

Uplink Fractional Power Control and Downlink Power Allocation for Cell-Free Networks

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Abstract—This paper proposes respective policies for uplink power control and downlink power allocation in cell-free wireless networks. Both policies rely only on large-scale quantities and are expressed in closed form, being therefore scalable. The uplink policy, which generalizes the fractional power control employed extensively in cellular networks, features a single parameter; by adjusting this parameter, the SIR distribution experienced by the users can be compressed or expanded, trading average performance for fairness. The downlink policy dualizes the uplink solution, featuring two parameters that again allow effecting a tradeoff between average performance and fairness.

Index Terms—Cell-free networks, ultradense networks, power control, power allocation, beamforming

I. INTRODUCTION

Cell-free networks consist of a dense infrastructure of access points (APs), each potentially communicating with every user (see [2]–[7] and references therein). This paradigm inherits ideas from network MIMO [8] and cloud radio access [9], with the possibility of having substantially more antennas than users per time-frequency resource unit so as to render matched-filter beamforming effective.

One of the challenges posed by cell-free networks is to devise policies for uplink power control and downlink power allocation. Given the major differences in pathloss and shadowing among links, such policies are essential to avoid huge performance disparities. This is illustrated in [2], where max-min policies are seen to equalize the performance across users relative to fixed-power situations. However, max-min policies entail centralized iterative procedures whose steps, in turn, involve a sequence of convex optimizations; the implementation of these procedures does not scale to large networks [10]. Moreover, max-min solutions suffer from excessive rigidity in that they are dragged down by worst-case users [1].

Alternative schemes based on heuristics are propounded in [5], [10] and, while these do scale, their performance is decidedly inferior. In this context, we seek new policies that comply with the following desiderata.

- Retaining the virtue, exhibited by the max-min solutions in [2], of depending only on large-scale quantities.
- Being directly computable from such large-scale quantities so as to ensure scalability.
- Allowing for a tradeoff between equalizing the performance across users (i.e., ensuring fairness) and maximizing the average performance.

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For cellular networks, an uplink power control policy that satisfies the foregoing desiderata was devised in the form of fractional power control, variants of which have been adopted by standards such as LTE and NR. Inspired by this highly successful approach, we seek a fractional power control formulation for the cell-free uplink and, subsequently, a dual power allocation solution for the corresponding downlink.

II. NETWORK AND CHANNEL MODELS

The networks under consideration feature N single-antenna APs and K single-antenna users. Every AP can communicate with every user on each time-frequency resource unit.

Under the premise that the AP locations are agnostic to the radio propagation, shadowing makes those locations appear Poisson-distributed from the vantage of any user [11]. Although asymptotic in the shadowing strength, this phenomenon is well manifested for shadowing intensities of interest [11], [12]. Capitalizing on this result, we draw the AP positions uniformly, avoiding the need for explicit modeling of the shadowing as it is then already implicitly captured by the geometry. Likewise, the user positions are drawn uniformly.

The signals are subject to pathloss with exponent η , which yields a large-scale channel gain $G_{n,k}$ between the k th user and the n th AP. Besides $G_{n,k}$, the channel between the k th user and the n th AP features a small-scale fading coefficient $h_{n,k} \sim \mathcal{N}_{\mathbb{C}}(0, 1)$, independent across users and APs.

Interference-limited conditions are considered, with noise negligible relative to the interference. Pilot contamination is also disregarded, as it can be kept at bay with procedures such as the ones described in [2, Sec. IV] or in [13], [14]. It follows that, based on uplink pilot transmissions from the users, the n th AP can perfectly estimate $h_{n,0}, \dots, h_{n,K-1}$.

