willingness to pay in equilibrium, $p_i^* > v$, as i would then profit from deviating to a lower price $p_i^* \le v$ in order to serve (at least) loyal consumers. Moreover, it cannot be the case that firms set different prices, $p_i^* < p_j^* \le v$, because firm i would then deviate to a higher price thereby increasing profits from loyals without sacrificing any profits derived from shoppers. Hence, in a pure strategy equilibrium both firms must quote the same price in the range $p_i^* = p_j^* \le v$.

We next argue that an equilibrium cannot hold if $p_i^* = p_j^* < v$. If $c_1 = c_2$ such that both firms are splitting shopper demand, either firm will profit from marginally undercutting the other and taking over all shoppers. If $c_1 \neq c_2$ such that one firm is serving all shoppers, the remaining firm will profit from deviating to v and charging loyals a higher price.

The only remaining candidate equilibrium is $p_i^* = p_j^* = v$. However, this cannot be an equilibrium either, as $\max(\tilde{c}_1, \hat{c}_2) < v$ implies that either firm has incentives to undercut the other. Hence no pure strategy equilibrium exists. □

We proceed to characterize the mixed strategy equilibrium of the game. It is useful to define

$$\Omega \equiv \frac{\left(1 - \left(\frac{L_1}{1-L_2}\right)^{1-\tau_p}\right)}{\left(1 - \frac{L_2}{1-L_1}\right)} \cdot \frac{(v - \tilde{c}_1)}{(v - \hat{c}_2)}. \tag{3}$$

The value of Ω describes the comparative willingness to undercut of both firms. In the standard Bertrand model $\Omega = 1$ and firms are equally willing to undercut the rival (down to their marginal cost). In our model, due to the stake as well as loyalty and cost asymmetries, firms differ in their willingness to undercut. When $\Omega < 1$, Firm 2 exhibits a stronger willingness to undercut than Firm 1, in which case we refer to Firm 1 as the *soft* competitor and to Firm 2 as the *tough* competitor. Conversely, if $\Omega > 1$, Firm 1 has a stronger willingness to undercut. In this second case Firm 1 is tough and Firm 2 is soft.

Inspection of Ω reveals that the comparative willingness to undercut depends on how the size of the loyal customer base and marginal cost compare across both firms. A firm becomes a softer competitor when its loyal customer base is larger and its marginal cost is higher, because undercutting in order to sell to shoppers becomes less profitable. Furthermore, Ω depends on stake parameters τ_p and τ_c (via $\tilde{c}_1$ and $\hat{c}_2$), so it is also affected by the type and the size of the stake. A higher revenue or profit stake increases Firm 1's profits when Firm 2 serves shoppers, weakening Firm 1's incentives to undercut relative to Firm 2 (reduces Ω).

The mixed strategy equilibrium of the game is characterized as follows.

Proposition 2. *If* $\max(\tilde{c}_1, \hat{c}_2) < v$ *there is a unique mixed strategy equilibrium where each firm randomizes over price range* $[\underline{p}, v]$ *according to cumulative density function* $G_i(\cdot)$ *where*

(i) *If* $\Omega < 1$ *then*

$$\underline{p} = \left(\frac{L_1}{1-L_2}\right)^{1-\tau_p} \cdot v + \left[1 - \left(\frac{L_1}{1-L_2}\right)^{1-\tau_p}\right] \cdot \tilde{c}_1$$

$$G_1(p_1) = \frac{(1-L_1)}{1-L_1-L_2} \cdot \frac{(p_1 - \underline{p})}{\left(p_1 - \hat{c}_2\right)}$$

$$G_2(p_2) = \frac{1-L_2}{1-L_1-L_2} - \frac{L_1}{1-L_1-L_2}\left(\frac{v - \tilde{c}_1}{p_2 - \tilde{c}_1}\right)^{\frac{1}{1-\tau_p}}$$

(ii) *If* $\Omega \ge 1$ *then*

$$\underline{p} = \frac{L_2}{1-L_1} \cdot v + \left[1 - \frac{L_2}{1-L_1}\right] \cdot \hat{c}_2$$

$$G_1(p_1) = 1 - \frac{L_2}{1 - L_1 - L_2} \cdot \frac{(v - p_1)}{\left(p_1 - \hat{c}_2\right)}$$
$$G_2(p_2) = \frac{1 - L_2}{1 - L_1 - L_2}\left[1 - \left(\frac{\underline{p} - \tilde{c}_1}{p_2 - \tilde{c}_1}\right)^{\frac{1}{1-\tau_p}}\right]$$

Proof. By Lemma 1 there is no pure strategy equilibrium. In any mixed strategy equilibrium, following standard arguments, firms will set their prices by randomizing continuously over the same price interval. Denote the lower bound of this interval by $\underline{p}$. The upper bound of the interval must also be common and equal to v for both firms because else, at the highest price, one or both firms would not serve shoppers and thus would deviate to v.

