Which inflation to target? A small open economy with sticky wages indexed to past inflation*

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Abstract

In a closed economy context there is common agreement on price inflation stabilization being one of the objects of monetary policy. Moving to an open economy context gives rise to the coexistence of two measures of inflation: domestic inflation (DI) and consumer price inflation (CPI). Which one of the two measures should be the target variable? This is the question addressed in this paper. In particular, I use a small open economy model to show that once sticky wages indexed to past CPI inflation are introduced, a complete inward looking monetary policy is no more optimal. I first, derive a loss function from a second order approximation of the utility function and then, I compute the fully optimal monetary policy under commitment. Then, I use the optimal monetary policy as a benchmark to compare the performance of different monetary policy rules. The main result is that once a positive degree of indexation is introduced in the model the rule performing better (among the Taylor type rules considered) is the one targeting wage inflation and CPI inflation. Moreover this rule delivers results very close to the one obtained under the fully optimal monetary policy with commitment.

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1 Introduction

The purpose of this paper is to analyse which measure of inflation should be taken as target variable in an open economy framework. In a closed economy context there is common agreement on price inflation stabilization being one of the objects of monetary policy. From the ad-hoc interest rate rule proposed by Taylor (1993), to the more recent New Keynesian literature deriving optimal monetary policy rules from the minimization of loss functions obtained via second order approximation of the utility of the representative consumer, the monetary instrument has to be chosen in order to match a given inflation target (among with other targets). However, moving to an open economy context gives rise to the coexistence of two measures of inflation: domestic inflation (DI) and consumer price inflation (CPI). Which one of the two measures should be the target variable? This is the question addressed in this paper.

For this purpose I develop a small open economy model similar to the one used by Galí and Monacelli (2005). In addition to the standard assumption of sticky prices, I also assume sticky wages indexed to past CPI inflation. In each period only a fraction of workers is allowed to reoptimize while for the others I allowed for a partial indexation to past CPI inflation. This will be the main difference with the existing open economy literature. The main idea is that the volatility of CPI and the impossibility for some workers to adjust their wages in order to keep their mark up constant make the stabilization of CPI inflation relevant in this context. In particular the assumptions on wages will produce two main consequences: first, given the presence of wage rigidities, strict inflation targeting will no more be optimal (as Erceg, Henderson and Levin (2000) show in a closed economy setup); second, if at time t there is an exogenous increase in foreign prices such that CPI increases, this leads to an increase of nominal wages in $t+1$ (at least for workers that will not reoptimize, via the indexation channel) therefore, other things equals, marginal cost for firms will increase in $t+1$ and so do prices. Therefore an increase of CPI inflation in period t will induce an increase in DI in period $t+1$. This link between CPI and DI makes it desirable to also stabilize CPI.

As underlined by Christiano, Eichenbaum and Evans (2005) and by Smets and Wouters (2003), there is strong empirical evidence of wage rigidities in the economy. Moreover Smets and Wouters (2003) estimate of the degree of wage indexation to past inflation for the EURO area is around 0.65. Consequently there is empirical evidence in favour of the importance of modelling also wage rigidities and wage indexation.

From a practical point of view there seems to be a unanimous consensus among central banks on CPI being the correct target. In particular, as stressed by Bernanke and Mishkin (1997), starting from 1990 the following countries have adopted an explicit target to CPI inflation: Australia, Canada, Finland, Israel, New Zealand, Spain, Sweden, UK. In the EMU the European Central Bank's object is to stabilize the Harmonized Index of Consumer Prices (HCPI) below 2%.

In contrast, from a theoretical point of view, the answer to this question has not yet been established. There is a related open economy literature that has tried to address the question of which inflation should be taken as target variable by the central bank. Clarida, Galí and Gertler (2001) analyse a small open economy model with price rigidities and frictions in the labour market and they find that, as long as there is perfect exchange rate pass-through, the target of the central bank should be DI. They do not, however, model explicitly the frictions in the labour market and they just assume an exogenous stochastic process for the wage mark-up. This is an important difference with the model I developed because assuming an exogenous process for the wage mark-up makes price stability no more optimal (like in my model) but omit the link that there might be between domestic wages and foreign prices. A similar result is obtained in Galí and Monacelli (2005)[1] where strict DI targeting turns out to be the optimal monetary policy, consequently outperforming a CPI targeting rule. In Clarida, Galí and Gertler (2002) they present a two country model with sticky prices and they show that in the case of no coordination the two monetary authorities should adjust the interest rate in response to DI. On the other side Corsetti and Pesenti (2005) use a two country model with firm's prices set one period in advance to show that "inward-looking policy of domestic price stabilization is not optimal when firms' markups are exposed to currency fluctuations". This argument would became less convincing in a currency area given that firms there would be less exposed to currency fluctuations (only firms trading with countries outside the currency area would be affected by currency fluctuations). In contrast, my results do not depend on fluctuations of the exchange rate and therefore can be extended to the case of a currency area and are more suitable to explain the behaviour of the ECB. Svensson (2000) uses a small open economy framework to analyse inflation targeting monetary policies and he underlines that "all inflation-targeting countries have chosen to target CPI inflation...None of them has chosen to target domestic inflation". Given this consideration he assumed an ad-hoc loss function that includes both CPI and DI in addition to other variables. The result of the model (that is not fully microfounded) is that flexible CPI inflation targeting is better than flexible DI targeting.

The first result of the paper is that from the Phillips Curve (computed both for wage inflation and for DI) it is clear that there is a link between DI, CPI and wage inflation. Given this link it is clearly difficult to stabilize DI without stabilizing also CPI and wage inflation. In order to get a more precise analysis of what should a Central Bank do I derived the loss function as a second order approximation of the utility function and I compute the fully optimal monetary policy under commitment. Differently from Galí and Monacelli (2005) I obtain a loss function that, in addition to output gap and home inflation, depends also on CPI inflation and wage inflation. Given the presence of both price and wage rigidities, price stabilization is no more the optimal monetary policy. The object

[1]Under the assumptions of log utility in consumption and unit elasticity of substitution among foreign goods.

of the paper is to use the optimal monetary policy as a benchmark to compare different, implementable, monetary policy rules. In the choice of possible targets for monetary policy I disregard the output gap, that cannot be considered a feasible target since it is not clear how to estimate the natural level of output, and I concentrate on the other three variables that appear in the loss function. I focus on Taylor's type rules targeting two of the three variables. I simulate the model under these monetary policy rules in order to make a ranking among them. I do this exercise for different degrees of wage indexation in order to analyse how this feature of the model affects the results. In the case of no indexation an inward-looking monetary policy targeting DI and wage inflation is, among the policy rules considered, the one performing better. But as soon as a positive degree of indexation is introduced in the model this is no more true and the policy rule performing better is a Taylor rule targeting CPI inflation and wage inflation. Increasing the level of indexation reinforces the results. Simulating the model under the optimal monetary policy rule and under the Taylor type rules and looking at the correlations among the series simulated in the different scenarios, it is clear that, at least for a positive degree of indexation, the Taylor rule targeting at CPI and wage inflation delivers a behaviour of the economy that is very close to the one obtained under the optimal rule.

