Short-Term Investment and Equilibrium Multiplicity. *

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Abstract

I study the effects of the heterogeneity of traders' horizons in a 2-period NREE model where all traders are risk averse. Owing to risk premia, short termism generates multiple equilibria. In particular two distinct patterns arise. Along the "low trading intensity equilibrium," short termists anticipate a thinner second period market and, owing to risk aversion, scale back their trades. This reduces both risk sharing and information impounding into prices, enforcing a high returns' volatility-low price informativeness equilibrium. Along the "high trading intensity equilibrium," the opposite happens and a low volatility-high price informativeness equilibrium arises. Thus, in the presence of short-term behavior and traders' risk aversion, periods of high volatility are a signal of poor price informativeness.

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1 Introduction

This paper analyzes the effects of short-term behavior on stock market patterns in an environment where traders have asymmetric information. Both empirical and theoretical considerations motivate the analysis. On the one hand, instances of short term behavior abound in financial markets. From day traders [1] to institutional investors [2] short termism seems to characterize the behavior of an increasing proportion of market participants. On the other hand, intuitive reasoning suggests that in a realistic market where price movements are due to both information arrival and supply shocks, the risk borne by an agent holding a positive inventory of the traded asset should have a different effect on his behavior depending on his time preferences. Indeed, short-term traders, faced with the need of liquidating their position in the short run, have fewer opportunities to smooth their inventory holdings' decisions. As a consequence, their behavior is strongly influenced by the anticipation of price reaction to future order flows. On the contrary, thanks to their longer horizon, long termists can attain a better intertemporal asset allocation and react less intensely to the expected fluctuations of asset prices. This difference, in turn, should make market patterns dependent on the *composition* of the market.

Following this insight, I analyze a 2-period noisy, rational expectations equilibrium model of stock-market trading based on Vives (1995). In the model, two classes of traders interact: a sector of short-term, risk averse informed traders of measure $\mu > 0$, and a sector of long-term, risk averse informed traders of measure $1 - \mu$. [3] Risk aversion has two effects on the market: first, informed agents, besides speculating on their private information, also *act* as market makers; second - and consequently - equilibrium prices are influenced by *both* information arrival and liquidity supply.

This last effect, depending on the trading horizon of informed speculators, has a different impact on their trading activity. Long-termists, when in period 1 choose their position,

[1] "In the 1980s, it was Wall Street's takeover barbarians. Today it is the amateurs in jeans and sneakers who sit in front of a computer and trade 40-50 times in a day." *The Economist,* "In praise of day traders," May 13th 1999.

[2] For instance, Kahn and Winton (1998) argue that "...traditionally [Institutional Investors] were *stock pickers* who tried to beat the market through trading; if a firm whose stock they held seemed headed for trouble, these investors headed for the door (*the Wall Street rule*)." Also, Wermers (1999) "Many newsmedia commentators ...tend to believe that institutional investors focus excessively on short-term trading strategies..." Finally, Tirole (2001) "...institutions shy away from sitting on boards and mostly act as short-term players."

[3] A number of authors have analyzed dynamic rational expectations equilibrium models, see e.g. Singleton (1987), Brown and Jennings (1989), Grundy and McNichols (1989), Vives (1995), and He and Wang (1995).

anticipate the volatility of the *asset* value using both the private and public information available (i.e. the equilibrium price). Short-termists, on the contrary, cannot hold the asset until the liquidation date and are, therefore, interested in anticipating the *second period price* which in turn depends on their first period behavior. As a consequence, two possible equilibria arise. If short term traders anticipate that second period price *overreacts* to the order flow they reduce the risk of their position by scaling back their trades. Thus, the market's risk bearing capacity decreases in the second period, making the market thinner and the second period price overreactive to the order flow. The opposite happens if they anticipate second period price *underreaction*. Therefore, in the presence of risk averse traders, short-term behavior induces multiple equilibria.

The consequences for market performance depend on which of the two equilibria arises. Along the low trading intensity equilibrium, short term traders speculate less aggressively on their private signal. This reduces price informativeness in both periods and increases returns' volatility. Along the high trading intensity equilibrium, the opposite happens and a low volatility, high price informativeness equilibrium arises. In spite of its "good" properties, the high trading intensity equilibrium is, however, unstable since the slope of its aggregate excess demand function is positive. Therefore, a price decline, e.g. spurred by a selling pressure, drives the market *away* from equilibrium. [4]

Summarizing, in the presence of short-term behavior and traders' risk aversion *(i)* markets may become *un*stable ad *(ii)* periods of high volatility are a signal of poor price informativeness. Hence, the usual explanation of a *more* volatile market as one where *more* information is gathered (see e.g. Admati and Pfleiderer, 1988) breaks down.

A number of authors have analyzed causes and effects of short-term behavior in financial markets. Holden and Subrahmanyam (1996), provide a foundation for short termism. They analyze a two period model where risk averse traders can collect either short or long lived information (and thus trade once or twice) and a public signal, unrelated to their information, is periodically released to the market. In such a context traders' decisions about information collection depend on their degree of risk aversion. Indeed, as the public signal *buffets* traders' position - negatively affecting their expected utility - the more risk averse a trader is the less willing to collect long lived information he becomes. As a consequence, for a sufficiently high degree of risk aversion, all traders concentrate on *short lived* information and long termists disappear from the market. Vives (1995) analyzes the effects of short term behavior on price informativeness. In a $N \geq 2$-period model, he

[4] This peculiar feature of the high trading intensity equilibrium is reminiscent of traders' behavior around market crashes (see Barlevy and Veronesi, 2002, and Gennotte and Leland, 1990).

shows that controlling for patterns of information arrival, price informativeness depends on speculators' trading horizons. In particular, when information arrival is concentrated in the first period, the price in period N is *more* informative in the market with long term traders than in the one with short termists. Conversely, when traders receive information at a constant rate the reverse happens and short term trading delivers a more informative last period price. Differently from the present context, in his model prices are set by a competitive, risk neutral market making sector and this renders the equilibrium in the market with short term traders unique. [5]

Others have investigated the multiplicity issue in market microstructure models. Pagano (1989), in a OLG model with symmetric information, shows that in the presence of transaction costs, anticipated high volatility levels make traders unwilling to enter the market, reducing risk sharing and leading to thin markets. Dennert (1991), shows that *high* volatility equilibria can be self-fulfilling in the steady state of a OLG market with differential information. Dow (1999) shows that thin markets, by crowding out the liquidity supply of risk hedgers, can be a self-fulfilling phenomenon. In his model, a risk averse agent trades-off the advantages of hedging a shock to his wealth, with the costs of trading in a market where a bid-ask spread arises owing to asymmetric information. For some parameter configuration, the model displays a high and a low liquidity equilibrium. In the thin market equilibrium, only more risk averse traders enter, while in the deep market equilibrium more risk tolerant traders participate. This analysis is clearly related to the present paper. However, in Dow (1999) equilibrium multiplicity relies on the heterogeneity of traders' risk aversion, while in the present context it is the result of traders' risk aversion *and* short term investment horizons. Finally, Admati and Pfleiderer (1988) in their analysis of trading patterns show that informed and discretionary liquidity traders' entry decisions in the market are strategic complements. As a consequence multiple equilibria arise. In particular, equilibria where liquidity traders cluster are also those where returns are more volatile and prices *more* informative. Notice, however, that in their case informed traders are risk neutral. Thus, highly volatile returns do not shy informed away from the market. In the present case, on the contrary, risk averse informed traders are crowded out by highly volatile markets rendering the price *less* informative.

The paper is organized as follows: in the next section, I outline the model's assumptions, define notation and show existence and uniqueness of the equilibrium in the case in which only long term traders are in the market. In the third section I introduce a positive measure

[5] At least in the 2-period case. See further, remark 5.

of short-term traders and show existence and multiplicity of equilibria in this market. I then study both analytically (section 3.2) and numerically (section 4) the effects on market performance of an increase in the size of the short term trading sector. Section 5 tackles the issue of equilibrium stability. A final appendix collects most of the proofs.

2 The Model

Trading happens over 2 periods, and there are two types of agents: a continuum of informed speculators (when long termists, maximizing the expected utility of their final wealth $W_{i2} = \sum_{n=1}^{2} \pi_{in}$; when short-termists, maximizing the expected utility of each period's profits) and noise traders. The asset payoff v is normally distributed $v \sim N(\bar{v}, \tau_v^{-1})$. Every informed speculator i has CARA utility function with risk-tolerance parameter $\gamma > 0$ and receives a noisy signal of the asset liquidation value $s_{in} = v + \epsilon_{in}$ in each period n, where $\epsilon_{in} \sim N(0, \tau_{\epsilon_n}^{-1})$, ϵ_{in} and v are independent and errors are independent across agents and periods. I will make the assumption that the strong law of large numbers holds (SLLN), i.e. $\int_0^1 s_{in} di = v$, almost surely.

In period 1 informed agents have the private signal s_{i1} available, while in period 2 they have the vector $s_i^2 = (s_{i1}, s_{i2})$. It follows from normal theory that the statistic $\tilde{s}_{i2} = (\sum_{n=1}^{2} \tau_{\epsilon_n})^{-1}(\sum_{n=1}^{2} \tau_{\epsilon_n} s_{in})$ is sufficient for the sequence s_i^2 in the estimation of v. An informed agent i in period n submits a limit order $X_{in}(\tilde{s}_{in}, p^{n-1}, \cdot)$ indicating the position desired at every price p_n, contingent on the information available. Noise traders' demand is normally distributed $u_n \sim N(0, \tau_u^{-1})$, u_1 and u_2 are independent. [6] Finally, u_n, and ϵ_{in} are independent for all i, n. I restrict attention to linear equilibria where a centralized mechanism aggregates orders and sets the equilibrium price that clears the market for the asset.

2.1 The Benchmark

In this section I derive the unique linear equilibrium of the market with only long term traders. The result obtained in proposition 1 coincides with the unique linear equilibrium found by He and Wang (1995) in the absence of a public signal and when the correlation across noise shocks is null. [7] Alternatively, one can see it as a generalization of a two-period

[6]The random variables $\{u_1, u_2\}$ can equivalently be interpreted as the increments in the stock supply in the two trading periods as in He and Wang (1995).

[7]Using their notation, when $\sigma_\delta = 0$ and $a_\Theta = 1$. As one can verify, gross trading intensity in their equilibrium has a closed form solution given by $\mu_1^{-1} = (1/\lambda w_1)$ and $\mu_2^{-1} = (1/\lambda w_2)$ that in our notation

version of Vives (1995) to a market with risk averse dealers. [8]

Proposition 1 *In the market with long term, informed speculators there exists a unique linear equilibrium where prices are given by* $p_o = \bar{v}$, $p_3 = v$, *and for* $n = 1, 2$, $p_n = \lambda_n z_n + (1 - \lambda_n \Delta a_n) p_{n-1}$ *and strategies are given by:*

$$x_{in} = a_n(\tilde{s}_{in} - p_n) + \gamma\tau_n(E[v|z^n] - p_n), \tag{1}$$

where $a_n = \gamma(\sum_{t=1}^{n} \tau_{\epsilon t})$, $\Delta a_n = a_n - a_{n-1}$, $z_n = \Delta a_n v + u_n$, $z^n = \{z_t\}_{t=1}^{n}$, $\tau_n = (Var[v|z^n])^{-1}$, $\lambda_n = (1 + \gamma\tau_u \Delta a_n)/\gamma\tau_{in}$, $\tau_{in} = (Var[v|z^n, \tilde{s}_{in}])^{-1}$.