The pricing strategy of each firm can be characterized with a cumulative density function $G_i(\cdot)$ with support over $[\underline{p}, v]$. There can be no gaps within the support for the usual reasons. Namely, if there was such a gap, then a firm would readily redistribute probability mass from close to the low limit of the gap to its upper bound. Moreover, $G_i(\cdot)$ cannot exhibit point masses in the range $p \in [\underline{p}, v)$ because if one firm places a point mass on some $p < v$, then the other would redistribute probability mass from above the point mass to just below it. However, there may be a point mass at v by at most one firm. Let the point mass by firm i on v be denoted by $\alpha_i \geq 0$.

Consider first the case where $\alpha_2 = 0$ so that Firm 1 (potentially) places a point mass of size $\alpha_1 \geq 0$ on v. When Firm 1 sets $p_1 \in [\underline{p}, v)$ its expected profit in equilibrium is equal to

$$\Pi_1^* = \int_{p_1}^{v} ((1 - L_2)(p_1 - c_1) + L_2(\tau_p p_2 - \tau_c c_2))g_2(p_2)dp_2$$
$$+ \int_{\underline{p}}^{p_1} (L_1(p_1 - c_1) + (1 - L_1)(\tau_p p_2 - \tau_c c_2))g_2(p_2)dp_2. \quad (4)$$

Taking the derivative with respect to p_1 and equating it to zero identifies the differential equation that pins down $G_2(\cdot)$ up to a constant C. We obtain

$$G_2(p_2) = \frac{1 - L_2}{1 - L_1 - L_2} + C((1 - \tau_p)p_2 - c_1 + \tau_c c_2)^{-\frac{1}{1-\tau_p}}. \quad (5)$$

Since Firm 2 does not place a point mass on v by assumption, we have $G_2(v) = 1$, which pins down C. The equilibrium mixed strategy for Firm 2 is therefore

$$G_2(p_2) = \frac{1 - L_2}{1 - L_1 - L_2} - \frac{L_1}{1 - L_1 - L_2}\left(\frac{(1 - \tau_p)v - c_1 + \tau_c c_2}{(1 - \tau_p)p_2 - c_1 + \tau_c c_2}\right)^{\frac{1}{1-\tau_p}},$$

which can be rewritten as the expression provided in the proposition. For $G_2(p_2)$ to be well defined, $(1 - \tau_p)p_2 - c_1 + \tau_c c_2 > 0$ has to hold at $p_2 = v$, thus we require $(1 - \tau_p)v - c_1 + \tau_c c_2 > 0 \Longleftrightarrow v > \tilde{c}_1$ which is satisfied by assumption. Since there are no ties in equilibrium, we can now derive $\underline{p}$ from $G_2(\underline{p}) = 0$ where

$$\underline{p} = v\left(\frac{L_1}{1 - L_2}\right)^{1-\tau_p} + \left[1 - \left(\frac{L_1}{1 - L_2}\right)^{1-\tau_p}\right]\tilde{c}_1.$$

Firm 2 must derive the same expected profit at any price $p \in [\underline{p}, v]$ given that it is indifferent when randomizing over the support. We can write Firm 2's equilibrium profits by noting that when charging price $p_2 = \underline{p}$ it serves all consumers except Firm 1's loyals,

$$\Pi_2^* = (\underline{p}(1 - \tau_p) - c_2(1 - \tau_c))(1 - L_1).$$

We next derive $G_1(\cdot)$ from

$$\Pi_2^* = (p_2(1 - \tau_p) - c_2(1 - \tau_c))(L_2 + (1 - L_1 - L_2)(1 - G_1(p_2)))$$

as

$$G_1(p_1) = \frac{(1 - \tau_p)(1 - L_1)(p_1 - \underline{p})}{(1 - L_1 - L_2)((1 - \tau_p)p_1 - (1 - \tau_c)c_2)}.$$

For $G_1(p_1)$ to be positive at least at $p_1 = v$ we require $(1 - \tau_p)v - (1 - \tau_c)c_2 > 0 \Longleftrightarrow v > \hat{c}_2$, which holds by assumption. Furthermore, for $G_1(p_1)$ and $G_2(p_2)$ to constitute an equilibrium we require that Firm 1 places a point mass on v, thus $G_1(v) \leq 1$ needs to hold. It can be shown with some algebraic manipulations that this condition is equivalent to $\Omega \leq 1$.