These results therefore confirm the original hypothesis that the introduction of wage indexation would have affected the ranking among policy rules giving more importance to the stabilization of CPI inflation, i.e. making more desirable a monetary policy that is not completely inward-looking.

The structure of the paper is the following: section 2 introduces the open economy model, section 3 presents the analysis of the loss function, section 4 computes the optimal monetary policy under commitment, section 5 shows how different, implementable, monetary policy rules perform under different degrees of indexation and section 6 concludes.

2 The model

Like in Galí and Monacelli (2005), there is a continuum $[0, 1]$ of small, identical, countries. Differently from the original model, I introduce the assumption of monopolistic competition on the supply side of labour market. I also assume the presence of wage rigidities. It is worthy to note that one of the two basic assumptions of the model is that markets are complete, so that households differ in the amount of labour supplied (consequence of the presence of sticky wages) but share the same consumption. The second assumption is that the law of one price holds for individual goods at all times. From now on I will use "h" as index for a particular household, "i" to refer to a particular country and "j" as sector index. When no index is specified the variables refer to the home country.

2.1 Households

Household "h" maximizes:

$$E_0 \sum_{t=0}^{\infty} \beta^t \left[U(C_t) + V(N_t(h)) \right] \tag{1}$$

where C_t is an aggregate consumption index:

$$C_t \equiv \left[(1-\alpha)^{\frac{1}{\eta}} C_{H,t}^{\frac{\eta-1}{\eta}} + \alpha^{\frac{1}{\eta}} C_{F,t}^{\frac{\eta-1}{\eta}} \right]^{\frac{\eta}{\eta-1}} \tag{2}$$

where α represents the degree of openness, and $C_{H,t}$ and $C_{F,t}$ are two aggregate consumption index, respectively for domestic and imported goods:

$$C_{H,t} \equiv \left[\int_0^1 C_{H,t}(j)^{\frac{\theta_p - 1}{\theta_p}} dj \right]^{\frac{\theta_p}{\theta_p - 1}} \tag{3}$$

$$C_{F,t} \equiv \left[\int_0^1 C_{i,t}^{\frac{\eta-1}{\eta}} di \right]^{\frac{\eta}{\eta-1}} \tag{4}$$

$$C_{i,t} \equiv \left[\int_0^1 C_{i,t}(j)^{\frac{\theta_p - 1}{\theta_p}} dj \right]^{\frac{\theta_p}{\theta_p - 1}} \tag{5}$$

The parameter $\theta_p > 1$ represents the elasticity of substitution between two varieties of goods, while the parameter $\eta > 0$ represents the elasticity of substitution between home produced goods and goods produced abroad. Each household h maximizes (1) under a sequence of budget constraints. The results regarding the optimal allocation of expenditure across goods are not affected by the introduction of monopolistic competition in the labour market so, using the results of Galí and Monacelli (2005), I can directly write the budget constraint after having aggregated over goods:

$$P_t C_t + E_t \left[Q_{t,t+1} D_{t+1} \right] \leq D_t + (1+\tau_w) W_t(h) N_t(h) + T_t \tag{6}$$

where $Q_{t,t+1}$ is the stochastic discount factor, D_t is the payoff in t of the portfolio held in $t-1$, T_t is a lump-sum transfer (or tax), τ_w is a subsidy to labour income and P_t is the aggregate price index:

$$P_t \equiv \left[(1-\alpha)(P_{H,t})^{1-\eta} + \alpha (P_{F,t})^{1-\eta} \right]^{\frac{1}{1-\eta}} \tag{7}$$

$$P_{H,t} \equiv \left[\int_0^1 P_{H,t}(j)^{1-\theta_p} dj \right]^{\frac{1}{1-\theta_p}} \tag{8}$$

$$P_{F,t} \equiv \left[\int_0^1 P_{i,t}^{1-\eta} di\right]^{\frac{1}{1-\eta}} \tag{9}$$

$$P_{i,t} \equiv \left[\int_0^1 P_{i,t}(j)^{1-\theta_p} dj\right]^{\frac{1}{1-\theta_p}} \tag{10}$$

Each household supplies a differentiated labour service in each sector, so that the total labour supplied by household h is given by $N_t(h) = \int_0^1 N_{t,h}(j)dj$. Consequently, he will maximize (1) subject to the demand for labour. Given that the production function in each sector j is given by $Y_t(j) = A_t N_t(j)$ with $N_t(j) = \left[\int_0^1 N_{t,j}(h)^{\frac{\theta_w - 1}{\theta_w}} dh\right]^{\frac{\theta_w}{\theta_w - 1}}$, the cost minimization problem of firms yields to the following demand for labour faced by individual h:

$$N_t(h) = \left[\frac{W_t(h)}{W_t}\right]^{-\theta_w} N_t \tag{11}$$

where $\theta_w > 1$ represents the elasticity of substitutions between workers and the aggregate wage index is given by $W_t \equiv \left[\int_0^1 W_t(h)^{1-\theta_w} dh\right]^{\frac{1}{1-\theta_w}}$.