III. UPLINK

Upon payload data transmission from the users, the n th AP observes

$$y_n = \sum_{k=0}^{K-1} \sqrt{G_{n,k}} h_{n,k} \sqrt{p_k} s_k, \quad (1)$$

where s_k is the unit-power symbol transmitted by user k while $p_k \in [0, 1]$ is its power control coefficient. With matched filtering, the signal of user k is recovered from

$$\begin{aligned} \sum_{n=0}^{N-1} \sqrt{G_{n,k}} h_{n,k}^* y_n &= \sum_{n=0}^{N-1} G_{n,k} \sqrt{p_k} |h_{n,k}|^2 s_k \\ &+ \sum_{n=0}^{N-1} \sqrt{G_{n,k}} h_{n,k}^* \sum_{\ell \neq k} \sqrt{G_{n,\ell}} \sqrt{p_\ell} h_{n,\ell} s_\ell \end{aligned} \quad (2)$$

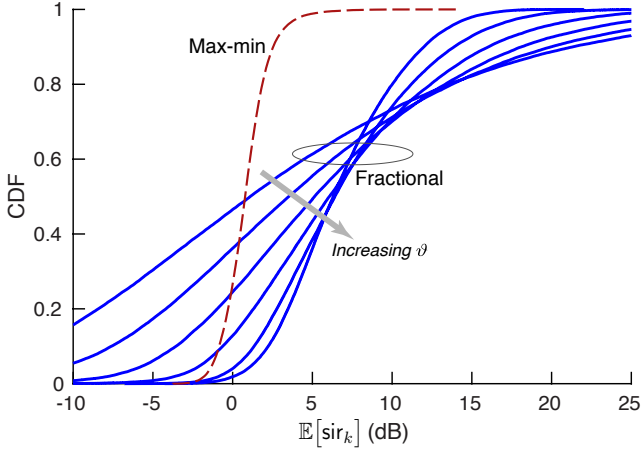


Fig. 1. CDF of $\mathbb{E}[\text{sir}_k]$ for the uplink with $\eta = 3.8$ and $N/K = 2.5$, parameterized by $\vartheta = \{0, 0.2, 0.4, 0.6, 0.8, 1\}$. Also shown is the max-min solution.

such that the signal-to-interference ratio (SIR) of user k , expected over the known $\{h_{n,k}\}$, equals [1], [2]

$$\mathbb{E}[\text{sir}_k] = \mathbb{E} \left[\frac{p_k \left(\sum_{n=0}^{N-1} G_{n,k} |h_{n,k}|^2 \right)^2}{\sum_{\ell \neq k} p_\ell \left| \sum_{n=0}^{N-1} \sqrt{G_{n,k} G_{n,\ell}} h_{n,k}^* h_{n,\ell} \right|^2} \right]. \quad (3)$$

A. Fractional Power Control

The origins of fractional power control can be traced back to [15]. The objective of that initial derivation was to minimize the variance of the large-scale SIR distribution (in dB) experienced by two interfering cellular users, and the solution turned out to be $p_k \propto 1/\sqrt{G_k}$ where G_k was the large-scale gain to the serving cell and with the proportionality such that $p_k \in [0, 1]$. This was subsequently generalized to $p_k \propto 1/G_k^\vartheta$ where $\vartheta \in [0, 1]$ regulates the extent to which the range of received powers are compressed [16]. The values typically featured in LTE and NR are in the range $\vartheta \approx 0.5\text{--}0.7$, with lower values favoring the average SIR while higher values promote fairness and cell-edge performance.

Derived in Appendix A, our proposed generalization of the above to cell-free networks is

$$p_k \propto \frac{1}{\left(\sum_{n=0}^{N-1} G_{n,k} \right)^\vartheta}, \quad (4)$$

which now depends on all the large-scale channel gains that involve a given user, reflecting the effective connection between such user and the network.

B. Performance Evaluation

To exemplify the performance, we consider a wrapped-around universe with $N = 200$ APs, $K = 80$ users, and a pathloss exponent of $\eta = 3.8$. The number of network snapshots is such that, with 95% confidence, the CDFs do not deviate from their true value by more than 0.3%.