We next turn to the case where Firm 1 places no point mass on v so that $\alpha_1 = 0 \leq \alpha_2$. In this case, we can write Firm 2's equilibrium profits by noting that it will only serve loyals when charging $p_2 = v$,

$$\Pi_2^* = (v(1 - \tau_p) - c_2(1 - \tau_c))L_2,$$

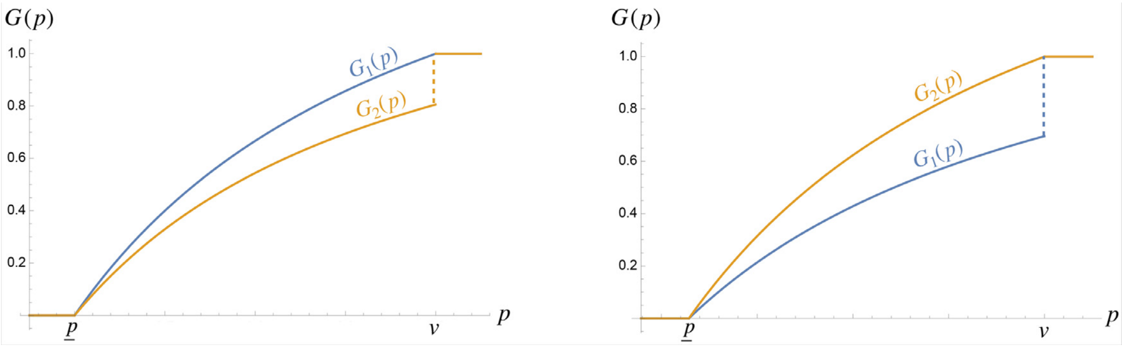

Fig. 1. Equilibrium pricing strategies. Both firms randomize their price over the support $p \in [\underline{p}, v]$ and only one firm exhibits a mass point at $p = v$. Plotted for $\Omega > 1$ in the left panel (Firm 1 is the tough competitor) and $\Omega < 1$ in the right panel (Firm 2 is tough), where the CDF of Firm 1 is plotted in blue and that of Firm 2 in orange, with parameter values $v = 1$, $c_1 = c_2 = 0.2$, $L_1 = 0.2$ and $L_2 = 0.4$ (left panel), $L_1 = 0.4$ and $L_2 = 0.2$ (right panel), $\tau_p = 0.1$, $\tau_c = 0$. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

which will constitute an equilibrium if $\Pi_2^* > 0 \Longleftrightarrow (1-\tau_p)v - (1-\tau_c)c_2 > 0$, which is equivalent to $v > \hat{c}_2$ and holds by assumption.

Firm 2's derives profit $(p_2(1-\tau_p) - c_2(1-\tau_c))$ from each consumer served, and with probability $(1 - G_1(p_2))$ serves all shoppers as well as its loyals. Equating these expected profits to the equilibrium profit expression above allows us to derive $G_1(p_1)$ from

$$(p_2(1-\tau_p) - c_2(1-\tau_c))(L_2 + (1 - L_1 - L_2)(1 - G_1(p_2))) = \Pi_2^*$$

as

$$G_1(p_1) = 1 - \frac{L_2}{1 - L_1 - L_2} \cdot \frac{(v - p_1)}{\left(p_1 - \frac{(1-\tau_c)}{(1-\tau_p)}c_2\right)}.$$

We can solve for $\underline{p}$ by noting that $G_1(\underline{p}) = 0$,

$$\underline{p} = \frac{L_2}{1 - L_1}v - \frac{1 - L_1 - L_2}{(1 - L_1)} \cdot \frac{(1-\tau_c)}{(1-\tau_p)}c_2.$$

We proceed to identify $G_2(p_2)$ from (5) by pinning down C with $G_2(\underline{p}) = 0$, which yields

$$G_2(p_2) = \frac{1 - L_2}{1 - L_1 - L_2}\left(1 - \left(\frac{\underline{p}(1-\tau_p) - c_1 + \tau_c c_2}{p_2(1-\tau_p) - c_1 + \tau_c c_2}\right)^{\frac{1}{1-\tau_p}}\right).$$