Proof. See appendix. QED

x_{in} indicates a trader i's position in period n. It has two components. The first one reflects the trader's *speculative* position and depends on the difference between his private signal and the equilibrium price weighted by *private* precision. The second one captures i's *market making* position and depends on the difference between the market expectation and the equilibrium price weighted by *public* precision. The more risk tolerant the trader is, the more aggressively he trades. Δa_n is the *net* trading intensity of period n and indicates the net change in traders' desired speculative positions across period $n-1$ and n. In particular, using the convention $a_o = 0$, $\Delta a_1 = a_1 > 0$ and $\Delta a_2 = a_2 - a_1 \geq 0$. z_n is the *informational content* of period n order flow. It conveys a signal about the change in traders' aggregate speculative position due to private information (across periods $n-1$ and n) garbled by the net demand of liquidity traders in period n. Finally, λ_n is the reciprocal of market depth in period n and measures the period n price reaction to the public order flow.

As in each period speculators trade on private information and absorb liquidity shocks, asset prices react both to information arrival and liquidity supply.[9] To see this, suppose $\Delta a_n > 0$ and $z_n > 0$. Observing this signal, traders infer that informed speculators are *increasing* their speculative position. This signals good news about the asset pay-off leading to an upward revision of its conditional expectation. Suppose now that $\Delta a_n = 0$. In this case no new private information arrives to traders and their aggregate speculative position does not change. However, selling the asset at the previous period equilibrium

correspond respectively to $a_1 \equiv \gamma\tau_{\epsilon_1}$ and $a_2 \equiv \gamma(\tau_{\epsilon_1} + \tau_{\epsilon_2})$. See He and Wang (1995), Corollary 2, p. 943.

[8]Vives (1995) considers a market where prices are set by a sector of uninformed, competitive, *risk neutral* market makers, while in the present model informed traders "price" the asset. However, it is easy to show that adding a sector of uninformed, competitive, *risk averse* market makers does not qualitatively affect the results of the paper (see Cespa, 1999).

[9]As e.g. in Subrahmanyam (1991), Admati and Pfleiderer (1991), and Brown and Zhang (1997)

price exposes traders to the risk of netting a price lower than its actual liquidation value. To be compensated for this risk they revise upwards the equilibrium price. On the basis of this intuition, λ_n can be decomposed in the following way:

$$\lambda_n = \underbrace{\frac{\tau_u \Delta a_n}{\tau_{in}}}_{(a)} + \underbrace{\frac{1}{\gamma \tau_{in}}}_{(b)}.$$

Part (a) of the above expression is the OLS coefficient of the *news* contained in the order flow (z_n) in the regression of the pay-off (v) over informed speculators' information set $(\{z^n, \tilde{s}_{in}\})$. It captures the *adverse selection* component of market depth i.e. price movements due to the presence of informed traders in the market. [10] Part (b) is the product of traders' risk aversion and the conditional variance of the asset pay-off given their information set. It captures the *risk premium* traders require to take a position in the asset.

If the asset is priced by a sector of risk neutral, competitive market makers, risk averse informed traders only speculate on private information, the risk premium disappears and prices only react to the arrival of new information (as e.g. in Vives, 1995, and Dow and Rahi, 2000 and 2002). The relevant second period depth measure in such a market is then given by $\beta = \Delta a_2 \tau_u / \tau_2$. [11] Let's define a measure of the effect of traders' risk aversion on second period depth as follows:

$$\alpha = \frac{\beta}{\lambda_2}.$$

Proposition 2 *In every equilibrium of the market with long term, informed speculators: (1) price precision in period n is given by $\tau_n = \tau_v + \tau_u \sum_{t=1}^{n} (\Delta a_t)^2$ and (2) the conditional volatility of returns is given by:*

$$Var[p_2|p_1] = \begin{cases} \alpha^{-2}\left(\tau_1^{-1} - \tau_2^{-1}\right) & \text{for } \Delta a_2 > 0 \\ \lambda_2^2 \tau_u^{-1} & \text{otherwise,} \end{cases}$$

where $\alpha = \beta/\lambda_2$.

[10] Note that its sign coincides with the sign of the net change in traders' desired speculative positions between dates $n-1$ and n, i.e. Δa_n.

[11] See Vives (1995). In the present model as informed traders "price" the asset the *adverse selection* effect is weaker than when they only speculate on private information; this is so because informed traders have *two* sources of information to disentangle noise from fundamentals in the order flow. When a sector of competitive, risk neutral, uninformed market makers is added to the model, informed traders are "crowded-out" from the market making activity and prices only react to new information. The adverse selection effect (in the second period) is then $\beta = (\Delta a_2 \tau_u / \tau_2) > (\Delta a_2 \tau_u / \tau_{i2})$.

Proof. For part 1, in every linear equilibrium p^n is informationally equivalent to z^n. Hence, $\mathrm{Var}[v|p^n] = \mathrm{Var}[v|z^n] = \tau_n^{-1}$. For part 2, $\mathrm{Var}[p_2 - p_1|p_1] = \mathrm{Var}[p_2 - p_1|z_1] = \lambda_2^2 \mathrm{Var}[z_2|z_1] = \lambda_2^2 \tau_2 \tau_1^{-1} \tau_u^{-1}$. If $\Delta a_2 \neq 0$, multiplying numerator and denominator of the previous expression by $(\Delta a_2)^2 \tau_u \tau_2$ and collecting parameters, I obtain the result in the proposition. QED

The conditional volatility of returns is the result of the composite effect of *(a)* the reduction in the conditional variance of the asset liquidation value due to the arrival of *news* (the factor $(\tau_1^{-1} - \tau_2^{-1})$) and *(b)* the effect that traders' risk aversion induces on second period depth (the factor α^{-2}). Point *(a)* above refers to the standard explanation of returns' volatility in a semi-strong efficient market where the only source of price movements is information arrival; point *(b)* is peculiar to the present market where risk premia also play a role in affecting returns' volatility.

Remark 1 Owing to speculators' risk aversion, p_2 is not a sufficient statistic for $\{z_1, z_2\}$ in the estimation of v. Thus, traders condition second period demand on both z_1 and z_2 (and their private information). In other words, as in He and Wang (1995), they are *chartists*.

In the rest of the paper, two different patterns of private information arrival will be considered: the "concentrated" arrival of information case, where traders receive a private signal in the first period only (i.e. $\tau_{\epsilon_1} > 0$ and $\tau_{\epsilon_2} = 0$); and the case of "constant" arrival of information, where traders receive a signal of constant precision across time (i.e. $\tau_{\epsilon_t} = c$ for $t \in \{1, 2\}$).

3 The Market with Short-Term Traders

In this section I study the effect of introducing a positive measure μ of short term traders on the equilibrium derived in proposition 1. Short termists are endowed with the same information as long term traders but maximize the expected utility of short run profits.

Indicating with $x_{is,1}$ ($x_{il,1}$) the first period position held by a short (long) termist, the following result applies:

Proposition 3 *Linear equilibria of the market where a measure $0 < \mu < 1$ of short termists and a measure $1 - \mu$ of long termists trade exist and are characterized by the following pair of prices and strategies:*

1. *prices: $p_o = \bar{v}$, $p_1 = \lambda_1 z_1 + (1 - \lambda_1 a_1)\bar{v}$, $p_2 = \lambda_2 z_2 + (1 - \lambda_2 \Delta a_2) p_{1l}$ and $p_3 = v$;*

2. *second period strategies are as in proposition 1; short and long term traders' first period strategies are given respectively by:*

$$x_{is,1} = a_{1s}(s_{i1} - p_1) + \gamma\rho\tau_1(E[v|z_1] - p_1) + \left(\frac{\gamma\rho\tau_{i1}(1 - \lambda_2\Delta a_2)}{\lambda_2\Delta a_2}\right)(p_{1l} - p_1), \quad (2)$$

$$x_{il,1} = a_{1l}(s_{i1} - p_1) + \gamma\tau_1(E[v|z_1] - p_1) + \left(\frac{\gamma\tau_u(1 - \lambda_2\Delta a_2)^2}{\lambda_2^2}\right)(p_{1l} - p_1), \quad (3)$$

where $p_{1l} = \lambda_{1l}z_1 + (1-\lambda_{1l}a_1)\bar{v}$, $\lambda_{1l} = (1+\gamma\tau_u a_1)/(a_1+\gamma\tau_1)$, $\lambda_2 = (1+\gamma\tau_u\Delta a_2)/(a_2+\gamma\tau_2)$, $a_1 = \mu a_{1s} + (1-\mu)a_{1l}$, $a_{1l} = \gamma\tau_{\epsilon_1}$, $\rho = (a_{1s}/a_{1l})$,

$$a_{1s} = \gamma\alpha(\tau_{\epsilon_1}^{-1} + \tau_2^{-1})^{-1}, \quad (4)$$

$\alpha = (\beta/\lambda_2)$*, and an explicit expression for* λ_1 *is given in the appendix.*

Proof. See the appendix. QED

As in proposition 1 in the first period long termists speculate on private information the more aggressively (a) the higher is the precision of their private signal and (b) the more risk tolerant they are. [12]

Two factors affect short termists' trading intensity. First, they react positively to γ and τ_{ϵ_1} and take into account the informativeness of second period price (τ_2). The reason is as in Vives (1995): given that they liquidate their position in the second period, they try to predict p_2. However, their signal is about v, therefore the closer is p_2 to v (the higher is τ_2) the more informative is their private signal about p_2 and the more intensely they trade. Second, to the extent that speculators' risk aversion affects second period depth, they scale up (down) their trading intensity depending on the value of α.

The second and third terms in (2) and (3) capture traders' market making activity. Differently from proposition 1, traders now have two market making motives. On the one hand, they absorb the liquidity shock. On the other hand, they stand ready to absorb those inventories that, owing to the different horizons traders have, are unloaded in the market.

Notice that (4) implicitly defines a_{1s}. Indeed, when a short termist chooses his position in the first period, he anticipates that he will unload it in the next period. This makes first period trading intensity depend on second period depth. However, to the extent that λ_2 is a function of a_{1s}, price reaction to second period order flow in turn depends on traders' first period behavior. Therefore, *second period depth* and *first period trading intensity* are simultaneously determined in equilibrium by the solution of (4).

[12] The variables p_{1l} and λ_{1l} represent, respectively, the equilibrium price and the market depth in a market with only long term traders computed with the parameters of the heterogeneous horizons market.

Corollary 1 *In every equilibrium of the market with short term traders: (1)* $a_{1l} > 0$ *and* $a_{1s} > 0$*, and (2)* $\alpha > 0$.

Proof. See the appendix QED

In equilibrium both long and short term traders put a positive weight on first period private information. Differently from what happens in the market with long term traders, here $\alpha > 0$ *even* if no new information arrives to traders in the second period (i.e. when $\Delta a_2 = 0$).

To understand the effect of short term horizons on the market, it is useful to start by considering the extreme case where $\mu = 1$.

3.1 The Case $\mu = 1$

Suppose that $\mu = 1$. When the arrival of information is concentrated in the first period (i.e. $\tau_{\epsilon_2} = 0$) a closed form solution to (4) can be obtained as shown by the following proposition.