Shown in Fig. 1 is the CDF of $\mathbb{E}[\text{sir}_k]$, parameterized by ϑ . Sweeping this parameter from $\vartheta = 0$ (fixed transmit powers)

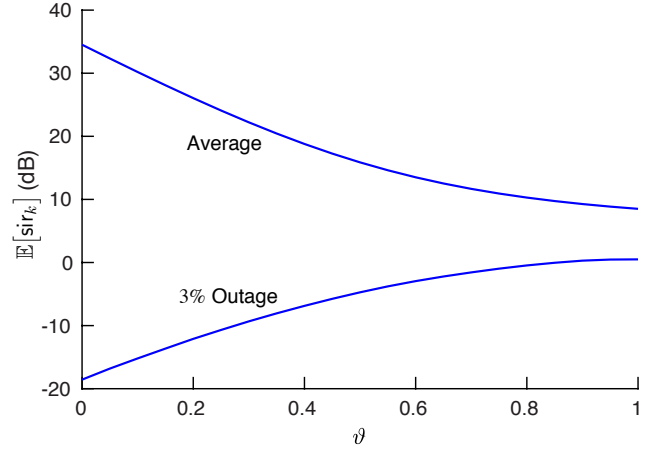


Fig. 2. Average and 3%-outage values of $\mathbb{E}[\text{sir}_k]$ for the uplink as a function of ϑ , with $\eta = 3.8$ and $N/K = 2.5$.

to $\vartheta = 1$ (complete large-scale channel inversion), we observe a progressive compression of the CDF. The tradeoff that ϑ enables between the average and the 3%-outage (as a proxy for fairness) can be appreciated in Fig. 2. Moving from $\vartheta = 0$ to $\vartheta = 1$, the 3%-outage point increases steadily at the expense of the average. Fig. 1 also contrasts (4) with the max-min solution: the $\{p_k\}$ that maximize the lowest SIR as per (14) are obtained through geometric programming and plugged into (3). Fractional power control is more elastic than the max-min solution, which seems to be dragged down by worst-case users, paying a price for its uncompromising equalization of all SIRs.

IV. DOWNLINK

With conjugate beamforming, the precoder applied by the n th AP to transmit to user k is $h_{n,k}$, based on which the n th AP generates the transmit signal

$$x_n = \sum_{k=0}^{K-1} \sqrt{p_{n,k}} h_{n,k} s_k \quad (5)$$

where s_k is the unit-power symbol meant for user k while $p_{n,k} \in [0, 1]$ is the share of power that the n th AP devotes to such user, with $\sum_{k=0}^{K-1} p_{n,k} \leq 1$. User k then observes

$$\begin{aligned} y_k &= \sum_{n=0}^{N-1} \sqrt{G_{n,k}} h_{n,k}^* x_n \\ &= \underbrace{\sum_{n=0}^{N-1} \sqrt{G_{n,k}} p_{n,k} |h_{n,k}|^2 s_k}_{\text{Signal: } S_k} \\ &\quad + \underbrace{\sum_{n=0}^{N-1} \sqrt{G_{n,k}} h_{n,k}^* \sum_{\ell \neq k} \sqrt{p_{n,\ell}} h_{n,\ell} s_\ell}_{\text{Interference: } I_k}. \end{aligned} \quad (6)$$

The performance achievable on the basis of y_k hinges on the knowledge at user k of the effective channel

$$c_k = \sum_{n=0}^{N-1} \sqrt{G_{n,k} p_{n,k}} |h_{n,k}|^2 \quad (8)$$

that relates s_k with y_k . In cellular massive MIMO, it is effectual to rely solely on the mean of c_k since, because of hardening, the actual value never departs significantly from such mean [17, Ch. 10]. In single-antenna cell-free networks, however, the hardening is only partial [4], [6]. The effective channels fluctuate markedly, which gives rise to self-interference if the k th receiver is only privy to $\mathbb{E}[c_k]$.