2.1.1 Wages decisions

In each period only a fraction $(1 - \xi_w)$ of households can reset wages optimally. For the fraction ξ_w of households that cannot reset wages optimally I allow for a partial indexation to past CPI inflation. Another way to think about it is that in each period there is a fraction of workers that find it easier, instead of fully reoptimize, just to follow a simple rule (like assumed in Christiano et al. (2005) for firms) trying to preserve their real wages. That is why the indexation is to CPI and not to domestic inflation. Like Smets and Wouters (2003), I have introduced the parameter γ_w so that will be possible to study, later on, how certain results may be affected by different degrees of indexation. Therefore the wage of the fraction ξ_w of households that can not reoptimize in t is given by:

$$W_t(h) = \Pi_{t-1}^{\gamma_w} W_{t-1}(h) \tag{12}$$

where Π_t is the CPI inflation. Each household that can reoptimise in t will choose $W_t(h)$ considering the hypothesis that he will not be able to reoptimise any more in the future. Consequently he will maximize (1) under (6) and (11) taking into account the probability of not being allowed to reoptimise in the future. The FOC of this optimisation problem is:

$$E_t \sum_{T=0}^{\infty} (\beta\xi_w)^T \left[U_C[C_{t+T}] \frac{W_t(h)\Pi_{tT}^{\gamma_w}}{P_{t+T}}(1+\tau_w)\frac{\theta_w - 1}{\theta_w} + V_N[N_{t+T}(h)]\right] N_{t+T}(h) = 0 \tag{13}$$

with $\Pi_{tT} = \Pi_t \Pi_{t+1} .... \Pi_{T-1} = \frac{P_{t+T-1}}{P_{t-1}}$. From (13) it is clear that the solution $\widetilde{W}_t(h)$ will be the same for all households that are allowed to reoptimise in t. To solve for the optimal wage we need first to log linearize (13) around the steady state:

$$E_t \sum_{T=0}^{\infty} (\beta\xi_w)^T \left[\widehat{\Psi}_{t+T} - \widehat{MRS}_{t+T}(h)\right] = 0 \tag{14}$$

where $\Psi_{t+T} = \frac{\widetilde{W}_t \Pi_{tT}^{\gamma_w}}{P_{t+T}}$ is the real wage, $MRS_t = -\frac{V_{N,t}}{U_{C,t}}$ and $\widehat{\Psi}_{t+T}$ and $\widehat{MRS}_{t+T}(h)$ are the log deviations from their levels with flexible prices. Rearranging terms I get the following equation for the optimal wage:

$$\log \widetilde{W}_t = -log(1-\Phi_w) + (1-\beta\xi_w) E_t \sum_{T=0}^{\infty} (\beta\xi_w)^T \left[\log MRS_{t+T}(h) + \log P_{t+T} - \gamma_w \log \Pi_{tT}\right] \tag{15}$$

where $\log(1-\Phi_w) = \log(1+\tau_w) - \log(\mu_w)$ and $\mu_w = \frac{\theta_w}{\theta_w - 1}$ is the wage markup. Whenever $\tau_w = \frac{1}{\theta_w - 1}$ then $\Phi_w = 0$ and the fiscal policy completely eliminates the distortion caused by the presence of monopolistic competition in the supply of labour. When instead $\tau_w < \frac{1}{\theta_w - 1}$, then $-\log(1-\Phi_w) > 0$ and a distortion is present in the economy[2]. From now on I will assume the following specification for the utility function:

$$U(C) + V(N) = \frac{C^{1-\sigma}}{1-\sigma} - \frac{N^{1+\varphi}}{1+\varphi} \tag{16}$$

Given this specification, and with some algebra, I get the following expression:

$$\begin{aligned}
\log \widetilde{W}_t = & \\
& -\frac{(1-\beta\xi_w)}{1+\varphi\theta_w} \sum_{T=0}^{\infty} (\beta\xi_w)^T E_t[\widehat{\mu}^w_{t+T}] + \log(W_t) + \\
& + \sum_{T=1}^{\infty} (\beta\xi_w)^T E_t \log \Pi^w_{t+T} + \\
& -\gamma_w (1-\beta\xi_w) \sum_{T=0}^{\infty} (\beta\xi_w)^T E_t \log \Pi_{tT}
\end{aligned} \tag{17}$$

where $\widehat{\mu}^w_t = \log(W_t) - \log(P_t) - \log(MRS_t) + \log(1-\Phi_w)$. The optimal wage today will be higher the higher the expectations about future wages. Future CPI

[2]Note that if $\frac{\theta_w}{\theta_w - 1} = 1 + \tau_w$ the fiscal policy is able to completely eliminate the distortion arising from labour markets. Following Woodford (2003) I'm defining $1 - \Phi_w = (1+\tau_w)\frac{\theta_w - 1}{\theta_w}$ where Φ_w represents the distortion in the economy. Whenever $\Phi_w > 0$ the level of employment in the flexible price equilibrium will be lower than the one that we would have without distortions. When doing welfare analysis I'll assume for simplicity $\Phi_w = 0$ but now I can consider the more general case.

inflation has instead a negative impact because of indexation. In particular, the higher the level of indexation and the higher the expected future CPI inflation, the lower will be the optimal wage today. This is because agents know that even if they will not be allowed to reoptimise in the near future, their wages will increase anyway because of indexation. This effect would disappear with $\gamma = 0$. Note that with the labour subsidy in place the distortion in the labour market is lower than the one that we would have without subsidy, indeed $-\log(\mu_w) < log(1-\Phi_w) \leq 0$. Still, $\widehat{\mu}_t^w = 0$ means that the wage charged is higher then the one that would be charged with perfect competition on the labour market. So, even if the monetary authority manages to eliminate the distortions arising from the nominal rigidities, the level of employment will be lower then the natural one, unless $\Phi_w = 0$.

The next step is to analyse the corresponding Phillips Curve on wage inflation. Given that the fraction $(1 - \xi_w)$ of households that is allowed to reoptimise will choose the same wage, while the others will follow the indexation rule, I can rewrite the aggregate wage index as:

$$W_t = \left[(1-\xi_w)\widetilde{W}_t^{1-\theta_w} + \xi_w (W_{t-1}\Pi_{t-1}^{\gamma_w})^{1-\theta_w}\right]^{\frac{1}{1-\theta_w}} \tag{18}$$

The log linearized version of this equation is given by:

$$logW_t = (1-\xi_w)\log\widetilde{W}_t + \xi_w \log W_{t-1} + \gamma_w \xi_w \log \Pi_{t-1} \tag{19}$$

It is useful to rewrite (17) in the following way:

$$log\widetilde{W}_t - \beta\xi_w E_t \log \widetilde{W}_{t+1} = -\frac{1-\beta\xi_w}{1+\varphi\theta_w}\widehat{\mu}_t^w + (1-\beta\xi_w)\log W_t \tag{20}$$