Proposition 4 ***(Multiplicity of Equilibria)*** *In the market with only short term traders, when* $\tau_{\epsilon_2} = 0$*, there exist two linear equilibria where: (1) prices are given by* $p_3 = v$*,* $p_o = \bar{v}$*,* $p_2 = \lambda_2 z_2 + (1 - \lambda_2 \Delta a_2) p_{1l}$ *and*

$$p_1 = (1 - \lambda_2 \Delta a_2) p_{1l} + \lambda_2 \Delta a_2 \left(\frac{z_1 + \gamma \rho \tau_1 E[v|z_1]}{\gamma \rho \tau_{i1}} \right); \tag{5}$$

(2) strategies are as in proposition 3, and (3) first period trading intensities are given by $a_{11s} = ((1 + 2\gamma^2 \tau_{\epsilon_1} \tau_u) + (1 + 4\gamma^2 \tau_{\epsilon_1} \tau_u)^{1/2})/(2\gamma \tau_u)$*,* $a_{12s} = ((1 + 2\gamma^2 \tau_{\epsilon_1} \tau_u) - (1 + 4\gamma^2 \tau_{\epsilon_1} \tau_u)^{1/2})/(2\gamma \tau_u)$*, and satisfy:* $a_{12s} < a_{1l} < a_{11s}$*, and* $\alpha(a_{12s}) < 1 < \alpha(a_{11s})$.

Proof. See the appendix. QED

With concentrated arrival of information, short term horizons induce equilibrium multiplicity: a *high* and a *low* trading intensity equilibrium arise (respectively HTIE, designated by subindex 1, and LTIE, designated by subindex 2). The intuition is as follows. In choosing their first period speculative position, traders need to forecast second period price. However, p_2 is in turn a function of first period speculative trading intensity. Thus, to improve their forecast of p_2 traders need to *guess* whether second period price will over or underreact to the order flow (respectively whether α will be lower or higher than 1). [13] If

[13] Price over and underreaction here refers to how prices would react in a market with risk neutral market makers where depth incorporates no risk premium.

they anticipate $\alpha < 1$ their first period position becomes riskier and they underreact to their first period signal ($a_{12s} < a_{1l}$). As $a_2 = a_{1l} > a_{12s}$, the aggregate speculative position increases in the second period ($\Delta a_2 > 0$). This leads to a positive *adverse selection* effect that adds to the *risk premium* in making the price overreact ($\alpha < 1$). Conversely, if they anticipate $\alpha > 1$, they overreact to their first period signal ($a_{11s} > a_{1l}$). In this case $a_2 = a_{1l} < a_{11s}$ and traders' aggregate speculative position *decreases* in the second period ($\Delta a_2 < 0$). This leads to a *negative* adverse selection effect [14] that offsets the positive *risk premium* and increases second period market depth. As a consequence, $\alpha > 1$. [15]

A consequence of the above result is that along the HTIE, traders' speculative position *decreases* across period 1 and 2 ($\Delta a_2 < 0$). This is due to the different estimation problem that agents face in the two periods. In period 1 agents forecast a price that is *endogenous* to their trading activity. Along the HTIE, they anticipate that the market underreacts in the second period, hence their speculative position in the first period becomes less risky and they *overreact* to their private signal, choosing a *high* $a_{1\cdot s}$. Conversely, second period traders only need to forecast the asset pay-off when choosing their position. Hence, the intensity with which they speculate on private information ($a_2 = a_{1l}$) is lower than $a_{1\cdot s}$. As a consequence $\Delta a_2 = a_2 - a_{1\cdot s} < 0$. [16]

Indicate with $\lambda_2^{-1}(a_{11s})$ ($\lambda_2^{-1}(a_{12s})$) second period depth along the HTIE (LTIE). The next result characterizes depth along the two equilibria.

Proposition 5 ***(Market Depth)*** *With concentrated arrival of information, in the market with short term traders: (1) $\lambda_2(a_{11s}) < 0 < \lambda_2(a_{12s})$, and (2) $|\lambda_2(a_{11s})| < \lambda_2(a_{12s})$.*

Proof. See the appendix. QED

Along the HTIE $\lambda_2 < 0$, i.e. the price reacts negatively to the informational content of the order flow. This type of anomaly, typical in a multi-asset framework (see e.g. Admati,

[14] Notice that Δa_2 can only be negative in the presence of short term traders since, as shown in proposition 1, $a_2 \geq a_{1l}$.

[15] An anonymous referee suggested the following alternative intuition to proposition 4. From proposition 3 we observe (a) $a_{1\cdot s}$ decreases with λ_2 and (b) λ_2 decreases with $a_{1\cdot s}$. Therefore, these two variables are interdependent and simultaneously determined in equilibrium. This gives rise to equilibrium multiplicity. In particular, two equilibria arise: (i) a high trading intensity equilibrium where both $a_{1\cdot s}$ and λ_2 are large and (ii) a low trading intensity equilibrium where both variables are small. The fact that these two variables are either jointly small or jointly large is due to points (a) and (b).

[16] Therefore, it is "as if" the impact of period 1 speculative position on second period depth created an incentive to speculate more intensely. In the second period, traders have no way to affect the liquidation price they face in period 3. Therefore, such an incentive disappears and makes (along the HTIE) second period desired speculative position smaller than the first period one.

1985), here depends on the fact that, as noted above, along the HTIE, the change in investors' desired speculative position across dates 1 and 2 (i.e Δa_2) is negative. When traders observe the public order flow in the second period, they know that along the HTIE, traders' desired speculative position is decreasing. Thus, if they observe $z_2 > 0$, they attribute the signal either to a negative pay off ($v < 0$) or to a positive demand from noise traders ($u_2 > 0$). [17] As a result, they revise downwards their expectation of the asset pay-off so that $\lambda_2 < 0$.

Notice also that along the HTIE owing to the compensation between the positive *risk premium* and the negative *adverse selection* effect, the market is deeper in the second period.

Indicate with $x_{is,1}(a_{11s})$ ($x_{is,1}(a_{12s})$) the first period position of a short term trader in the HTIE (LTIE) and with x_{i1} the first period position of a trader in a market with long termists only. The following result characterizes expected positions and volume along the two equilibria.

Proposition 6 ***(Expected Positions and Volume)*** *With concentrated arrival of information, in the market with short term traders: (1)* $E\left[|x_{is,1}(a_{12s})|\right] < E\left[|x_{i1}|\right] < E\left[|x_{is,1}(a_{11s})|\right]$, *and (2)* $E\left[|x_{i2} - x_{is,1}(a_{11s})|\right] > E\left[|x_{i2} - x_{is,1}(a_{12s})|\right] > E\left[|x_{i2} - x_{i1}|\right]$.

Proof. See the appendix. QED

To build the intuition for the above result define $x^*_{il,1}$ the position of a long term trader in a market with $\mu = 1$; then (2) can be expressed as follows: $x_{is,1} = \rho x^*_{il,1} + (\gamma\tau_v(\rho - 1))/((a_2 + \gamma\tau_2)(z_1 - E[z_1]))$. Therefore, in the HTIE (i.e. when $\rho(\equiv a_{1s}/a_{1l}) > 1$), short term speculators scale up their trades with respect to a long termist and accommodate unexpectedly high order flows. The opposite happens along the LTIE (i.e. if $\rho < 1$). As in the second period both trader types' positions coincide, volume is always higher with short term traders than with long termists.

I conclude this section by studying the effects of short horizons on returns' volatility and price informativeness. Indicate with $\tau_n(a_{11s})$ ($\tau_n(a_{12s})$) and with $\tau_n(a_{1l})$ respectively, the period n price precision calculated in the HTIE (LTIE) and the one calculated in a market with long term traders only. Furthermore, define $\text{Var}[p_2|p_1; a_{11s}]$ ($\text{Var}[p_2|p_1; a_{12s}]$) as the conditional volatility of returns calculated along the HTIE (LTIE).

Proposition 7 ***(Price Informativeness and Volatility of Returns)*** *With concentrated arrival of information, in the market with short term traders: (1)* $\tau_n(a_{12s}) < \tau_n(a_{1l}) <$

[17] Indeed, since $z_2 = \Delta a_2 v + u_2$, if $z_2 > 0$, given that along the HTIE $\Delta a_2 < 0$, either $v < 0$ or $u_2 > 0$.

$\tau_n(a_{11s})$, *for* $n = 1, 2$, *and (2)* $Var[p_2|p_1; a_{11s}] < Var[p_2|p_1; a_{12s}]$.

Proof. For part 1 when $n = 1$, $\tau_1(a_{1\cdot s}) = \tau_v + a_{1\cdot s}^2 \tau_u$, while $\tau_1(a_{1l}) = \tau_v + a_{1l}^2 \tau_u$. Then, $\tau_1(a_{1\cdot s}) - \tau_1(a_{1l}) > 0$ if and only if $a_{1\cdot s} > a_{1l}$. When $n = 2$, if $\tau_{\epsilon_2} = 0$, $a_{2l} = a_{1l} = \gamma\tau_{\epsilon_1}$. Therefore, $\tau_2(a_{1\cdot s}) - \tau_2(a_{1l}) = 2a_{1l}\tau_u(a_{1\cdot s} - a_{1l}) > 0$ if and only if $a_{1\cdot s} > a_{1l}$. For part 2, see the appendix. QED

Thus, a market populated by short term traders only, along the HTIE delivers more informative prices than one where only long termists act. For $n = 1$, the reason is obvious. For $n = 2$, it is interesting to contrast the result with Vives (1995) who finds that when $\tau_{\epsilon_2} = 0$, the precision of the final price is always higher with long term traders. The intuition for his result is based on the following three facts: *(i)* price precision is a quadratic function of net trading intensities, [18] *(ii)* total trading intensity is the same independently of speculators' horizons and *(iii)* short term traders always trade less intensely than long termists. When $\tau_{\epsilon_2} = 0$ long termists concentrate all their trading activity in the first period. On the contrary, short termists spread it across both periods. Thus, in the market with long term traders, the intertemporal distribution of net trades is more unequal than in the market with short termists and price informativeness is higher. In the present model, this is what happens in the LTIE. However, owing to the compensating effect of the risk premium, the HTIE may also realize. Then, the degree of inequality in the distribution of net trades becomes higher when short term traders are in the market and price informativeness is higher.

The second result shows that along the LTIE, the conditional volatility of returns is *higher* than along the HTIE. The intuition is that if short term traders anticipate lower second period depth, they scale back their trading intensity. This reduces the total risk bearing capacity of the market and the second period depth leading to a *high* volatility equilibrium.

Remark 2 According to proposition 7, in the presence of short termists, a market with high conditional volatility of returns delivers less informative prices. [19] Other authors have related returns' volatility to price informativeness (see e.g. Admati and Pfleiderer, 1988, and Foster and Viswanathan, 1990). Admati and Pfleiderer (1988) find that a high returns' volatility is *positively* related to a high price informativeness. This is due to the "clustering"

[18] Thus, it works like an inequality index in the distribution of net trading intensities, being higher when their distribution across time is more unequal

[19] Numerical simulations show that the same results hold when the flow of information arrival is constant. See section 4.

effect implied by the presence of *discretionary* liquidity traders in the market: when more liquidity traders concentrate in a given period during the day, strategic informed speculators find it more profitable to trade. As a consequence, more information is impounded into the price and returns are more volatile. However, speculators in their model are risk neutral and do not bear the consequences of trading in a more volatile market. By contrast, in this model, *risk averse* informed traders may be *crowded out* by highly volatile markets rendering prices less informative. Hence, proposition 7 shows that the usual explanation of a more volatile market as one where more information is gathered may break down in the presence of risk averse, short term informed traders.