For $G_2(p_2)$ to be well-defined we require $v(1-\tau_p) - c_1 + \tau_c c_2 > 0 \Longleftrightarrow v > \tilde{c}_1$, which again holds by assumption. We also require Firm 2 to place a mass point on v, which requires $G_2(v) \leq 1$ and this can be shown to be equivalent to $\Omega \geq 1$. Moreover, given that the above two cases exhaust all parameters, we conclude that the mixed strategy equilibrium is unique. □

In the mixed strategy equilibrium both firms price according to cumulative density functions $G_1(p_1)$ and $G_2(p_2)$ with common support $[\underline{p}, v]$ characterized in Proposition 2. The aggressiveness with which each firm prices is reflected in how it allocates probability over the price support. One of the two firms (the soft competitor) places a probability mass on the monopoly price $p = v$ and the other firm (the tough competitor) places instead higher probability on lower prices $p \in [\underline{p}, v)$ to undercut more often. We note that the solution is equivalent to that derived by Narasimhan (1988) for the case where there is no stake and marginal costs are equalized to zero.

Fig. 1 depicts the pricing strategies of both firms by plotting $G_1(p)$ and $G_2(p)$ for two different values of Ω. Recall that Ω describes the comparative willingnes to undercut of both firms. The left panel corresponds to a case where $\Omega > 1$ so that Firm 1 is the tough competitor and Firm 2 is the soft competitor. In this case, Firm 2 places a point mass on the monopoly price while Firm 1 allocates higher probability to lower prices.[3] In the right panel, $\Omega < 1$ so that Firm 1 is soft and Firm 2 is tough. In this case the roles are reversed; Firm 1 places a mass point on the monopoly price and Firm 2 allocates higher probability to lower prices. When $\Omega = 1$ such that both firms are matched in their willingness to undercut (a case not plotted in the figure) neither firm places a point mass on v though their pricing strategies will generally differ.

[3] The parameter values used in Fig. 1 ensure that Firm 1 prices higher than Firm 2 in the first order stochastic sense when $\Omega > 1$ (and vice versa). This is convenient to illustrate key equilibrium properties but we note that it need not be the case in general.

Table 1
Summary table of comparative statics. Column $\tau_p = \tau_c$ reflects a simultaneous shift in both stake parameters (a profit stake). In cells with two vertically arranged entries, the top entry corresponds to the case $\Omega < 1$ and the bottom entry to $\Omega > 1$. An entry $+-$ denotes that the sign of the impact varies with other parameters.

	L_1	L_2	c_1	c_2	τ_p	τ_c	$\tau_p = \tau_c$
$\underline{p}$	+	+	+	−	+	−	+
			0	+			0
G_1	−	+−	−	+	−	+	−
		−	0	−			0
G_2	−	−	−	+	−	+	−
	+−		+	−	+−		+

Both firms face a trade-off between serving a captive market of loyals at a high price or lowering their price in a bid to undercut the rival and serve shoppers. Equilibrium pricing strategies imply that the soft competitor is more likely to quote a high price (often, the monopoly price) while the tough competitor is more likely to quote a low price. That is, firms resolve the tradeoff by adjusting the likelihood with which they quote high and low prices depending on their competitive standing as described by Ω. In doing so, both firms exhibit willingness to undercut the rival, derive positive expected market share from shoppers, and generate positive profits.

Our discussion has focused so far on how firms price relative to each other. We next examine the determinants of the absolute level of prices in equilibrium. The comparative statics of the mixed strategy equilibrium are complex due to the fact that pricing strategies $G_1(p)$ and $G_2(p)$ depend on all parameters, either directly or via the lower bound of the price support $\underline{p}$. We report the statics that can be resolved analytically in the following proposition and provide an overview of all parameters in Table 1. Entries in the table not covered in the proposition are based on numerical analysis.