From now on all the lower case letters denote the log of the variables. Combining (20) with (19), I obtain:

$$\pi_t^w = -\lambda_w \widehat{\mu}_t^w + \beta E_t[\pi_{t+1}^w] - \xi_w \gamma_w \beta \pi_t + \gamma_w \pi_{t-1} \tag{21}$$

where $\lambda_w = \frac{1-\xi_w}{\xi_w}\frac{1-\beta\xi_w}{1+\varphi\theta_w}$. As in the case of no indexation, current wage inflation depends positively on the expected future wage inflation and negatively on the deviation of the markup from its frictionless level. In particular when $\widehat{\mu}_t^w > 0$ the markup charged is higher then its optimal level, that's way wages respond negatively to a positive $\widehat{\mu}_t^w$. This result is consistent with the one obtained in Galí (2003) in the closed economy case with no indexation. The presence of indexation introduces two new elements: a negative impact of current CPI inflation and a positive impact of past CPI inflation. For what concern present inflation, because of indexation households know that, even if they will not be able to change wages in the next period, their wages will increase because of the link with current inflation, so there is no need to increase them today. Past inflation, instead, has a positive impact on current wage inflation because agents that are not allowed to reoptimize in t will see their wages increase because of indexation.

2.1.2 Consumption Decisions

Maximizing (1) with respect to consumption, under the constraints, leads to the standard Euler Equation:

$$\beta R_t E_t \left[\left(\frac{C_{t+1}}{C_t} \right)^{-\sigma} \frac{P_t}{P_{t+1}} \right] = 1 \tag{22}$$

with $R_t = \frac{1}{E_t[Q_{t,t+1}]}$.

2.2 Firms

The production function of an home firm in sector j is given by:

$$Y_t(j) = A_t N_t(j) \tag{23}$$

with $a_t \equiv \log(A_t)$ and

$$a_{t+1} = \rho_a a_t + \varepsilon_{A,t}. \tag{24}$$

The aggregate domestic output is given by:

$$Y_t = \left[\int_0^1 Y_t(j)^{\frac{\theta_p - 1}{\theta_p}} dj \right]^{\frac{\theta_p}{\theta_p - 1}} \tag{25}$$

Up to a first order approximation Galí and Monacelli (2005) demonstrate that:

$$y_t = a_t + n_t \tag{26}$$

In each period only a fraction $(1 - \xi_p)$ of firms can reset prices optimally.

Given that the elasticity of substitution between varieties of final goods is $\theta_p > 1$, the markup that each firm would like to charge is $\mu_p = \frac{\theta_p}{\theta_p - 1}$. Assuming the presence of a subsidy τ_p to the firm's output, optimal price setting of a home firm j must satisfy the following FOC:

$$E_t \sum_{T=0}^{\infty} \xi_p^T Q_{t,t+T} Y_{t+T}(j) \left[(1 + \tau_p) \frac{\theta_p - 1}{\theta_p} P_{H,t}(j) - MC_{t+T} \right] = 0 \tag{27}$$

where MC_t represents the nominal marginal cost. Like for wages, it is useful to define $1 - \Phi_p \equiv (1 + \tau_p) \frac{\theta_p - 1}{\theta_p}$, where Φ_p indicates the distortion due to monopoly power on the firm side that is still present in the economy after the intervention of the fiscal authority. If the fiscal authority optimally choose τ_p in order to exactly offset the monopoly distortion then $\Phi_p = 0$. If $\Phi_w > 0$ and/or $\Phi_p > 0$ then the flexible price allocation will deliver an output and an employment level lower then the natural ones.

From the log-linear approximation of (27) around the steady state it is possible to derive the standard log-linear optimal price rule:

$$\widetilde{p}_{H,t} = -\log(1-\Phi_p) + (1-\beta\xi_p)E_t \sum_{T=0}^{\infty} (\beta\xi_p)^T [mc_{t+T} + p_{H,t}] \tag{28}$$

where $\widetilde{p}_{H,t}$ represents the (log of) price chosen by the firms that are allowed to reoptimise in t, and mc_t represents the (log of) real marginal cost.

2.3 Equilibrium Conditions

To close the model some relations between home and foreign variables are needed. A "star" will be used to denote world variables. The following equations[3] are the ones obtained by Galí and Monacelli (2005):

$$C_t^* = Y_t^* \tag{29}$$

$$c_t = c_t^* + \frac{1-\alpha}{\sigma} s_t \tag{30}$$

where $S_t \equiv \frac{P_{F,t}}{P_{H,t}}$ are the effective terms of trade and (30) represents the international risk sharing condition. The market clearing condition is given by:

$$Y_t = C_t S_t^{\alpha} \tag{31}$$

The world output is assumed to follow an exogenous law of motion:

$$y_{t+1}^* = \rho_y y_t^* + \varepsilon_{y,t}. \tag{32}$$

The terms of trade can be expressed also in function of the aggregate and the home price indexes:

$$\alpha s_t = p_t - p_{H,t} \tag{33}$$

The relation between the home output and the world output is given by:

$$s_t = \sigma_\alpha (y_t - y_t^*) \tag{34}$$

with $\sigma_\alpha \equiv \frac{\sigma}{1-\alpha+\alpha\omega} > 0$ and $\omega \equiv \sigma\eta + (1-\alpha)(\sigma\eta - 1)$

2.4 The New Keynesian Phillips Curve

The relation between domestic inflation and real marginal cost is not affected by the presence of sticky wages:

$$\pi_{H,t} = \beta E_t[\pi_{H,t+1}] + \lambda \widehat{mc}_t \tag{35}$$

[3]All these relations, with the only exception of (29) that is an exact relation, hold exactly only under the assumption that $\sigma = \eta = 1$. Otherwise they hold up to a first order approximation.

with $\lambda \equiv \frac{(1-\beta\xi_p)(1-\xi_p)}{\xi_p}$ and with $\widehat{mc}_t$, log deviation of the real marginal cost from its level in absence of nominal rigidities (i.e. $\widehat{mc}_t = mc_t - mc$ with $mc = \log(1-\Phi_p)$). The presence of sticky wages leads to an additional term in the standard equation relating the marginal cost with the output gap (the derivation is in the appendix):

$$\widehat{mc}_t = (\sigma_\alpha + \varphi)(y_t - \overline{y}_t) + \widehat{\mu}_t^w \tag{36}$$