Self-interference can be avoided by inserting precoded pilots within the downlink transmissions, enabling the explicit estimation by the users of their effective channels [3]. Recalling S_k and I_k as defined in (7), the k th user can then operate at

$$\text{sir}_k = \frac{\mathbb{E}[|S_k|^2 |c_k]}{\mathbb{E}[|I_k|^2 |c_k]} \quad (9)$$

$$= \frac{\left(\sum_{n=0}^{N-1} \sqrt{G_{n,k} p_{n,k}} |h_{n,k}|^2\right)^2}{\sum_{n=0}^{N-1} G_{n,k} \mathbb{E}[|h_{n,k}|^2 |c_k] \sum_{\ell \neq k} p_{n,\ell}}. \quad (10)$$

Since c_k is a function of $h_{0,k}, \dots, h_{N-1,k}$, the realization of c_k conditions an individual $h_{n,k}$ only very weakly, hence $\mathbb{E}[|h_{n,k}|^2 |c_k] \approx \mathbb{E}[|h_{n,k}|^2] = 1$. It follows that the expectation of sir_k over the small-scale fading, and consequently over c_k , satisfies

$$\mathbb{E}[\text{sir}_k] \approx \frac{\mathbb{E}\left[\left(\sum_{n=0}^{N-1} \sqrt{G_{n,k} p_{n,k}} |h_{n,k}|^2\right)^2\right]}{\sum_{n=0}^{N-1} G_{n,k} \sum_{\ell \neq k} p_{n,\ell}}. \quad (11)$$

A. Fractional Power Allocation

Capitalizing on the fractional power control derived for the uplink, we seek a downlink counterpart that can play a similar role. The policy we propose, derived in Appendix B, is

$$p_{n,k} \propto \frac{G_{n,k}}{\left(\sum_{m=0}^{N-1} G_{m,k}\right)^\vartheta \left(\sum_{\ell=0}^{K-1} \frac{G_{n,\ell}}{\left(\sum_{m=0}^{N-1} G_{m,\ell}\right)^\vartheta}\right)^\gamma}, \quad (12)$$

where, as in the uplink, $\vartheta \in [0, 1]$ while γ is an additional parameter best set in the range $[0.4, 1.6]$.

B. Performance Evaluation

Besides the max-min solution, in the downlink we consider as an additional benchmark the heuristic policy suggested in [5, Sec. III-D]. Fig. 3 shows the CDF of $\mathbb{E}[\text{sir}_k]$, parameterized by ϑ and γ . Equipped with these parameters, (12) is seen to be highly versatile: not only can it match the performance of [5, Sec. III-D], but its upper tail can surpass it (when configured for average performance) or its lower tail can amply outperform it (when configured for fairness). In this case, though, the lower tail falls short of the max-min solution.

V. SUMMARY

The proposed fractional power control and power allocation policies, embodied by (4) and (12), are closed-form functions of the large-scale channel gains and allow regulating the tradeoff between average performance and fairness. The regulation is exerted through one (uplink) or two (downlink) parameters, hence it does not enable favoring individual users or maximizing specific metrics (e.g., a weighted sum). If such is the goal, learning-based solutions offer an attractive compromise between performance and scalability [18].

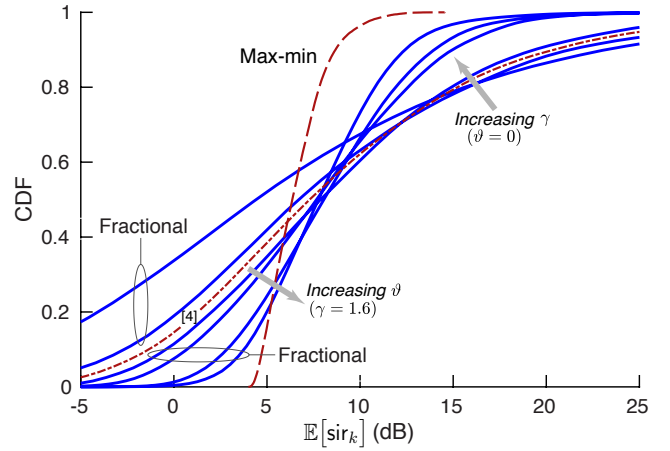


Fig. 3. CDF of $\mathbb{E}[\text{sir}_k]$ for the downlink with $\eta = 3.8$ and $N/K = 2.5$, parameterized by $\vartheta = \{0, 0.4, 0.8\}$ and $\gamma \in \{0.4, 0.8, 1.2, 1.6\}$. Also shown are the max-min solution and the benchmark from [5, Sec. III-D].