Proposition 3. *The mixed strategy equilibrium of the game exhibits the following properties*

- *If $\Omega < 1$ then $G_2(p_2)$ is decreasing (thus p_2 FOSD increasing) in L_1, L_2, c_1, and τ_p, increasing in τ_c and c_2.*
- *If $\Omega > 1$ then $G_1(p_1)$ is decreasing (thus p_1 FOSD increasing) in L_1, L_2, c_2, and τ_p, increasing in τ_c, and independent of c_1.*
- *Lower bound $\underline{p}$ is increasing in v, L_1, L_2, and τ_p and decreasing in τ_c. If $\Omega < 1$ then $\underline{p}$ is increasing in c_1 and decreasing in c_2, if $\Omega > 1$ then $\underline{p}$ is independent of c_1 and increasing in c_2.*

Proof. Follows from direct differentiation of relevant equilibrium expressions. □

In what follows, we refer to the average price quoted by each firm as its *quoted price* (and use the plural when referring to both firms). Consider first the impact of consumer loyalty on quoted prices. The lower bound of price support $\underline{p}$ increases with L_1 and L_2 in all cases; a larger segment of loyal consumers (and a smaller addressable segment of shoppers) drives up the minimum price firms are willing to quote. An increase in the size of either firm's loyal customer base drives up the quoted price by that firm (G_1 is decreasing in L_1 and G_2 is decreasing in L_2) and can also drive up the rival's quoted price. The latter is guaranteed to happen with a tough competitor, but we cannot rule out that a soft competitor responds by lowering its quoted price.

Marginal costs have a complex effect on quoted prices. On the one hand, marginal costs have a non-monotonic effect on price lower bound $\underline{p}$. An increase in c_1 weakly increases $\underline{p}$, however an increase in c_2 may in fact reduce $\underline{p}$ because it increases Firm 1's undercutting incentives (due to cost-sharing τ_c). On the other hand, an increase in the marginal cost of one firm can either increase or reduce G_1 and G_2. Therefore, while an increase in a firm's marginal cost will generally increase its quoted price, we cannot rule out that small increases can in some cases drive down the quoted prices of both firms. Nonetheless, it is clear that $\underline{p} \geq \min(\tilde{c}_1, \hat{c}_2)$ must hold, so a large enough increase will drive up quoted prices.

Consider next the impact of stake parameters τ_p and τ_c on quoted prices. As noted above, an increase in revenue-sharing τ_p weakens the stakeholder's incentives to undercut. This is reflected in a higher $\underline{p}$ and a decrease in G_1, both of which increase the price quoted by Firm 1. When $\Omega < 1$ such that Firm 1 is the soft competitor, Firm 2 responds in the same fashion and a higher revenue stake τ_p also increases Firm 2's quoted price. This is not always the case when $\Omega > 1$, given that G_2 can increase when τ_p is small and reduce Firm 2's quoted price. Thus a revenue stake has a non-monotonic effect on prices when $\Omega > 1$.

An increase in cost-sharing τ_c (keeping τ_p constant) raises the share of Firm 2's costs that are carried by Firm 1. This increases the markup derived by Firm 2 at any given price and strengthens Firm 1's incentives to undercut, because allowing Firm 2 to serve non-loyals becomes more costly. As a result, an increase in τ_c reduces $\underline{p}$ and increases G_1 and G_2, reducing the quoted prices of both firms.

We should stress that the impact of τ_c in isolation is not equivalent to a profit stake, given that this requires $\tau_p = \tau_c$ and thus combines the two effects described above. To determine which effect prevails, column $\tau_p = \tau_c$ in Table 1 presents the comparative statics for this case. When $\Omega < 1$ so that Firm 1 is the soft competitor, the effect of τ_p prevails over that of τ_c