When wages are fully flexible $\widehat{\mu}_t^w = 0$. When wages are sticky this is no more true and in particular, when $\widehat{\mu}_t^w > 0$, the markup charged by workers is higher then the optimal one and firms bear a higher real marginal cost. Consequently the NKPC for a small open economy with both price and wage rigidities is:

$$\pi_{H,t} = \beta E_t[\pi_{H,t+1}] + \lambda(\sigma_\alpha + \varphi)(y_t - \overline{y}_t) + \lambda\widehat{\mu}_t^w \tag{37}$$

Even assuming that the only distortions left in the economy are the ones generated by the presence of nominal rigidities, clearly as in Erceg et al. (2000), since it is not possible to stabilize at the same time the home inflation, the wage inflation and the output gap, the flexible price allocation is no longer a feasible target. Is it still true then, that a Taylor rule targeting the home inflation is the one that performs better? It can be interesting to analyse the impact of an increase in p_t on $\pi_{H,t}$. To keep the wage markup constant wages should increase to offset the change in prices but, because of stickiness, this is not possible for all households, so some of them will charge a wage that is lower than the desired one and $\widehat{\mu}_t^w$ will become negative. This will have a negative impact on home inflation. On the other hand, because of indexation to past inflation, in $t+1$ the aggregate wage index will increase and so will do $\widehat{\mu}_{t+1}^w$. This will lead to an increase of $E_t\pi_{H,t+1}$. So, other things equal, an increase in p_t will for sure cause an increase of $\pi_{H,t+1}$, whereas the impact on current home inflation is not clear. Given this link between home inflation, CPI inflation and wage inflation, it seems reasonable to postulate that targeting only one of these variables may not be optimal because, if CPI and wage inflation are very volatile, it will be hard to stabilize only home inflation.

To prove this conjecture, in the next section, I will derive the welfare function from a second order approximation of the utility of the representative household. I will then use the welfare function to study the behavior of the economy under optimal monetary policy. Finally, using the results under optimal monetary policy as benchmark, I will compare different welfare losses obtained using different, implementable, policy rules.

3 Welfare function

Before starting with the welfare analysis it is important to underline that in the open economy model there are 5 distortions: monopolistic power in both goods and labour markets; nominal rigidities in both wages and prices; incentives to generate exchange rate appreciation. In a closed economy framework to require $\Phi_w = \Phi_p = 0$ is enough to ensure that the flexible price allocation will coincide with the optimal one, but this is no more true in an open economy. As emphasised by Corsetti and Pesenti (2001), a monetary expansion has two consequences in this context: it increases the demand for domestically produced goods and it deteriorates the terms of trade of domestic consumers. So in some cases the monetary authority may have the incentive to generate an exchange rate appreciation, even at the cost of a level of output (employment) lower than the optimal one. From now on I will assume $\sigma = \eta = 1$ (i.e. log utility in consumption and unit elasticity of substitution between home produced goods and goods produced abroad). In this case the equilibrium conditions derived in 2.3 hold exactly and maximizing (1) under the production function $Y_t = A_t N_t$, (31) and (30) leads to the following FOC:

$$-\frac{U_N}{U_C} = (1-\alpha)A^{1-\alpha}N^{-\alpha}(Y^*)^{\alpha} \tag{38}$$

The solution is a constant, optimal, level of employment $N = (1-\alpha)^{\frac{1}{1+\varphi}}$. Let's now analyse under which conditions the flexible price equilibrium delivers the optimal allocation. Under flexible prices in every period $\widehat{\mu}_t^w = \widehat{mc}_t = 0$ holds. Combining these two conditions and using some of the equilibrium conditions I get:

$$N_t^{1+\varphi}\frac{\mu_w}{1+\tau_w} = \frac{1+\tau_p}{\mu_p} \tag{39}$$

Once having substituted for the optimal level of N, (39) tells how the two subsidies should be set in order to attain the optimal allocation in the flexible prices equilibrium. From now on I will assume that the subsidies are set such that the flexible price equilibrium coincides with the Pareto optimum[4].

All households have the same level of consumption but different levels of labour. For this reason, when computing the welfare function, we need to average the disutility of labour across agents:

$$W_t = U(C_t) + \int_0^1 V(N_t(h))dh \tag{40}$$

From now on all the variables of the type $\widehat{a}_t$ represent log deviations from the steady state.
The second order approximation of the welfare function leads to[5]:

$$\begin{aligned} W_t - \overline{W} = \\ (1-\alpha)\widehat{y}_t + \overline{V}_N\overline{N}E_h[\widehat{n}_t(h) + \frac{1+\varphi}{2}\widehat{n}_t^2(h)] + o(\|a\|^3) \end{aligned} \tag{41}$$

[4]In the simulation I set $\Phi_w = 0$ and consequently, $1 - \Phi p = 1 - \alpha$.
[5]The derivations of the equations in this section are in appendix B.

The approximation of the two expected values leads to:

$$E_h[\widehat{n}_t(h)] = \\ \widehat{y}_t - a_t + \frac{1}{2\theta_p} Var_f[\widehat{y}_t(f)] - \frac{1}{2}\frac{\theta_w - 1}{\theta_w} Var_h[\widehat{n}_t(h)] + o(\|a\|^3) \tag{42}$$

$$E_h[\widehat{n}_t^2(h)] = Var_h[\widehat{n}_t(h)] + [E_h[\widehat{n}_t(h)]]^2 \tag{43}$$

Having chosen optimally τ_p and τ_w, the following holds $-\overline{V}_N \overline{N} = (1-\alpha)$. Then using this relation and substituting (42) and (43) into (41) the second order approximation of the welfare function around the steady state become:

$$W_t - \overline{W} = \\ (1-\alpha)a_t - \frac{(1-\alpha)}{2\theta_p} Var_j[\widehat{y}_t(j)] - \frac{(1-\alpha)(1+\varphi\theta_w)}{2\theta_w} Var_h[\widehat{n}_t(h)] + \\ -\frac{(1-\alpha)(1+\varphi)}{2}(\widehat{y}_t - a_t)^2 + o(\|a\|^3) \tag{44}$$

Computing the approximation around the steady state of the welfare function in absence of nominal rigidities leads to[6]:

$$W_t^n - \overline{W} = \\ (1-\alpha)a_t - \frac{(1-\alpha)(1+\varphi)}{2}(\widehat{y}_t^n - a_t)^2 + o(\|a\|^3) \tag{45}$$

Consequently,

$$W_t - W_t^n = \\ -\frac{(1-\alpha)(1+\varphi)}{2}(\widehat{y}_t^2 - (\widehat{y}_t^n)^2) + (1-\alpha)(1+\varphi)(\widehat{y}_t - \widehat{y}_t^n)a_t + \\ -\frac{(1-\alpha)}{2\theta_p} Var_j[\widehat{y}_t(j)] - \frac{(1-\alpha)(1+\varphi\theta_w)}{2\theta_w} Var_h[\widehat{n}_t(h)] + o(\|a\|^3) \tag{46}$$

Log-linearizing equation (38) I get $a_t = \widehat{y}_t^n$.