Remark 3 Comparison of the level of volatility with short term trading to that with long term trading is difficult to handle analytically and numerically does not give clear-cut results. Simulations have been run for the concentrated arrival of information case with $\tau_v, \tau_u, \tau_{\epsilon_1}, \gamma \in \{.1, .4, .7, 1\}$. In each set of simulations the value of the variance has been computed letting in turn τ_v, τ_u, τ_{ϵ_1} or γ vary between $\{1, 2, \ldots, 10\}$. When $\tau_v \in \{1, 2, \ldots, 10\}$, $\text{Var}[p_2|p_1; a_{11s}] < \text{Var}[p_2|p_1; a_{1l}] < \text{Var}[p_2|p_1; a_{12s}]$. However, for other parameterizations, this result is no longer true. [20]

Remark 4 The result on volatility is reminiscent of Dennert (1991) who shows that in the *steady state* solution of a stock market model with OLG and asymmetric information two equilibria arise: one with high and one with low price volatility. [21]

Remark 5 The multiplicity result relies on the assumptions of traders' *risk aversion* and *perfectly competitive* behavior. Adding to the model a sector of uninformed, competitive, risk averse dealers with risk tolerance γ^U, it is possible to show that as $\gamma^U \to \infty$, the set of equilibria reduces to a singleton (Cespa, 1999). In particular, as in Vives (1995), [22] the HTIE cannot arise due to dealers' risk neutrality. Indeed, even if traders overreacted to their first period signal, there would not be any positive *risk premium* in the second period to compensate the negative *adverse selection* effect and make the price underreactive.

Turning to the competitive behavior assumption, intuitively, the introduction of a "large" informed trader in the market should eliminate the HTIE. Indeed, Caballé and

[20] For example, letting $\tau_{\epsilon_1} \in \{1, 2, \ldots, 10\}$, for high values of the signal precision $\text{Var}[p_2|p_1; a_{11s}] > \text{Var}[p_2|p_1; a_{1l}]$. Similar patterns arise for the other parameter values and for the case of constant arrival of information.

[21] According to Dennert (1991), an economy is in a *steady state*, if prices are identically distributed i.e. $p_t \sim p \sim N(E[p], \text{Var}[p]), \forall t$.

[22] See Vives (1995), Remark 3.1, p. 139.

Krishnan (1992) in their multi-asset extension of Kyle (1985) show that to avoid price manipulation the matrix that maps order flows into prices must be *positive definite*. As a consequence, each price *positively* reacts to its own order flow. Thus, imperfect competition on traders' side should rule out the possibility of having a *negative* depth.

3.2 The Case $\mu < 1$

Building on the insight gained by studying the market with only short term traders, in this section I focus on the *general* model with short and long termists. The next result generalizes proposition 4 to the case $0 < \mu < 1$.

Corollary 2 *In the market where a sector of short term traders and one of long termists (respectively of measure $0 < \mu < 1$ and $1 - \mu$) interact, when the arrival of information is concentrated in the first period (i.e. when $\tau_{\epsilon_2} = 0$), there exist two linear equilibria. Traders' first period trading intensities are such that $a_{12s} < a_{1l} < a_{11s}$ and $a_{1l} = \gamma\tau_{\epsilon_1}$.*

Proof. See the appendix. QED

Indicate with $a_{1k} \equiv \mu a_{1ks} + (1 - \mu)a_{1l}$, $\lambda_2^{-1}(a_{1k})$, $\text{Var}[p_2|p_1; a_{1k}]$ and $\tau_n(a_{1k})$, $k, n = 1, 2$, respectively total first period trading intensity, second period depth, returns' volatility and period n price informativeness along equilibrium k.

With concentrated arrival of information, a straightforward generalization of propositions 5 and 7 gives the following corollary:

Corollary 3 *In the market where a sector of short term traders and one of long termists (respectively of measure $0 < \mu < 1$ and $1 - \mu$) interact, when the arrival of information is concentrated in the first period $\forall \mu \in (0, 1)$: (1) $\lambda_2(a_{12}) > 0$ and $\lambda_2(a_{11}) < 0$; (2) $Var[p_2|p_1; a_{11}] < Var[p_2|p_1; a_{12}]$, $|\lambda_2(a_{11})| < \lambda_2(a_{12})$, and $\tau_n(a_{11}) > \tau_n(a_{12})$, $k, n = 1, 2$.*

Proof. See the appendix. QED

All the results in the above corollary mirror what has been shown for the case $\mu = 1$ and the intuitions given for that case apply here. [23]

The next proposition characterizes the effects of an increase in the size of the short-term trading sector on price informativeness.

[23] Numerical simulations were run for the case of constant arrival of information (i.e. when $\tau_{\epsilon_2} = \tau_{\epsilon_1}$) and confirmed the results of corollaries 2 and 3. See section 4.

Proposition 8 *In every equilibrium of the market with short and long term traders, when the arrival of information is concentrated in the first period: (1)* $\partial\tau_1(a_{11})/\partial\mu > 0$, $\partial\tau_1(a_{12})/\partial\mu < 0$*; (2)* $\partial\tau_2(a_{11})/\partial\mu > 0$*, and* $\partial\tau_2(a_{12})/\partial\mu < 0$ *for* $0 < \mu < 1/2 + \gamma a_{1l}\tau_u/4$ *while* $(\partial\tau_2(a_{12})/\partial\mu) > 0$ *otherwise.*

Proof. See the appendix. QED

Along the HTIE, an increase in the measure of short term traders induces more informative second period equilibrium prices. This follows directly from proposition 7. Along the LTIE, an increase in μ may lead to more as well as less informative second period prices. The reason is that the inequality in the intertemporal distribution of net trades is high for μ close to zero (remember that in this case Δa_2 is close to zero) and decreases as μ increases; for $\mu = 1/2 + \gamma a_{1l}\tau_u/4$ it reaches its minimum ($\Delta a_2 = a_{1l}$ and $a_1 = \gamma a_{1l}^2/(2 + \gamma a_{1l}\tau_u)$) and then increases again.

4 Numerical Simulations

In this section I collect the results of numerical simulations. Three groups of results are presented. The first two verify that the results obtained in sections 3.1 and 3.2 carry over to the case of constant arrival of information. The third group analyzes volume patterns in the general model of section 3. [24]

4.1 The Model with Constant Arrival of Information

First I run simulations to verify whether the results obtained in propositions 4, 5, 6 (2) and 7 also hold when $\mu = 1$ and traders receive information of a constant precision in both periods with $\tau_v, \tau_u, \tau_{\epsilon_1} = \tau_{\epsilon_2}, \gamma \in \{.1, .4, .7, 1\}$. In each set of simulations the values of the trading intensities, price precisions, depth, volume and volatility in the market with short term traders and in the one with long term traders, have been computed letting in turn τ_v, τ_u, $\tau_{\epsilon_1} = \tau_{\epsilon_2}$ or γ vary between $\{1, 2, \ldots, 10\}$ (for example, when γ varied in the set $\{1, 2, \ldots, 10\}$, the remaining parameters varied in the set $\{.1, .4, .7, 1\}$). With this parameters choice: *(1)* $a_{12s} < a_{1l} < a_{11s}$; *(2a)* $\tau_1(a_{11s}) > \tau_1(a_{1l}) > \tau_1(a_{12s})$, *(2b)* $\tau_2(a_{11s}) > \tau_2(a_{12s}) > \tau_2(a_{1l})$; *(3)* $|\lambda_2(a_{11s})| < \lambda_2(a_{12s})$; *(4)* $E\left[|x_{i2} - x_{is,1}(a_{11s})|\right] > E\left[|x_{i2} - x_{is,1}(a_{12s})|\right] > E\left[|x_{i2} - x_{i1}|\right]$; *(5)* $\text{Var}\left[p_2|p_1; a_{11s}\right] > \text{Var}\left[p_2|p_1; a_{12s}\right]$. As in the case

[24] Simulations were run with the aid of Mathematica®.

of concentrated arrival of information, in the first period a high and a low trading intensity equilibrium arise. This makes the first period price more informative with short term traders than with long term ones along the HTIE (and the reverse happen along the LTIE). In the second period, as in Vives (1995), along the LTIE the sequence $\{p_1, p_2\}$ is more informative owing to the higher inequality in the intertemporal distribution of net trades that short term trading implies. Furthermore, in the HTIE this inequality increases rendering prices even more informative. Points 3, 4 and 5 above confirm the results obtained for the case of concentrated arrival of information.

Figures 1 and 2 (a) show the results of one of these simulations.

Please insert figures 1 and 2 here.

Second, I run simulations to check whether corollaries 2 and 3 hold when the flow of information arrival is constant. In this case too numerical simulations confirm analytical results showing that two equilibria exist where $|\lambda_2(a_{11})| < \lambda_2(a_{12})$, $\tau_n(a_{11}) > \tau_n(a_{12})$, $n = 1, 2$ and $\text{Var}[p_2|p_1; a_{11}] < \text{Var}[p_2|p_1; a_{12}]$. Figures 2 (b–d) and 3 (a) show the results of one of these simulations.

Second period depth decreases with μ both along the HTIE and the LTIE. Indeed, along the HTIE when μ tends to 1, the risk premium decreases. As for the LTIE lower risk sharing in the second period increases the risk premium, increasing λ_2.

Price precision increases with μ along the HTIE in both periods. In period 1 this is just the result of the *increased* aggregate trading intensity; in period 2 a more unequal intertemporal distribution of net trades arises. Along the LTIE price precision decreases in period 1 and increases in period 2 with μ. In the first case this is the result of the *decreased* aggregate trading intensity. In the second case, this mirrors the effect found in Vives (1995).

Please insert figure 3 here.

4.2 Volume

The third group of results compares expected total volume across equilibria using the general model of section 3. [25] Figure 3 (b,c) depicts the evolution of volume along the HTIE (solid curve) and the LTIE as μ goes from .1 to .9 both when the arrival of information is concentrated in the first period and when it is constant across periods. As one can verify, expected total volume is higher along the HTIE than along the LTIE. While the pictures are the result of a given parameter configuration, [26] results do not change qualitatively if one considers other parameter values. Short termists trading activity has an externality on long termists market making behavior. In particular, along the HTIE since the market in the second period is deeper, long term traders increase the size of their market making activity. This, coupled with the increased trading activity of short termists, generates a *high* volume equilibrium result. Along the LTIE, the second period price overreacts to the order flow. This increases the riskiness of the long term traders' first period positions, reducing their market making activity. As a result, a *low* volume equilibrium realizes.

Notice also that, in line with proposition 6, both along the HTIE and the LTIE volume increases as μ tends to 1.

Remark 6 The evolution of market patterns displayed in the numerical simulations highlights the existence of a discontinuity at $\mu = 0$. [27] The intuition is as follows: when $\mu = 0$, a short term trader anticipates that his trading behavior won't have any effect on second period price. Therefore, he scales down his position with respect to a long term trader. As μ increases, trading horizons *do* influence market patterns and, depending on which equilibrium realizes, lead short termists to overreact (underreact) to their signal. In particular, even for a small value of μ the effect of short termists' overreaction (underreaction) is sufficiently strong to substantially affect first period total trading intensity a_1 and produce the observed discontinuity.

5 Equilibrium Stability

Given that in the presence of short term traders the model displays equilibrium multiplicity, two natural questions arise. Namely, which equilibrium is more "plausible"? Which one is

[25] Computations of the volume formula are available from the author upon request.

[26] In particular, $\tau_{\epsilon_1} = \tau_v = \tau_u = \gamma = 1$ and $\tau_{\epsilon_2} \in \{0, 1\}$.

[27] In other words, as $\mu \to 0$, endogenous variables along the HTIE and the LTIE (like market depth or price informativeness) do not converge to the same value (i.e. their equilibrium value in the market with long term traders).

"stable"? To answer the first question, one has to determine on which of the two equilibria short term traders are *more likely* to coordinate from an ex-ante point of view. To answer the second question one has to find out which of the two equilibria resists shocks to the fundamental.