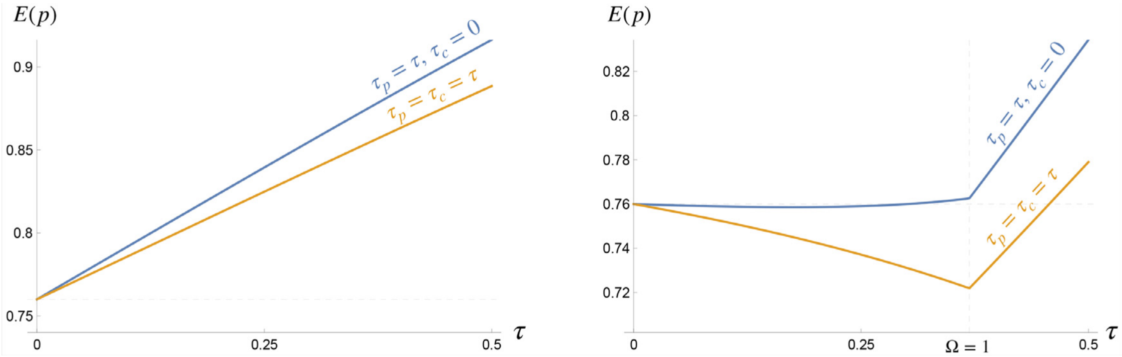

Fig. 2. Average price paid by consumers. A stake can reduce or increase the purchase price, and a revenue stake always leads to a higher purchase price than a profit stake. Plotted for a parameter trajectory where $\Omega < 1$ (left panel) and a trajectory where either $\Omega > 1$ or $\Omega < 1$ as a function of τ (right panel), where the blue curve corresponds to a revenue stake, orange to a profit stake, and the horizontal dotted line to the case where there is no stake ($\tau_p = \tau_c = 0$), with parameter values $v = 1$, $c_1 = c_2 = 0.2$, $L_1 = 0.4$ and $L_2 = 0.2$ (left panel), $L_1 = 0.2$ and $L_2 = 0.4$ (right panel). (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

and a higher profit stake always leads firms to increase their quoted prices. The opposite is true when $\Omega > 1$ so that Firm 2 is the soft competitor, in which case a higher profit stake reduces quoted prices.

We next examine the mechanism driving Firm 2 to lower its quoted price in response to a higher revenue stake or profit stake when $\Omega > 1$. To gain some insight, let $c_1 = c_2 = c$ and $\tau_p = \tau_c = \tau$, such that $\hat{c}_2 = c_2$ and therefore $\underline{p}$ and $G_1(p_1)$ do not depend on τ. Because Firm 2's profits are proportionally reduced by τ in this case, Firm 1's pricing strategy (which ensures Firm 2 is indifferent when randomizing over the price range) is unaffected by the stake. How does Firm 2's pricing strategy respond to the stake's effect on Firm 1's profits? Inspection of Π_1^* in (4) reveals that Firm 1's incentives to raise its price increase with τ. When Firm 1 holds a larger stake in Firm 2's profits, it earns more profits for any price p_1 it may charge, but more so for higher prices because these result in lost sales to Firm 2 when the latter's price and therefore profits are high. Because firms must be indifferent in equilibrium when randomizing over price range $[\underline{p}, v)$, this drives Firm 2 to shift its price distribution toward lower prices to disincentivize Firm 1 from increasing its own. As a result, increasing τ increases $G_2(p_2)$. This effect is muted when $\Omega < 1$ because in that case both $\underline{p}$ and $G_1(p_1)$ depend on the stake and move in the opposite direction.

5. The effect of a stake on market outcomes

We next examine the market outcomes generated by firm pricing strategies in the presence of a stake. To compare market outcomes under different stakes, we focus on the average price paid by consumers in equilibrium and refer to this as the *purchase price*. Note that the purchase price differs from quoted prices in the mixed strategy equilibrium because different consumers purchase from different firms. Moreover, the purchase price provides a direct measure of consumer welfare given our demand specification and the fact that the market is covered in equilibrium. It also provides a direct measure of firm profits in the case where $c_1 = c_2$ (if costs differ across firms, then in principle the purchase price can fall but the total profits of both firms increase due to demand reallocation toward the more efficient firm).

Fig. 2 plots the purchase price in three different scenarios: with a profit stake, with a revenue stake, and in the absence of a stake. The left panel plots a parameter trajectory where Firm 1, the stakeholder, always exhibits weaker incentives to undercut the rival ($\Omega < 1$). The right panel plots a trajectory where Firm 2 (for low τ values) and Firm 1 (for higher τ values) have weaker incentives to undercut.

There are two main points to draw from Fig. 2. The first is that the purchase price is always higher with a revenue stake than with a profit stake. This can be observed directly in the plots and holds across the full parameter space. Clearly, a revenue stake is more effective than a profit stake as a tool to relax price competition between firms. This implies that a revenue stake is more detrimental to consumer welfare than a profit stake.