Consequently, from (46) I get the following loss function:

$$W \equiv \sum_{t=0}^{\infty} \beta^t (W_t - W_t^n) = \\ -\frac{1-\alpha}{2} \sum_{t=0}^{\infty} \beta^t \left[(1+\varphi)x_t^2 + \frac{1}{\theta_p} Var_j\left[\widehat{y}_t(j)\right] + \frac{1+\varphi\theta_w}{\theta_w} Var_h\left[\widehat{n}_t(h)\right] \right] \tag{47}$$

[6] With flexible prices and wages there are no differences across workers and firms so $Var_f = Var_h = 0$

where $x_t = \widehat{y}_t - \widehat{y}_t^n = y_t - y_t^n$. As proved by Woodford (2001),

$$\sum_{t=0}^{\infty} \frac{\beta^t}{\theta_p} Var_j\left[\widehat{y}_t(j)\right] = \frac{\theta_p}{\lambda} \sum_{t=0}^{\infty} \beta^t \pi_{h,t}^2 \tag{48}$$

It remains to study $Var_h[\widehat{n}_t(h)]$. Let's first write the log linear labour demand faced by each household:

$$\widehat{n}_t(h) = -\theta_w \log(W_t(h)) + \theta_w \log(W_t) + \widehat{n}_t + o(\|a\|^2) \tag{49}$$

consequently:

$$Var_h[\widehat{n}_t(h)] = \theta_w^2 Var_h[w_t(h)] \tag{50}$$

with $w_t(h) = \log(W_t(h))$.
Following the same procedure used in Woodford (2001) for the variance of prices the following holds:

$$\sum_{t=0}^{\infty} \beta^t Var_h[\widehat{w}_t(h)] = \frac{\xi_w}{(1-\beta\xi_w)(1-\xi_w)} \sum_{t=0}^{\infty} \beta^t \pi_{w,t}^2 + \\ + \gamma_w^2 \frac{\xi_w}{(1-\beta\xi_w)(1-\xi_w)} \beta \sum_{t=0}^{\infty} \beta^t \pi_t^2 + t.i.p. + o(\|a\|^3) \tag{51}$$

Consequently,

$$\frac{1+\varphi\theta_w}{\theta_w} \sum_{t=0}^{\infty} \beta^t Var_h[\widehat{n}_t(h)] = \frac{\theta_w}{\lambda w} \sum_{t=0}^{\infty} \beta^t \pi_{w,t}^2 + \gamma_w^2 \beta \frac{\theta_w}{\lambda_w} \sum_{t=0}^{\infty} \beta^t \pi_t^2 \tag{52}$$

With the previous results I can rewrite the loss function:

$$W = -\frac{1-\alpha}{2} \sum_{t=0}^{\infty} \beta^t \left[(1+\varphi) x_t^2 + \frac{\theta_p}{\lambda} \pi_{h,t}^2 + \frac{\theta_w}{\lambda_w} \pi_{w,t}^2 + \beta \gamma_w^2 \frac{\theta_w}{\lambda_w} \pi_t^2 \right] \tag{53}$$

Taking unconditional expectation of (53) and letting $\beta \to 1$ the expected welfare loss is:

$$L = -\frac{1-\alpha}{2} \left[(1+\varphi) Var(x_t) + \frac{\theta_p}{\lambda} Var(\pi_{h,t}) + \frac{\theta_w}{\lambda_w} Var(\pi_{w,t}) + \beta \gamma_w^2 \frac{\theta_w}{\lambda_w} Var(\pi_t) \right] \tag{54}$$

From the comparison between this equation and the one obtained by Galí and Monacelli (2005) it emerges that the loss function is affected by two extra terms: the variance of wage inflation and the variance of CPI inflation. It is interesting to compare the coefficients of the two price inflations. Assuming that $\xi_p = \xi_w$ and that $\theta_p = \theta_w$, CPI inflation has a higher weight than domestic inflation in the loss function whenever $\gamma_w > \sqrt{\frac{1}{\beta(1+\varphi\theta_w)}}$. Clearly, the higher the level of wage indexation, the more important it will be to stabilize CPI inflation. Also, the higher φ and θ_w (and θ_p) the more likely it is that the previous condition will be satisfied. So depending on the calibration of the model and in particular depending on the level of wage indexation, it may be that the loss function is more sensible to variation in CPI inflation than to variation in DI. Clearly this is not enough to say which variable to target. For this reason I first analyse the behaviour of the economy under fully optimal monetary policy with commitment. Afterward, using the results with optimal monetary policy as a benchmark, I simulate the model under different, ad-hoc, policy rules, to make a ranking among them (section 5).

4 Optimal monetary policy with commitment

In this section, following Clarida, Galí and Gertler (1999) and Giannoni and Woodford (2002), I compute the fully optimal monetary policy under commitment.

The first step, in order to make optimal monetary policy easier to compute, is to reduce the original system of equations fully characterizing the model (see appendix C) as much as possible. The system can be reduced to the following equations:

$$\alpha(x_t + \frac{\log(1-\alpha)}{1+\varphi} + a_t - y_t^*) = \alpha(x_{t-1} + \frac{\log(1-\alpha)}{1+\varphi} + a_{t-1} - y_{t-1}^*) + \pi_t - \pi_{h,t} \quad (55)$$

$$\pi_{w,t} = w_t + \pi_t - w_{t-1} \quad (56)$$

$$\pi_{w,t} = \beta E_t \pi_{w,t+1} - \lambda_w \left[w_t - \alpha y_t^* + \varphi a_t - (1+\varphi-\alpha)(x_t + \frac{\log(1-\alpha)}{1+\varphi} + a_t) \right] - \xi_w \gamma_w \beta \pi_t + \gamma_w \pi_{t-1} \quad (57)$$

$$\pi_{h,t} = \beta E_t \pi_{h,t+1} + \lambda(1+\varphi) x_t + \lambda \left[w_t - \alpha y_t^* + \varphi a_t - (1+\varphi-\alpha)(x_t + \frac{\log(1-\alpha)}{1+\varphi} + a_t) \right] \quad (58)$$

$$y_{t+1}^* = \rho_y y_t^* + \varepsilon_{y,t}. \quad (59)$$

$$a_{t+1} = \rho_a a_t + \varepsilon_{A,t}. \quad (60)$$

With the inclusion of a monetary policy rule, equations (55), (56), (57) and (58) define the variables x_t, $\pi_{h,t}$, $\pi_{w,t}$, π_t and w_t, while the last two equations define the low of motion of the two exogenous shocks.