To address the plausibility issue I compare the ex-ante expected utility of short termists along the two equilibria. Suppose an unexpected shock in period 1 hits a proportion μ of traders in the market, forcing them to liquidate their position. If the ex-ante expected utility of a trader along the HTIE (LTIE) is higher than along the LTIE (HTIE), then traders are more likely to coordinate on the HTIE (LTIE).

Indicate with $\pi_{is,1}$ the first period profit of a short termist. Because of normality assumptions $E[-\exp\{-(\pi_{is,1}/\gamma)\}] = -(\text{Var}[x_{is,1}](\gamma^{-2}\text{Var}[p_2|z_1, s_{i1}] + \text{Var}[x_{is,1}]^{-1}))^{-(1/2)}$, and given that $\text{Var}[x_{is,1}] = \gamma^2\ \text{Var}[p_2|s_{i1}, z_1]^{-2}\text{Var}[E[p_2 - p_1|z_1, s_{i1}]]$, we can conclude that the LTIE is more plausible if and only if

$$\frac{\text{Var}[E[p_2 - p_1|z_1, s_{i1}; a_{12}]]}{\text{Var}[p_2|z_1, s_{i1}; a_{12}]} \geq \frac{\text{Var}[E[p_2 - p_1|z_1, s_{i1}; a_{11}]]}{\text{Var}[p_2|z_1, s_{i1}; a_{11}]}, \tag{6}$$

where, $\text{Var}[E[p_2 - p_1|z_1, s_{i1}; a_{12}]]$ and $\text{Var}[p_2|z_1, s_{i1}; a_{12}]$ indicate respectively the variance of short termists' conditional expected returns and the conditional volatility of short termists' returns along the LTIE. Thus, short term traders rather coordinate on the LTIE if the variance of their expected returns, taking into account the associated risk, is larger along the LTIE than along the HTIE.

Whether condition (6) holds crucially depends on the size of the short term trading sector. If $\mu = 1$ for a wide range of parameter values the LTIE is more plausible. [28] When $\mu < 1$, the effect of short-term traders on prices is mitigated by long termists' trading behavior. Hence, for low values of μ, when either τ_u, τ_v or γ are "high," the HTIE is more plausible. [29]

Turning to the stability issue. Assume, for simplicity, that $\bar{v} = E[v|z_1], \mu = 1$ and define the second period aggregate excess demand function along equilibrium $k = 1, 2$ as follows

$$XD_k \equiv z_2(a_{1k}) + \lambda_2^{-1}(a_{1k})(1 - \lambda_2(a_{1k})\Delta a_{2k})\bar{v} - \lambda_2^{-1}(a_{1k})p_2(a_{1k}), \tag{7}$$

[28] Numerical simulations were run with the same set of parameter values used in section 4. Intuitively, along the HTIE both the numerator and the denominator in (6) are lower than along the LTIE. However, the risk reduction is not high enough to compensate for the reduced variance of expected returns. Hence, short term traders find it more profitable to coordinate on the LTIE.

[29] For these parameter values, the "compensation" that the risk premium has on second period depth is small. Hence, the market is less liquid in the second period and both the numerator and the denominator on the r.h.s. of (6) are higher. This effect coupled with the fact that for low values of μ short term traders' effect on prices is less intense, is enough to make the HTIE more plausible.

where $XD_k = 0$ when the market is in equilibrium and $XD_k \neq 0$ otherwise. [30] Figures 4 and 5 show the graph of (7) both along the LTIE and the HTIE (dotted lines).

Please insert figures 4 and 5 here.

While the excess demand function in the LTIE slopes downwards, owing to $\lambda_2(a_{11s}) < 0$ the one associated with the HTIE slopes upwards. Hence, if the price is above (below) its equilibrium level in the LTIE an excess supply (demand) forces it back to equilibrium. On the contrary, for a price above (below) its equilibrium level along the HTIE the value of the excess demand function increases (decreases) moving the price further away from equilibrium.

The intuition for this effect can be obtained by rewriting traders' second period strategies as follows

$$x_{i2} = a_2(\tilde{s}_{i2} - p_2) + \frac{\gamma\tau_2}{a_2 + \gamma\tau_2}(\beta^{-1} - a_2)(E[v|z_1] - E[v|z^2]). \qquad (8)$$

Consider the market making part of (8) and suppose $E[v|z_1] > E[v|z^2]$: the market believes that the value of the asset has decreased. The larger β^{-1} with respect to a_2, the more likely is that market inference is driven by the effect of noise shocks on second period order flow (rather than by informed traders). [31] Thus, traders take the other side of the market and buy the asset. Conversely, the smaller β^{-1} with respect to a_2, the more likely that second period order flow is information driven. Hence, traders align their trading to the market and sell the asset too. The same thing happens along the HTIE where, since $\Delta a_{21} < 0$, $\beta < 0$. Based on these considerations, the LTIE is stable while the HTIE is unstable.

Though market behavior along the HTIE may appear counterintuitive, a positively sloped excess demand function is in line with models of market crashes (see e.g. Gennotte and Leland, 1990, and Barlevy and Veronesi, 2002). [32] In these models traders facing a price

[30] Notation: $\Delta a_{2k} = a_2 - a_{1k}$, $z_2(a_{1k}) = \Delta a_{2k} v + u_2$ and $p_2(a_{1k})$ is the second period price along equilibrium k. To obtain (7), under the above assumptions the second period market clearing equation reads as follows $z_2(a_{1k}) + \lambda_2^{-1}(a_{1k})(1 - \lambda_2(a_{1k})\Delta a_{2k})\bar{v} = \lambda_2^{-1}(a_{1k})p_2(a_{1k})$. The l.h.s. of this equation represents the second period net aggregate demand while the r.h.s. represents the second period net aggregate supply. With this in mind, define the net aggregate excess demand (demand minus supply) as $XD_k \equiv z_2(a_{1k}) + \lambda_2^{-1}(a_{1k})(1 - \lambda_2(a_{1k})\Delta a_{2k})\bar{v} - \lambda_2^{-1}(a_{1k})p_2(a_{1k})$.

[31] A large value of β^{-1} means that the weight the market puts on z_2 is low, while a_2 low means that informed speculators trade with little aggressiveness on their signal.

[32] Notice, however, that in Gennotte and Leland's case the excess demand function slopes upwards only in the presence of unobservable hedging demand while in the present context, as in Barlevy and Veronesi, all rational traders are fully aware of other traders' behavior.

decrease may rationally choose to reduce their holdings of the risky asset, interpreting such a decline as "bad news" about the asset pay-off. Thus, an initial price decline (e.g. spurred by a shock to fundamentals) may lead to further price reductions under an increasing selling pressure. In the present context this is what happens in figure 5. A shock (unanticipated) to fundamentals moves the equilibrium price upwards (from the dotted to the continuous line). At the old equilibrium price the market experiences an excess supply of the asset which further pushes down the price. In this situation the old equilibrium price is *too low* to justify the new market's quote, traders do not *trust* the new quote and decide to sell, further moving the market away from equilibrium.

Notice that in the example given in figure 5, the price starts declining even though the shock to fundamentals is positive. [33] Episodes of this type are not uncommon as argued by Eichengreen (1990) in his comparison of the stock market crashes of 1929 and 1987. [34] Notice also that along the HTIE the model does not display a "real" *market crash*. Indeed for the price to tumble, a (even very small) shock to fundamentals is required. [35] However, the example still captures some of the features that characterized the 1987 crash. According to the Brady Report (1988), sellers suffered from an "illusion of liquidity" and to the market mutual fund behavior "looked much like that of portfolio insurers, that is, selling without primary regard to price." Along the HTIE short term traders in the first period anticipate a *liquid* market and overreact to their signal expecting a "small" price variation across the two periods. This makes the change in traders' desired speculative position negative leading to a negative adverse selection effect that compensated by the positive risk premium renders the second period market "deep." However, such a *higher* depth comes at the price of instability, since an unanticipated shock to fundamentals moves the market away from equilibrium.

[33] The actual values of the fundamentals have no particular role in the example. The important factor determining the price decline in the presence of a positive excess supply is the *negative* value of second period market depth.

[34] As quoted by Barlevy and Veronesi (2002), Eichengreen notes that "probably the crucial difference between the two episodes was the state of the economy immediately preceding the crash. In the first nine months of 1987, spending was strong. In October 1929, in contrast, a full-blown recession was already under way."

[35] A market crash refers to a situation where the price of a stock changes dramatically despite the absence of a change in the underlying value of its fundamentals. Technically, this requires the existence of a discontinuity between the price and the asset supply, something that in the present model does not happen.

6 Conclusions

In this paper, I have analyzed the effects of short-term behavior on stock market patterns in the context of a dynamic rational expectations equilibrium model. Owing to risk premia, short term horizons induce equilibrium multiplicity. In particular, two different outcomes are possible: in the LTIE, short term traders anticipate second period price overreaction to the order flow, scale back their trades and enforce a high volatility, low price informativeness equilibrium; along the HTIE the opposite happens and a low volatility, high price informativeness equilibrium arises. Therefore, in contrast to the usual explanation of return volatility in efficient markets (see e.g. Admati and Pfleiderer, 1988), short termism coupled with traders' risk aversion makes prices *less* informative in more volatile markets.

To address the multiplicity issue, I have studied the excess demand functions along the two equilibria. Owing to the negative second period depth, along the HTIE the excess demand function slopes upwards, making the HTIE *un*stable.

A number of issues are left for future research. First, the N-period extension should be considered. While analytical results exist for the case of long term traders only (He and Wang, 1995, and Vives, 1995), there is no general analysis for the case presented here. Next, introducing *hedgers*, welfare considerations could be addressed. Dow and Rahi (2000), show that in a static context a tax on speculation, by reducing the informativeness of the price, can improve both *hedgers* and speculators' welfare. In the present context, the final effect should depend on the structure of the equilibrium set. Effects on investment decisions could also be considered: to the extent that stock market prices at the same time accomplish the role of indicators for firms' decisions *and* aggregate information dispersed among traders in the economy, equilibria with low price informativeness should lead to sub-optimal decisions. Finally, by considering a multi-asset framework (Admati, 1985, and Cespa, 1999), one could characterize how risk premia interact across different assets.

APPENDIX

First, I state a well known result on multivariate normal random variables (see e.g. Danthine and Moresi 1992).