A second point is that the stake's effect on the purchase price hinges on the competitive standing of both firms. Inspection of purchase prices in Fig. 2 with and without a stake (the latter is depicted by the horizontal dotted line) reveals that the introduction of a stake can either increase or reduce the purchase price. To understand these outcomes, it is useful to reflect on how the competitive standing of both firms affects competition intensity in the absence of the stake. Competition is most intense when firms exhibit similar loyalty and marginal cost ($\Omega \approx 1$), and least intense when firms differ significantly on these parameters ($\Omega \to 0$ or $\Omega \to \infty$). The observation that player heterogeneity softens competition has been widely documented in the auction literature.[4]

[4] See Baye et al. (1993); Szech (2015); Franke et al. (2018) for examples of all-pay auctions that exhibit this property. Shelegia and Wilson (2021) explore when this property fails. Our price competition game can be remapped to a contest where prices are converted to bids in the form of offered surplus,

How does the introduction of a stake affect competition intensity? A stake alters the competitive standing of both firms in favor of the stakeholder, so it can either reduce or increase competition intensity depending on how this affects heterogeneity between firms. When Firm 1 is the soft competitor without a stake (higher loyalty and/or marginal costs), introducing a stake reinforces this and increases heterogeneity between the firms. This softens competition and increases the purchase price as shown in the left panel in Fig. 2. When Firm 2 is the soft competitor in the absence of a stake, a small stake has the contrary effect because it erodes the differential between both firms and reduces heterogeneity. This intensifies competition and reduces the purchase price, as shown in the right panel of Fig. 2 for low τ values ($\Omega > 1$). The effect is clearly observable with a profit stake, and is present but much weaker with a revenue stake. For higher values of τ, the purchase price increases once Firm 1 becomes the soft competitor ($\Omega < 1$) such that further increasing the stake increases heterogeneity.

Based on inspection of equilibria across the parameter space and the mechanisms described above, we conclude the following. Revenue stakes soften competition except for a small parameter range where $\Omega > 1$ and τ_p is small. In general, we expect most platforms and gatekeepers to soften competition when implementing a revenue stake as they enjoy a dominant position and thus have weaker incentives to undercut competitors (which suggests that $\Omega < 1$). Profit stakes intensify competition when $\Omega > 1$ and soften it when $\Omega < 1$. This implies that a stakeholder with incentives to compete aggressively on price may not benefit from implementing a profit stake.

The impact of profit stakes on prices has also been examined in the Cournot competition literature. Reynolds and Snapp (1986) and Farrell and Shapiro (1990) analyzed the impact of a profit stake under Cournot competition, and found that it raises price and reduces output (i.e., softens competition). In our model, output is constant due to our unit demand assumption and the fact that the market is covered in equilibrium. We find that a profit stake can in some cases raise the prices paid by consumers, as in the Cournot model, though in other cases it reduces them. This implies that the anticompetitive properties of profit stakes are not always preserved under price competition. The result is noteworthy because, unlike the Cournot framework, price competition provides a rationale for firms *not* to acquire minority participations in competitors under certain conditions.

Our analysis abstracts from the market structure that supports the creation of the stake. Firm 1 is the stakeholder in our model, and therefore the decision to implement a stake rests with Firm 1 in the first place. Firm 1 could create a revenue stake by implementing an ad valorem fee if it operates as a platform, gatekeeper, or licensor. Alternatively, Firm 1 could create a profit stake by acquiring partial ownership of Firm 2. It is important to recognize, however, that objection by Firm 2 could challenge Firm 1's ability to successfully create a stake. The profitability generated by a stake is distributed unevenly among firms, because it redistributes revenues and costs and also affects pricing strategies, shifting the markups and market shares of firms. Thus, in order to shed further light on the decision to create a stake we can use our model to determine when a stake is profitable for *both* firms.

To study the creation of a stake, we next endogenize the contractual agreement between firms by examining the scenario where the stakeholder, Firm 1, makes a take-it-or-leave-it (TIOLI) offer to Firm 2 consisting of a stake vector (τ_p, τ_c). We restrict the contractual space to offers such that $\max(\tau_p, \tau_c) < 1/2$, which rules out large stakes and in particular profit stakes that would confer Firm 1 control over Firm 2 and thus eliminate competition between both. If Firm 2 accepts the offer, the stake is implemented and competition unfolds as characterized above. If Firm 2 rejects, it earns zero profit. This assumption is convenient to simplify the problem and is most relevant if Firm 1 can punish Firm 2 for rejection.[5]

Proposition 4. *If Firm 1 makes a TIOLI offer to Firm 2, the unique optimal contract is* $\tau_p^* = \frac{v-c_2}{v}$ *and* $\tau_c^* = 0$.