To compute the optimal monetary policy under commitment the central bank has to choose $\{x_t, \pi_{h,t}, \pi_{w,t}, \pi_t, w_t\}_{t=0}^{\infty}$ in order to maximize[7]:

$$W = -\frac{1-\alpha}{2}\sum_{t=0}^{\infty}\beta^t\left[(1+\varphi)x_t^2 + \frac{\theta_p}{\lambda}\pi_{h,t}^2 + \frac{\theta_w}{\lambda_w}\pi_{w,t}^2 + \beta\gamma_w^2\frac{\theta_w}{\lambda_w}\pi_t^2\right] \tag{61}$$

subject to the sequence of constraints defined by equations (55), (56), (57) and (58).

Before presenting the first order conditions it is interesting to note that in a closed economy contest Giannoni and Woodford (2002) compute the optimal monetary policy in a model with sticky wages indexed to past inflation and in their case the model is fully characterized by equations (56), (57) and (58). On the other hand Galí and Monacelli (2005) have a small open economy without wage rigidities and to compute optimal monetary policy is enough to consider (58), therefore is not only the open economy aspect of this paper that makes (55) necessary. Actually is a combination of the open economy assumption plus the presence of wages rigidity. Like in Galí and Monacelli (2005), the behavior of DI is determined by the Phillips Curve while CPI is determined by equation (90) that links CPI with DI and the terms of trade. In Galí and Monacelli (2005) there is no need to consider this equation while studying optimal monetary policy because nor the welfare function neither the NKPC contain the CPI. Here instead, the CPI enters the NKPC on wage inflation both throw the indexation channel and throw the real wage and that's why I need to consider also equation (55) as a constraint (even in the case of no indexation).

The FOCs of this problem are ($\Phi_{i,t}$ is the lagrange multiplier associated to the constraint i):

- x_t :

$$-(1-\alpha)(1+\varphi)x_t - \alpha\Phi_{1,t} + \beta\alpha\Phi_{1,t+1} + \alpha\lambda\Phi_{4,t} + \lambda_w(1+\varphi-\alpha)\Phi_{3,t} = 0 \tag{62}$$

- $\pi_{h,t}$:

$$-(1-\alpha)\frac{\theta_p}{\lambda}\pi_{h,t} - \Phi_{1,t} - \Phi_{4,t} + \Phi_{4,t-1} = 0 \tag{63}$$

- $\pi_{w,t}$:

$$-(1-\alpha)\frac{\theta_w}{\lambda_w}\pi_{w,t} - \Phi_{2,t} - \Phi_{3,t} + \Phi_{3,t-1} = 0 \tag{64}$$

- π_t :

$$-(1-\alpha)\beta\gamma_w^2\frac{\theta_w}{\lambda_w}\pi_t + \Phi_{1,t} + \Phi_{2,t} - \xi_w\gamma_w\beta\Phi_{3,t} + \gamma_w\beta\Phi_{3,t+1} = 0 \tag{65}$$

[7]Giannoni and Woodford (2002) do the optimization including also the IS equation among the constraints and maximizing also with respect to the interest rate. Following Clarida et al. (1999) I have divided the problem in two steps. I first maximize the welfare with respect to $\{x_t, \pi_{h,t}, \pi_{w,t}, \pi_t, w_t\}_{t=0}^{\infty}$ without considering the IS. Then, once obtained the optimal responses of those variables to the exogenous shocks, I can use the IS in order to see how the interest rate has to be set under optimal monetary policy.

- w_t :

$$\Phi_{2,t} - \beta\Phi_{2,t+1} - \Phi_{3,t}\lambda_w + \lambda\Phi_{4,t} = 0 \tag{66}$$

Equations (62)-(66) plus the constraints (55)-(58) fully characterize the behaviour of the economy under optimal monetary policy. Using the Uhlig's toolkit[8] I can solve the system of equations and study the behavior of the variables under optimal monetary policy. In the next section I consider several, implementable, policy rules and I analyse how well they perform using the optimal monetary policy as the benchmark case.

5 Evaluation of different policy rules

The original question was if, once wage rigidities are introduced in a small open economy, it is better to have a inward-looking monetary policy (like in Galí and Monacelli (2005)),or if it is preferable an outward-looking monetary policy. To answer this question I will compare the performance of several rules.

5.1 Implementable policy rules

The welfare loss is function of π, π_H, π_w and the output gap. In the choice of possible targets for monetary policy I disregard the output gap, that cannot be considered a feasible target since it is not clear how to estimate the natural level of output. I therefore concentrate on the other three variables. I consider Taylor type rules targeting two of the three variables, i.e. I consider the following rules:

$$r_t = \rho + \phi_p\pi_t + \phi_{p,H}\pi_{H,t} \tag{67}$$

$$r_t = \rho + \phi_p\pi_t + \phi_w\pi_{w,t} \tag{68}$$

$$r_t = \rho + \phi_{p,H}\pi_{H,t} + \phi_w\pi_{w,t} \tag{69}$$

Instead of imposing *a priori* given coefficients for ϕ_p, $\phi_{p,H}$ and ϕ_w I chose the values minimizing the welfare loss for a given grid of parameters[9]. I did this exercise for different degrees of wage indexation in order to analyse how this feature of the model affects the results. The zero indexation case is the benchmark.

5.2 Calibration of the parameters

Most of the parameters have been calibrated like in Erceg et al. (2000). The average contract duration is one quarter, i.e. $\xi_p = \xi_w = 0.75$. The elasticity of substitution between workers and between goods are $\theta_p = \theta_w = 4$. The discount factor is $\beta = 0.99$. The productivity shock follows an AR(1) process with $\rho_a = 0.95$. The exogenous shock to productivity is an i.i.d with mean

[8] To simulate the model I used the Matlab program developed by Harald Uhlig. See Uhlig (1995).