Lemma 1 *Let $Q(\boldsymbol{w})$ be a quadratic function of the vector $\boldsymbol{w}$: $Q(\boldsymbol{w}) = D + \mathbf{b}'\boldsymbol{w} - \boldsymbol{w}'\mathbf{A}\boldsymbol{w}$, where $\boldsymbol{w} \sim N(\boldsymbol{\mu}, \boldsymbol{\Sigma})$ and $\boldsymbol{\Sigma}$ is non singular. We then have*

$$E[\exp(Q(\boldsymbol{w}))] = |\boldsymbol{\Sigma}|^{-1/2}|2\mathbf{A} + \boldsymbol{\Sigma}^{-1}|^{-1/2}\times \\ \exp\left\{D + \mathbf{b}'\boldsymbol{\mu} + \boldsymbol{\mu}'\mathbf{A}\boldsymbol{\mu} + \frac{1}{2}(\mathbf{b} - \mathbf{A}\boldsymbol{\mu})'(2\mathbf{A} + \boldsymbol{\Sigma}^{-1})^{-1}(\mathbf{b} - 2\mathbf{A}\boldsymbol{\mu})\right\}.$$

Proof of Proposition 1

Notice that in every linear equilibrium, the sequences z^n and p^n, $n = 1, 2$ are observationally equivalent (o.e.). To see this, assume a candidate linear symmetric equilibrium $x_{i1} = a_1 s_{i1} - \varphi_1(p_1)$, $x_{i2} = a_2\tilde{s}_{i2} - \varphi_2(p_1, p_2)$, where $\varphi_1(p_1)$ and $\varphi_2(p_1, p_2)$ are two linear functions. Market clearing in period 1 implies that $\int_0^1 a_1 s_{i1} di - \varphi_1(p_1) + u_1 = 0$. By the SLLN the previous condition is equivalent to $z_1 - \varphi_1(p_1) = 0$. Therefore, (owing to the linearity of $\varphi_1(\cdot)$) z_1 is o.e. to p_1. Consider now period 2. Market clearing gives $\int_0^1 a_2\tilde{s}_{i2}di - \varphi_2(p_1, p_2) + u_1 + u_2 = 0$, by the SLLN $a_2 v - \varphi_2(p_1, p_2) + u_1 + u_2 = 0$. Adding and subtracting $a_1 v$ to the previous expression gives $z_1 + z_2 - \varphi_2(p_1, p_2) = 0$. Therefore, as in period 2 z_1 and p_1 are known, p_2 is o.e. to z_2. Thus, $\{p_1, p_2\}$ and $\{z_1, z_2\}$ are o.e..

Next, in period $n = 2$ because of normality of the random variables and CARA utility functions, $x_{i2} = \gamma \left(\text{Var}\left[v|z^2, \tilde{s}_{i2}\right]\right)^{-1} \left(E\left[v|z^2, \tilde{s}_{i2}\right] - p_2\right)$. Given that z^2 and p^2 are observationally equivalent traders' expectations are given by $E\left[v|z^2\right]=\tau_2^{-1}\left(\tau_v\bar{v}+ \tau_u \sum_{t=1}^2(\Delta a_t)z_t\right)$ and $E\left[v|z^2, \tilde{s}_{i2}\right] = \tau_{i2}^{-1}\left(\tau_2 E\left[v|z^2\right] + \sum_{t=1}^2 \tau_{\epsilon t}\tilde{s}_{i2}\right)$, where $\tau_2 = \tau_v + \tau_u \sum_{t=1}^2(\Delta a_t)^2$, and $\tau_{i2} = \tau_2 + \sum_{t=1}^2 \tau_{\epsilon t}$. Hence, one can solve for traders' second period strategies and obtain

$$x_{i2} = \gamma\left(\sum_{t=1}^2 \tau_{\epsilon t}\right)(\tilde{s}_{i2} - p_2) + \gamma\tau_2(E[v|z^2] - p_2). \tag{9}$$

The second period market clearing equation reads as follows: $\int_0^1 x_{i2}\, di + u_1 + u_2 = 0$, where u_1 and u_2 are the (uncorrelated) supply increments of the traded asset. By the strong law of large numbers we can rewrite it as $a_2(v - p_2) + \gamma\tau_2(E[v|z^2] - p_2) + u_1 + u_2 = 0$. Adding and subtracting $a_1 v$ to the previous expression $z_1 + z_2 + \gamma\tau_2 E[v|z^2] = (a_2 + \gamma\tau_2)p_2$, and, by using the previously given definitions, $p_2 = (1 - \lambda_2\Delta a_2)(1 - \lambda_{1l}a_1)\bar{v} + (1 - \lambda_2\Delta a_2)\lambda_{1l}z_1 + \lambda_2 z_2$, where $\lambda_2 = (1 + \gamma\tau_u\Delta a_2)/(a_2 + \gamma\tau_2)$ and $\lambda_{1l} = (1 + \gamma\tau_u a_1)/(a_1 + \gamma\tau_1)$.

To obtain period 1 strategies, one substitutes period 2 strategy into the objective function of the informed, obtaining $E[-\exp\{-\gamma^{-1}\pi_{i2}\}|z^2,\tilde{s}_{i2}] = -\exp\{-x_{i2}^2/2\gamma^2\tau_{i2}\}$. Going back one step, the function that traders maximize is $E[-\exp\{-Q_{i1}/\gamma\}\ |z_1,s_{i1}]$, where $Q_{i1} = (p_2-p_1)x_{i1}+x_{i2}^2/2\gamma\tau_{i2}$. Applying lemma 1 as in Holden and Subrahmanyam (1996), the above optimization problem is solved by

$$x_{i1} = \frac{\gamma(E\left[p_2|z_1,s_{i1}\right]-p_1)}{G_1} + E[x_{i2}|z_1,s_{i1}]\frac{G_1-G_2}{G_1}, \tag{10}$$

where G_1 and G_2 are the elements in the first row of the matrix

$$G = \left(\left(\mathrm{Var}\left[p_2, E\left[v|z^2,\tilde{s}_{i2}\right]|z_1,s_{i1}\right]\right)^{-1} + \begin{pmatrix} \tau_{i2} & -\tau_{i2} \\ -\tau_{i2} & \tau_{i2} \end{pmatrix}\right)^{-1}.$$

Tedious calculations allow to obtain $G_1 = \lambda_2^2/(\tau_{i1}\lambda_2^2+\tau_u(1-\lambda_2\Delta a_2)^2)$, $G_2 = (\tau_u\Delta a_2\lambda_2+\tau_{\epsilon_2}\lambda_2^2)/(\tau_{i2}(\tau_{i1}\lambda_2^2+\tau_u(1-\lambda_2\Delta a_2)^2))$, $E[p_2|z_1,s_{i1}] = \lambda_2\Delta a_2 E[v|z_1,s_{i1}]+(1-\lambda_2\Delta a_2)p_{1l}$, and $E[x_{i2}|z_1,s_{i1}] = \gamma\tau_{i2}\ (1-\lambda_2\Delta a_2)\ (E[v|z_1,s_{i1}]-p_{1l})$, where $p_{1l} = \lambda_{1l}z_1+(1-\lambda_{1l}a_1)\bar{v}$. Identifying parameters, informed first period trading intensity is given by $a_1 = \gamma\tau_{\epsilon_1}$. Finally, setting $\lambda_{1l} = \lambda_1$ and rearranging traders' strategies, we can write $x_{i1} = a_1 s_{i1} - \lambda_1^{-1}p_1 + \lambda_1^{-1}(1-a_1\lambda_1)\bar{v} = a_1(s_{i1}-p_1)+\gamma\tau_1(E[v|z_1]-p_1)$, and $x_{i2} = a_2\tilde{s}_{i2}-\lambda_2^{-1}p_2+(\lambda_2^{-1}(1-a_1\lambda_1)-\lambda_1^{-1})p_1+\lambda_1^{-1}(1-a_1\lambda_1)\bar{v} = a_2(\tilde{s}_{i2}-p_2)+\gamma\tau_2(E[v|z^2]-p_2)$.

QED

Proof of Proposition 3

As in period 2 short and long term traders' horizons coincide, $x_{is,2}$ and $x_{il,2}$ are given by (9). Imposing market clearing, $\int_0^\mu x_{is,2}\,di + \int_\mu^1 x_{il,2}\,di + u_1 + u_2 = 0$. The second period market clearing price and depth are given by $p_2 = \lambda_2 z_2 + (1-\lambda_2\Delta a_2)p_{1l}$ and $\lambda_2 = (a_2+\gamma\tau_2)^{-1}(1+\gamma\tau_u\Delta a_2)$.

In the first period, trading horizons differ and the market clearing equation reads as follows $\int_0^\mu x_{is,1}\,di + \int_\mu^1 x_{il,1}\,di + u_1 = 0$. Long term traders' first period strategy is given by (10) and with an argument along the lines of the previous proof one can show that $a_{1l} = \gamma\tau_{\epsilon_1}$. For short term traders, because of short term horizons,

$$x_{is,1} = \gamma(\mathrm{Var}[p_2|p_1,s_{i1}])^{-1}(E[p_2|p_1,s_{i1}]-p_1), \tag{11}$$

where $\mathrm{Var}[p_2|p_1,s_{i1}] = \lambda_2^2((\tau_2+\tau_{\epsilon_1})/\tau_{i1}\tau_u)$. Using the formulas for the conditional expectation and the conditional variance obtained in the proof of proposition 1 and plugging them in (11) gives (2). Collecting parameters and identifying the unknown gives

$a_{1s} = \gamma\alpha(\tau_{\epsilon_1}^{-1} + \tau_2^{-1})^{-1}$, and $p_1 = \lambda_1 z_1 + (1 - \lambda_1 a_1)\bar{v}$, where

$$\begin{aligned}\lambda_1 &= \left(\frac{\lambda_2^2 \Delta a_2 (1 + (\mu\rho + (1-\mu))\gamma a_1 \tau_u)}{\lambda_2 \left(\mu\rho\gamma\tau_{i1} + (1-\mu)(\gamma\tau_2\phi + a_{1l}\lambda_2\Delta a_2)\right)}\right) \\ &+ \mu\lambda_{1l}\left(\frac{(1-\lambda_2\Delta a_2)\lambda_2(a_{1s} + \gamma\rho\tau_1)}{\lambda_2 \left(\mu\rho\gamma\tau_{i1} + (1-\mu)(\gamma\tau_2\phi + a_{1l}\lambda_2\Delta a_2)\right)}\right) \\ &+ (1-\mu)\lambda_{1l}\left(\frac{\gamma\Delta a_2 \tau_u (1-\lambda_2\Delta a_2)^2}{\lambda_2 \left(\mu\rho\gamma\tau_{i1} + (1-\mu)(\gamma\tau_2\phi + a_{1l}\lambda_2\Delta a_2)\right)}\right),\end{aligned}$$

and $\phi = \lambda_2\Delta a_2(1-\alpha) + (1-\lambda_2\Delta a_2)\alpha$.

Notice that (4) implicitly defines short termists' first period trading intensity. Therefore, existence of an equilibrium, depends on the existence of a real solution to (4). Given that $\alpha = \Delta a_2 \tau_u (a_2 + \gamma\tau_2)/(\tau_2(1 + \gamma\tau_u\Delta a_2))$, equation (4) can be rewritten as follows: $F(a_{1s}) \equiv a_{1s}(\tau_2 + \tau_{\epsilon_1})(1 + \gamma\tau_u\Delta a_2) - \gamma\tau_u\Delta a_2\tau_{\epsilon_1}(a_2 + \gamma\tau_2) = 0$. From here one can check that $F(0) < 0$ and that $\lim_{a_{1s}\to 1/\gamma\mu\tau_u + (a_2 - (1-\mu)a_{1l})/\mu} F(a_{1s}) = \tau_{\epsilon_1}(2 + 3a_2\gamma\tau_u + a_2^2\gamma^2\tau_u^2 + \gamma^2\tau_u\tau_v)/\gamma\tau_u^2 > 0$. As $F(\cdot)$ is continuous, there exists a a_{1s}^* in the interval $(0, 1/\gamma\mu\tau_u + (a_2 - (1-\mu)a_{1l})/\mu)$, such that $F(a_{1s}^*) = 0$.