Proof. We show that contract (τ_p^*, τ_c^*) enables Firm 1 to extract maximum possible profits from all consumers. Note that $\hat{c}_2(\tau_p^*, \tau_c^*) = v$, so the contract implements a pure strategy equilibrium with prices given by $p_1^* = p_2^* = v$ as described in Proposition 1. Consider first the case $c_1 > c_2$. Firm 1 earns $v - c_1$ from its loyals and earns $p_2\tau_p^* - c_2\tau_c^* = v \cdot \frac{v-c_2}{v} = v - c_2$ from all remaining consumers. Given that $c_1 > c_2$, this is the maximum profit that can be extracted. In the case $c_1 < c_2$, Firm 1 earns $v - c_1$ from its loyals and shoppers, and extracts $v - c_2$ from Firm 2's loyals. No more profit can be extracted. And in the case $c_1 = c_2$, Firm 1 earns $v - c_1$ or $v - c_2$ from every consumer. Again, this cannot be improved upon.

A contract that allows Firm 2 to earn positive profit cannot be optimal for Firm 1. Any contract that leads to $\hat{c}_2 > v$ is not optimal because Firm 1 will not appropriate any surplus from Firm 2's loyals. Among contracts where $\hat{c}_2 = v$, only $\tau_p^* = \frac{v-c_2}{v}$ and $\tau_c^* = 0$ achieves full profit extraction for Firm 1. The only exception is $\tau_p = \tau_c = 1$ which is not permitted by assumption. □

Firm 1's optimal offer is a revenue stake. The optimal offer dominates all feasible alternative offers across the full parameter space. The result underscores the effectiveness of revenue-sharing over profit-sharing to relax price competition. Moreover, the optimal stake offered by Firm 1 is just as effective as full profit-sharing in extracting Firm 2's profits (that is, hypothetical full ownership of Firm 2 that retains competition in the marketplace). We note however that this last point hinges on our unit demand assumption, given that revenue-sharing induces double-marginalization with downward-sloping

the winner serves non-loyal consumers (shoppers) and the loser pays his bid in the form of forgone profits on his loyals. Baye et al. (2012) present a framework similar to the contest we just described incorporating externalities among participants, though they consider only symmetric games.

[5] For example, if Firm 1 is a gatekeeper and offers a revenue stake based on an ad valorem fee, it can refuse to deal with Firm 2 if it does not pay the fee. Alternatively, if Firm 1 offers a profit stake and Firm 2 rejects, it can commit to intense price competition that is detrimental for Firm 2.

demand and profit-sharing does not. It can also be shown that a revenue stake is the optimal solution for Firm 2 if it makes a TIOLI offer to Firm 1 (albeit one that is optimal for Firm 2 and differs from the one characterized in Proposition 4) with the exception that, if Firm 1 is particularly inefficient or Firm 2 particularly efficient, then no stake is offered $\tau_p^* = \tau_c^* = 0$.

6. Concluding remarks

At the outset of this paper we set out to examine the impact of stakes under price competition. Our main findings can be summarized in two key points. First, a revenue stake has a stronger competition-dampening effect than a profit stake. For example, our model predicts that charging a competitor a 15% ad valorem fee on its sales is more effective to relax price competition than acquiring a 15% ownership stake. The superior performance of the revenue stake is explained by the omission of the cost factor; when the stakeholder is burdened by the costs of the competitor, it has an additional incentive to price undercut in order to displace its sales.

Second, a stake in a competitor can either soften or intensify price competition. This hinges on the type of the stake and the competitive standing of firms. In general, revenue stakes tend to soften competition. Profit stakes soften competition when, in the absence of the stake, the stakeholder has less incentives to price aggressively than the rival. However, when the stakeholder has incentives to price more aggressively than the rival, small profit stakes can intensify competition.

We conclude that firms have strong incentives to implement revenue-sharing mechanisms on their competitors where feasible. Both the stakeholder and the firm in which the stake is held stand to benefit when the alternative is intense price competition. This competition-dampening effect of revenue-sharing in our model is remarkable given the limited attention this mechanism has received in the competition policy literature. While profit stakes based on (minority) ownership of rivals are recognized to affect competitive interaction, revenue stakes implemented through ad valorem fees have received little attention in comparison. Our results, together with the observation that some of the most profitable businesses in the last decade apply revenue-sharing fees to competitors (Amazon's marketplace, Apple's App store, Google Play) suggest that this mechanism deserves further study and increased regulatory scrutiny.

CRediT authorship contribution statement

Andres Hervas-Drane: Conceptualization, Methodology, Writing – original draft, Writing – review & editing. **Sandro Shelegia:** Conceptualization, Methodology, Writing – original draft, Writing – review & editing.

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