APPENDIX A

Inspired by [15], we seek to minimize the variance of the large-scale SIR distribution (in dB), hence the starting point needs to be an expression for such SIR. In a cellular network, this equals the large-scale signal power divided by the large-scale other-cell interference power, namely [17]

$$\text{SIR}_k = \frac{p_k G_k}{\sum_{\ell \neq k} p_\ell G_\ell}, \quad (13)$$

which can be recovered from (3) by letting $N = 1$ and replacing $|h_k|^2$ and $|h_\ell|^2$ by their unit expected value. A counterpart to (13) for cell-free networks can be obtained by similarly replacing $|h_{n,k}|^2$ and $|h_{n,\ell}|^2$ by unity, and by zeroing out cross-terms containing $h_{n,k}^* h_{m,\ell}$ for $n \neq m$. This gives

$$\text{SIR}_k = \frac{p_k \left(\sum_{n=0}^{N-1} G_{n,k}\right)^2}{\sum_{\ell \neq k} p_\ell \sum_{n=0}^{N-1} G_{n,k} G_{n,\ell}}. \quad (14)$$

Specializing (14) to $K = 2$ and focusing, without loss of generality, on user 0,

$$\text{SIR}_0 = \frac{p_0 \left(\sum_{n=0}^{N-1} G_{n,0}\right)^2}{p_1 \sum_{n=0}^{N-1} G_{n,0} G_{n,1}}, \quad (15)$$

from which, introducing the notation $z|_{\text{dB}} = 10 \log_{10} z$, we can further write

$$\text{SIR}_0|_{\text{dB}} = p_0|_{\text{dB}} + 2 G_S|_{\text{dB}} - p_1|_{\text{dB}} - G_I|_{\text{dB}}, \quad (16)$$

where $G_S = \sum_{n=0}^{N-1} G_{n,0}$ and $G_I = \sum_{n=0}^{N-1} G_{n,0} G_{n,1}$.

We are interested in distributed power control policies where p_0 depends on $\{G_{n,0}\}_{n=0}^{N-1}$, but not on $\{G_{n,1}\}_{n=0}^{N-1}$, hence the following can be reasonably assumed to hold:

- p_0 and p_1 are independent, as they are controlled on the basis of distinct channel gains.
- p_0 is not independent of G_S , nor of G_I .
- p_1 is not independent of G_I , but it is independent of G_S .

With that, the variance of (16) can be developed into

$$\begin{aligned} \text{var}[\text{SIR}_0|_{\text{dB}}] &= \text{var}[p_0|_{\text{dB}} + 2G_S|_{\text{dB}}] + \text{var}[p_1|_{\text{dB}} + G_I|_{\text{dB}}] \\ &\quad - 2\text{cov}[p_0|_{\text{dB}} + 2G_S|_{\text{dB}}, p_1|_{\text{dB}} + G_I|_{\text{dB}}] \\ &= \text{var}[p_0|_{\text{dB}}] + 4\text{var}[G_S|_{\text{dB}}] + 4\text{cov}[p_0|_{\text{dB}}, G_S|_{\text{dB}}] \\ &\quad + \text{var}[p_1|_{\text{dB}}] + \text{var}[G_I|_{\text{dB}}] + 2\text{cov}[p_1|_{\text{dB}}, G_I|_{\text{dB}}] \\ &\quad - 2\text{cov}[p_0|_{\text{dB}}, G_I|_{\text{dB}}] - 4\text{cov}[G_S|_{\text{dB}}, G_I|_{\text{dB}}], \end{aligned}$$

which, by virtue of the fact that $\text{var}[p_0|_{\text{dB}}] = \text{var}[p_1|_{\text{dB}}]$ and $\text{cov}[p_0|_{\text{dB}}, G_I] = \text{cov}[p_1|_{\text{dB}}, G_I]$, simplifies into

$$\begin{aligned} \text{var}[\text{SIR}_0|_{\text{dB}}] &= 2\text{var}[p_0|_{\text{dB}}] + 4\text{cov}[p_0|_{\text{dB}}, G_S|_{\text{dB}}] \\ &\quad + 4\text{var}[G_S|_{\text{dB}}] + \text{var}[G_I|_{\text{dB}}] - 4\text{cov}[G_S|_{\text{dB}}, G_I|_{\text{dB}}]. \end{aligned} \quad (17)$$