QED

Proof of corollary 1

For part 1, given that $a_{1l} = \gamma\tau_{\epsilon_1}$, the result follows. Next, for a_{1s}, assume that $\Delta a_2 < -1/\gamma\tau_u$, in this case, $\Delta a_2 < 0$ and $\alpha > 0$, hence $a_{1s} > 0$. Alternatively, suppose $-1/\gamma\tau_u < \Delta a_2 < 0$, then $\alpha < 0$, hence $a_{1s} < 0$. However, in this case, $\Delta a_2 = a_2 - a_1 > 0$, contradicting the assumption. Therefore, $a_{1s} > 0$ and $\alpha > 0$. QED

Proof of proposition 4

Parts 1 and 2 directly follow from the proof of proposition 3. For part 3, expliciting (4) $a_{1s} = \gamma\Delta a_2\tau_u(\gamma\tau_2 + a_2)\tau_{\epsilon_1}/((\tau_2 + \tau_{\epsilon_1})(1 + \gamma\tau_u\Delta a_2))$. Assume that the denominator on the r.h.s. of the previous equation is non null (at equilibrium), then rearranging we obtain: $a_{1s}(\tau_2 + \tau_{\epsilon_1})(1 + \gamma\tau_u\Delta a_2) - \gamma\Delta a_2\tau_u\tau_{\epsilon_1}(a_2 + \gamma\tau_2) = 0$, which is a quartic in a_{1s}. If $\tau_{\epsilon_2} = 0$, the previous equilibrium condition becomes

$$-\underbrace{(a_{1s}^2\gamma\tau_u - a_{1s}(1 + 2\gamma^2\tau_{\epsilon_1}\tau_u) + \gamma^3\tau_{\epsilon_1}^2\tau_u)}_{(a)} \times \underbrace{(2a_{1s}^2\tau_u - 2a_{1s}\gamma\tau_{\epsilon_1}\tau_u + \tau_v + \gamma^2\tau_{\epsilon_1}\tau_u + \tau_{\epsilon_1})}_{(b)} = 0.$$

While equation (a) has two real roots, equation (b) only possesses imaginary solutions. In particular $a_{11s}, a_{12s} = ((1 + 2\gamma^2\tau_{\epsilon_1}\tau_u) \pm (1 + 4\gamma^2\tau_{\epsilon_1}\tau_u)^{1/2})/2\gamma\tau_u$. This solution clearly satisfies the condition $1 + \gamma\tau_u\Delta a_2 \neq 0$. Direct comparison of the obtained solutions with the long term case gives $a_{12s} < a_{1l} < a_{11s}$, and $\alpha(a_{12s}) < 1 < \alpha(a_{11s})$.

QED

Proof of proposition 5

For part 1, $\lambda_2(a_{11s}) < 0 \Leftrightarrow 4\gamma^2\tau_{\epsilon_1}\tau_u > 0$, which is always true. Next, as $a_{12s} < a_{1l}$, $\lambda_2(a_{12s}) > 0$. For part 2,$|\lambda_2(a_{11s})| < \lambda_2(a_{12s}) \Leftrightarrow \lambda_2^2(a_{11s}) < \lambda_2^2(a_{12s})$. Substituting equilibrium values for a_{11s} and a_{12s}, the last inequality is always satisfied. QED

Proof of proposition 6

Tedious computations lead to $x_{is,1} = a_{1\cdot s}s_{i1} - z_1$, $x_{i1} = a_{1l}s_{i1} - z_1$, and $x_{i2} = a_2\tilde{s}_{i2} - z_1 - z_2$. Since if $Y \sim N(0, \sigma^2)$, then $E[|Y|] = \sqrt{(2/\pi)\sigma^2}$, one obtains that $E[|x_{is,1}(a_{1\cdot s})|] = ((2/\pi)(\tau_u^{-1} + a_{1\cdot s}\tau_{\epsilon_1}^{-1}))^{1/2}$ and $E[|x_{i1}|] = ((2/\pi)(\tau_u^{-1} + a_{1l}\tau_{\epsilon_1}^{-1}))^{1/2}$. Hence, the result follows. For the second part a similar argument leads to $E[|x_{i2} - x_{is,1}(a_{1\cdot s})|] = ((2/\pi)(\tau_u^{-1} + \gamma a_{1l}(1-\rho)^2))^{1/2}$, $E[|x_{i2} - x_{i1}|] = (2/\pi\tau_u)^{1/2}$. Substituting the solutions of short term traders' first period trading intensities, the result follows. QED

Proof of proposition 7

For part 2, substitute the equilibrium values found in proposition 4 in the expression for $\text{Var}[p_2|p_1; a_{1\cdot s}]$ and check that $\text{Var}[p_2|p_1; a_{11s}] - \text{Var}[p_2|p_1; a_{12s}] < 0$ is always satisfied. QED

Proof of corollary 2

Follows immediately generalizing the last part of proposition 4's proof to the case $0 < \mu < 1$; $a_{11s}, a_{12s} = ((1 + 2\gamma^2\mu\tau_{\epsilon_1}\tau_u) \pm (1 + 4\gamma^2\mu\tau_{\epsilon_1}\tau_u)^{1/2})/(2\gamma\mu\tau_u)$. QED

Proof of corollary 3

For part 1, $\lambda_2(a_{11}) < 0 \Leftrightarrow 4\gamma^2\mu\tau_{\epsilon_1}\tau_u > 0$ which is always true. Next, since $\mu(a_{1l} - a_{12s}) > 0$, $\lambda_2(a_{12s}) > 0$. The last part of the proposition follows from generalizing the proofs of propositions 5 and 7 to the case $0 < \mu < 1$. QED

Proof of Proposition 8

For part 1, $\partial\tau_1(a_{1\cdot})/\partial\mu = 2a_{1\cdot}\tau_u((a_{1\cdot s} - a_{1l}) + \mu(\partial a_{1\cdot s}/\partial\mu))$ and substituting $\partial a_{1\cdot s}/\partial\mu$ gives the result. For part 2, $\partial\tau_2(a_{1\cdot})/\partial\mu = (\partial\tau_1(a_{1\cdot})/\partial\mu) - 2\tau_u\Delta a_2(\partial a_{1\cdot s}/\partial\mu)$. This expression is positive if $\rho > 1$. If $\rho < 1$, computing the derivative, $\partial\tau_2/\partial\mu = -(2\tau_{\epsilon_1}(1 + \gamma^2\tau_{\epsilon_1}\tau_u - \sqrt{1 + 4\mu\gamma^2\tau_{\epsilon_1}\tau_u}))/\sqrt{1 + 4\mu\gamma^2\tau_{\epsilon_1}\tau_u}$, which is positive if and only if $\mu \in (0, 1/2 + \gamma a_{1l}\tau_u/4)$ and negative otherwise. QED

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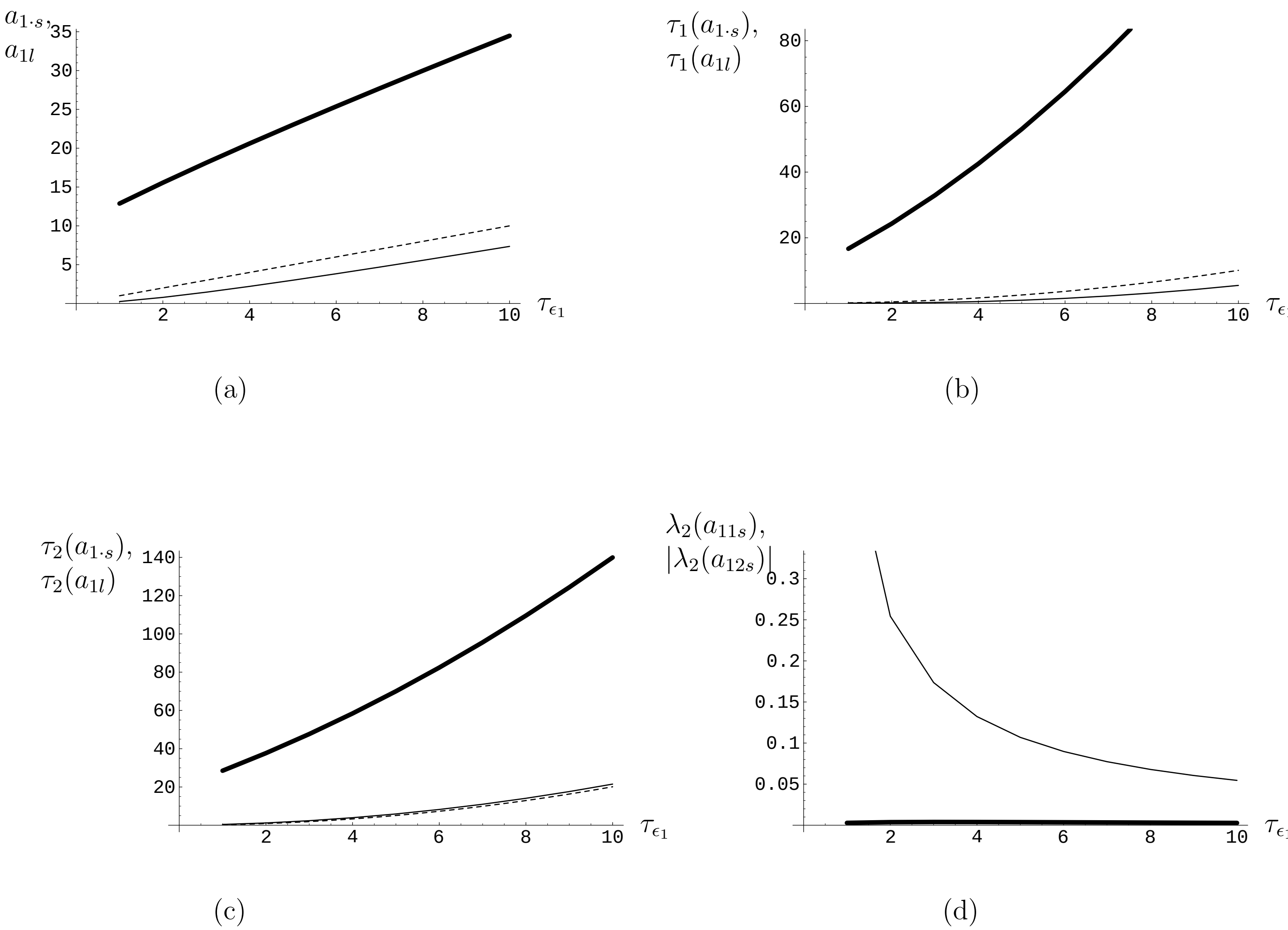

Figure 1: First period trading intensities (a), first and second period price informativeness (b,c) and second period market depth (d) as a function of τ_{ϵ_1}. Parameter values: $\gamma = 1$ and $\tau_v = \tau_u = 1$, while $\tau_{\epsilon_1} = \tau_{\epsilon_2} \in \{1, 2, \ldots, 10\}$. Solid (thin) curves refer to the HTIE (LTIE). Dotted curves refer to the unique equilibrium in the market with long term traders only.