Regrouping some terms, the above can be rewritten as

$$\begin{aligned} \text{var}[\text{SIR}_0|_{\text{dB}}] &= 2\text{var}[p_0|_{\text{dB}} + G_S|_{\text{dB}}] + 2\text{var}[G_S|_{\text{dB}}] \\ &\quad + \text{var}[G_I|_{\text{dB}}] - 4\text{cov}[G_S|_{\text{dB}}, G_I|_{\text{dB}}], \end{aligned} \quad (18)$$

where the last three terms do not depend on $p_0|_{\text{dB}}$, hence they are immaterial to the minimization with respect to it. The quantity to minimize is $\text{var}[p_0|_{\text{dB}} + G_S|_{\text{dB}}]$, and the power control policy that minimizes it is $p_0|_{\text{dB}} = -G_S|_{\text{dB}}$, i.e.,

$$p_0 = \frac{1}{\sum_{n=0}^{N-1} G_{n,0}}. \quad (19)$$

With a view to $K > 2$ users and to regulating the forcefulness of the policy, we generalize (19) into (4) by incorporating ϑ . Finally, all powers should be scaled to fit within $[0, 1]$.

APPENDIX B

Observe that, in the uplink, (4) depends only on the numerator of the SIR expression from which it derives. This is consistent with the nature of beamforming, whose sole purpose is to maximize the signal power from the intended user, with no regard for the interference. Moreover, with the exponent set to its highest value of $\vartheta = 1$, the power control in (4) sets the received signal power from user k to $\sum_{n=0}^{N-1} G_{n,k}$.

From the numerator of the downlink SIR expression in (10), with $|h_{n,k}|^2$ replaced once more by its unit expected value, the same behavior can be induced if $p_{0,k}, \dots, p_{N-1,k}$ ensure that

$$\sum_{n=0}^{N-1} \sqrt{G_{n,k} p_{n,k}} = \sqrt{\sum_{n=0}^{N-1} G_{n,k}}, \quad (20)$$

which has infinitely many solutions. As the left-hand side of (20) is concave on $p_{0,k}, \dots, p_{N-1,k}$, we can identify the one solution minimizing $\sum_{n=0}^{N-1} p_{n,k}$ by building the Lagrangian

$$L = \sum_{n=0}^{N-1} p_{n,k} + \lambda \left(\sum_{n=0}^{N-1} \sqrt{G_{n,k} p_{n,k}} - \sqrt{\sum_{n=0}^{N-1} G_{n,k}} \right). \quad (21)$$

Setting $\partial L / \partial p_{n,k} = 0$, what emerges is $p_{n,k} = \alpha G_{n,k}$ with α upholding (20). This gives $p_{n,k} = G_{n,k} / \sum_{m=0}^{N-1} G_{m,k}$, which, proceeding as in the uplink, we broaden to

$$p_{n,k} = \frac{G_{n,k}}{\left(\sum_{m=0}^{N-1} G_{m,k} \right)^\vartheta}. \quad (22)$$

The expression in (22), which can be regarded as the downlink dual of (4), can be further generalized to account for the additional issue that appears in the downlink, namely that the total power per AP is constrained. In order to equalize the per-AP total powers, we can multiply $p_{n,k}$ by α_n with

$$\alpha_n = \frac{1}{\sum_{\ell=0}^{K-1} \frac{G_{n,\ell}}{\left(\sum_{m=0}^{N-1} G_{m,\ell} \right)^\vartheta}} \quad (23)$$

ensuring equal total power per AP. Finally, introducing a new parameter γ , we can relax the per-AP power equalization into a partial equalization as per (12).

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