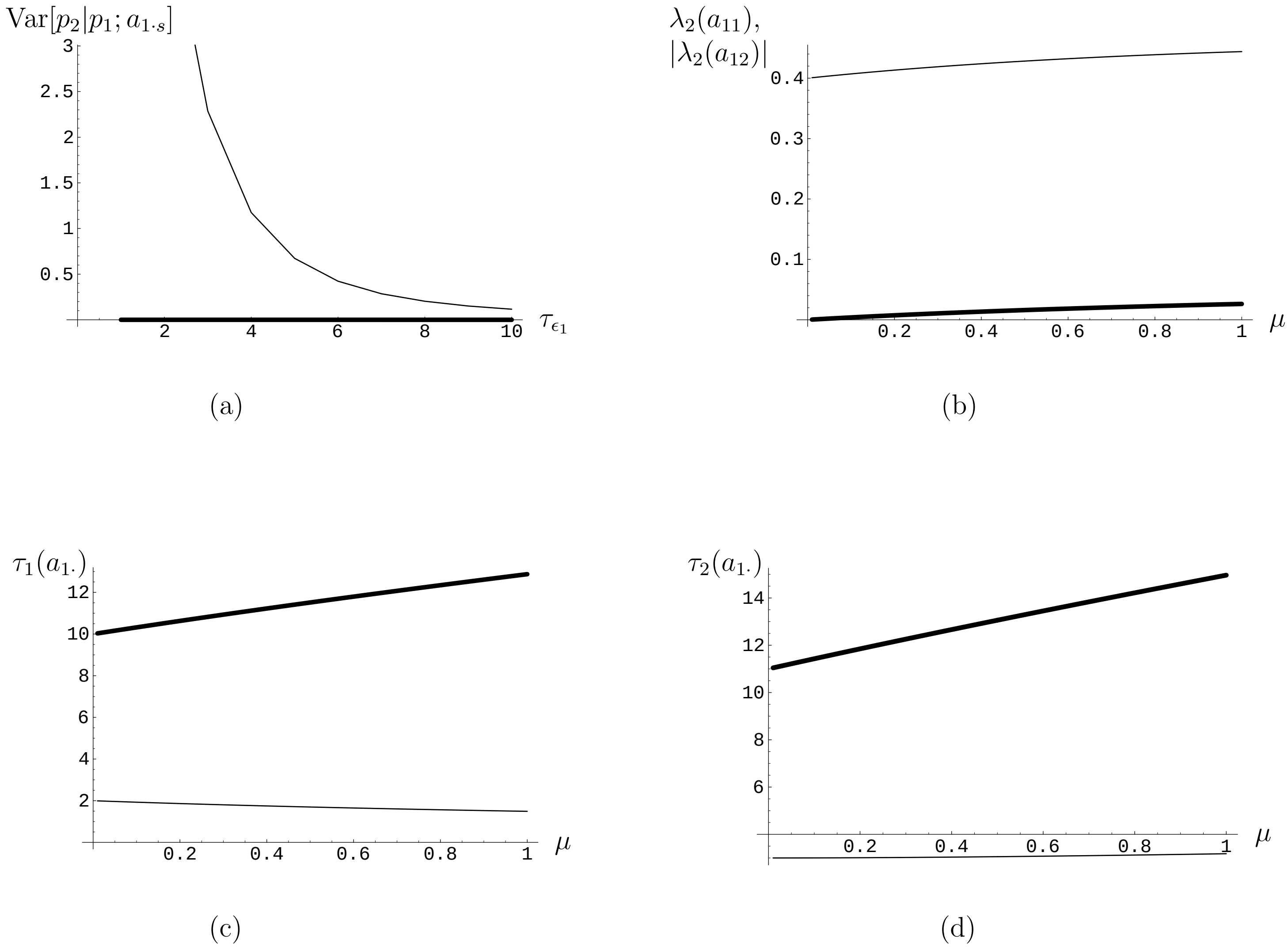

Figure 2: Returns' conditional volatility as a function of τ_{ϵ_1} (a). Parameter values: $\gamma = 1$ and $\tau_v = \tau_u = 1$, while $\tau_{\epsilon_1} = \tau_{\epsilon_2} \in \{1, 2, \ldots, 10\}$. Second period market depth, first and second period price informativeness as a function of μ (b,c and d). Parameter values: $\tau_{\epsilon_1} = \tau_{\epsilon_2}, \tau_v, \tau_u, \gamma \in \{.1, .4, .7, 1\}$ and $\mu \in \{.1, .2, \ldots, .9\}$. Solid (thin) curves refer to the HTIE (LTIE).

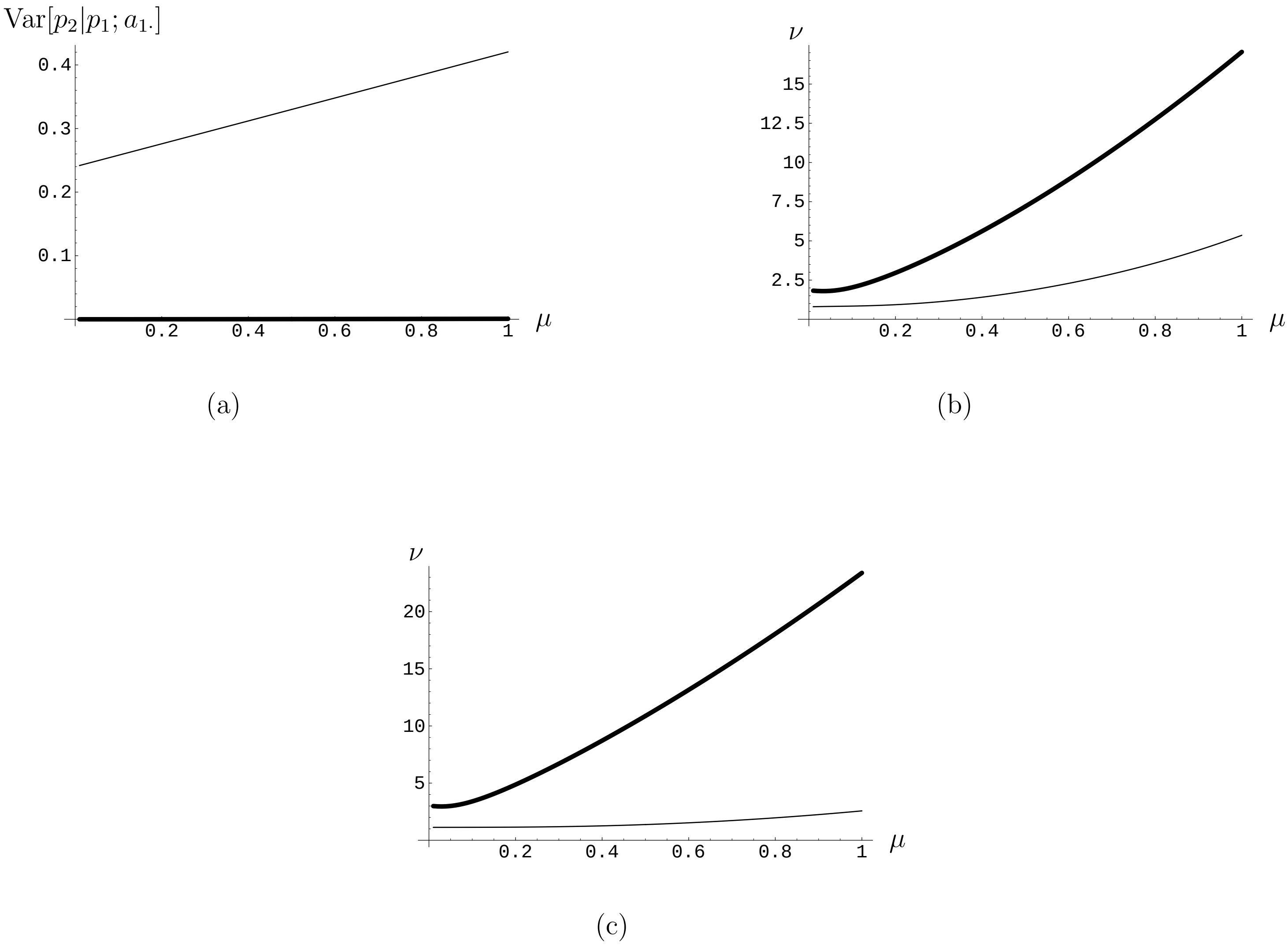

Figure 3: Conditional volatility of returns as a function of μ (a); expected total volume as a function of μ with concentrated and constant arrival of information (b and c). Parameter values as in figure 2 (b–d). Solid (thin) curves refer to the HTIE (LTIE).

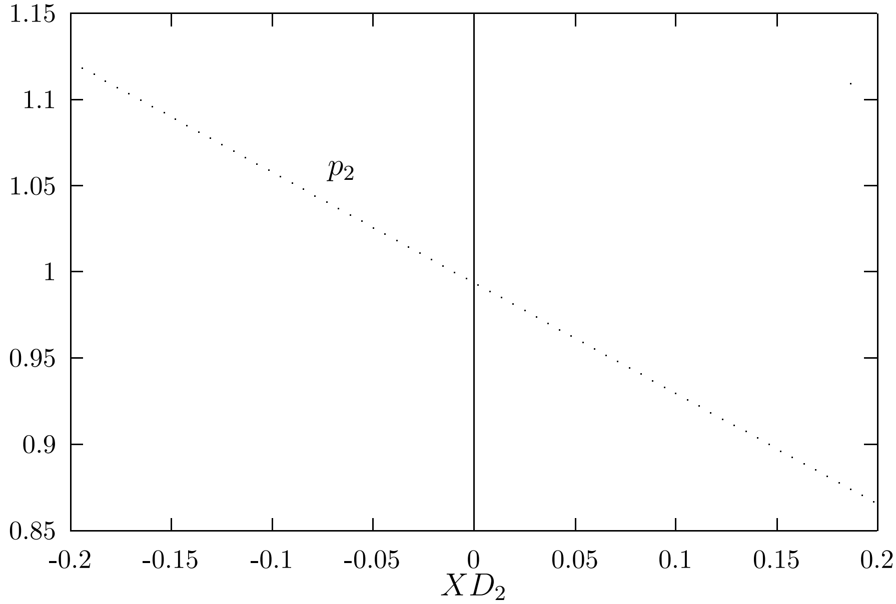

Figure 4: Excess demand function along the LTIE. Parameter values: $\bar{v} = v = \tau_{\epsilon_2} = 0$, $\tau_{\epsilon_1} = \tau_v = \tau_u = \gamma = 1$, and $u_2 = 1.56$.

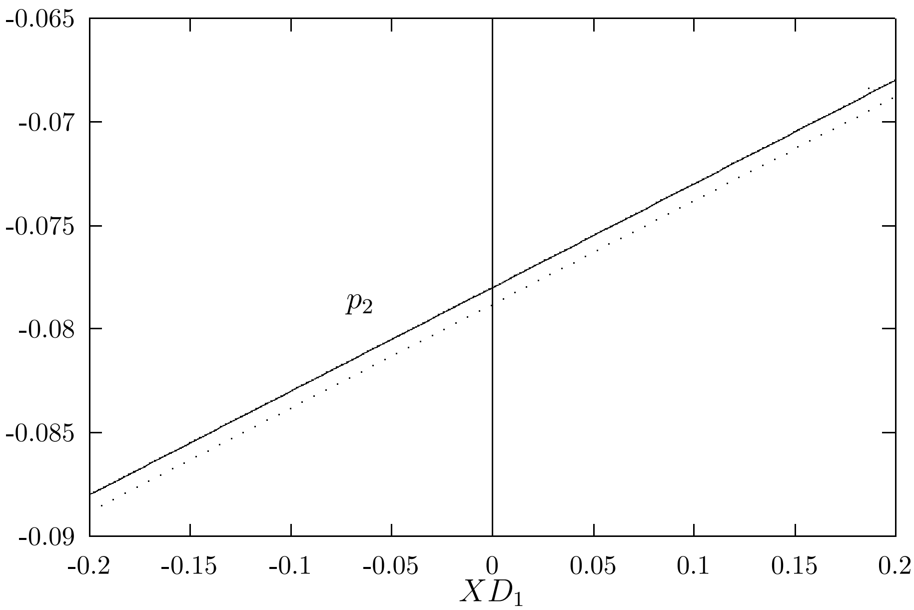

Figure 5: Excess demand function along the HTIE. Parameter values: $\bar{v} = v = \tau_{\epsilon_2} = 0$, $\tau_{\epsilon_1} = \tau_v = \tau_u = \gamma = 1$, and $u_2 = 1.56$.