[9] I used a grid from 1 to 5 with intervals of 0.25 for both the parameters in the Taylor rule.

zero and standard deviation $\sigma_a = 0.0071$. The parameters related to the open economy part are calibrated following Galí and Monacelli (2005): $\alpha = 0.4$ and the world output follows an AR(1) process with $\rho_y = 0.86$. The exogenous shock to world output is i.i.d with zero mean and with standard deviation $\sigma_y = 0.0078$. The correlation between the two exogenous shocks is $corr_{a,y} = 0.3$. Since the loss function has been derived under the assumption $\sigma = \eta = 1$ I keep this assumption in the simulation. Finally $\varphi = 3$, i.e. the labour supply elasticity is set equal to $\frac{1}{3}$. For what concern the level of wage indexation I simulated the model under different parameter values for γ_w in order to be able to evaluate the impact of different degrees of indexation on the results.

5.3 Performance of different monetary policy rules

The purpose of this section is twofold: first, I want to make a ranking among the ad-hoc policy rules defined before in order to find the one performing better; second, I want to quantitatively evaluate how close they are to the optimal monetary policy. To this end, the first step is to simulate the model under different policy rules and to compute the welfare losses associated to each of them. This allows me to make a ranking among the policy rules. Clearly, in general, two rules could deliver exactly the same loss and, nonetheless, be different, i.e. they could generate very different impulse responses to the exogenous shocks. Therefore, to have more conclusive results, I study not only the losses associated to different policy rules, but also the standard deviations of the variable of interest and the correlations between the simulated series obtained under optimal monetary policy and the ones obtained under the different Taylor rules. This last measure is particularly interesting because the more the correlation is close to one, the more I can "safely" say that the rule is close to the optimal one.

In table 1 are reported the welfare losses associated both to the optimal policy rule and to the Taylor rules. With no indexation the rule performing best (among the Taylor type rules) is the one targeting both DI and wage inflation. The loss associated to this rule is very close to the one obtained under optimal monetary policy. Anyway, the Taylor targeting at CPI and wage inflation delivers a loss only slightly bigger while the Taylor targeting at CPI and DI delivers a loss that is roughly 6.6 times bigger than the one observed under optimal monetary policy. Therefore, only looking at the welfare losses, when there is no indexation, the inward-looking monetary policy (i.e. the one targeting at DI and wage inflation) seems to perform slightly better than the outward-looking one (i.e. the one targeting at CPI and wage inflation). But as soon as a positive degree of indexation is introduced this result drastically changes because the rule performing better is the one targeting CPI and wage inflation. In particular, it can be observed that even with high degrees of wage indexation, the loss delivered by this rule is always around 1.2 times the one obtained with the optimal rule, whereas when targeting DI and wage inflation the loss can be even 3.8 times the one obtained under optimal monetary policy. Therefore this first analysis support the original intuition that the introduction of wage indexation makes it desirable for monetary policy to move from an inward-looking monetary policy to a rule targeting the CPI inflation. The second step is to understand how much this rule is close to the optimal one.

In table 2 are reported the standard deviations of output gap, DI, CPI and wage inflation under different rules, for different degrees of wage indexation. The original intuition that, given the link between CPI inflation and wage inflation trough the indexation mechanism, it would have been difficult to stabilize DI inflation without stabilizing CPI, is confirmed by the analysis of the standard deviations. Indeed, when the monetary authority uses the Taylor rule targeting CPI and wage inflation, the standard deviation of DI is lower than when the target variables are DI and wage inflation. Also, the standard deviation of wage inflation is always lower when the target variable is CPI than when is DI. Therefore, from the analysis of table 2, it can be concluded that the Taylor rule targeting at CPI and wage inflation delivers a welfare loss lower than the ones obtained with the other two rules because it reduces the overall variance of the main variables.

The analysis of the variances is useful in understanding where the losses come from. Still, it could be the case that two rules deliver exactly the same variances but generate very different responses to the exogenous shocks. Therefore the last step is the study of the correlations among the series simulated using the fully optimal monetary policy rule and the ones simulated using the Taylor rules. When there is no wage indexation none of the three Taylor rules seems to deliver a behaviour of the economy close to the one observed under the fully optimal monetary policy, indeed the correlations computed for the series of output gap, CPI, DI and wage inflation are between 0.16 and 0.28. While the analysis of the welfare loss and of the standard deviations does not really allow to discriminate between the inward-looking and the outward-looking Taylor rule, looking at the correlations it is clear the the inward-looking Taylor rule is the closest to the optimal rule, even if the correlations are quite small. Things change drastically as soon as a positive degree of wage indexation is introduced. Indeed now using the Taylor rule with CPI and wage inflation, the correlations of CPI, DI and output gap are between 0.95 and 0.98. The wage inflation series seem instead to be uncorrelated under this rule. For higher levels of wage indexation instead, the correlation computed for wage inflation is around 0.80 and there is no doubt that under the outward-looking Taylor rule, the reaction of the variables to the exogenous shocks is very close to what it would be observed under the fully optimal monetary policy rule with commitment.

6 Conclusions

The starting point of this paper was to analyse whether the introduction of wage rigidities in a small open economy model was enough to justify why a central bank should care not only about domestic inflation but also about CPI inflation. As in the closed economy case, once both kinds of nominal rigidities are present, it is no more possible to reach the flexible price allocation because it is not possible to stabilize at the same time price inflation, wage inflation and the output gap. Given this, an interesting question was if it were still true that targeting home inflation is the best that a central bank can do and, if not, how the new results are affected by the presence of wage indexation. To this purpose I derived the loss function from a second order approximation of the welfare function. Compared with the one obtained by Galí and Monacelli (2005), the presence of sticky wages makes the loss function depending also on $Var[\pi_{w,t}]$ while the presence of indexation introduces, in addition to $Var[\pi_{H,t}]$, also the volatility of CPI, $Var[\pi_t]$. Then, I used the loss function to derive the optimal monetary policy with commitment.The next step has been to simulate the model under different, implementable, monetary policy rules, in order to make a ranking among them, using the optimal monetary policy as a benchmark. With zero indexation a Taylor type rule targeting wage inflation and DI is the rule performing best among the one considered. Still, computing the correlations among the simulated series of the main variables under this rule and under the optimal monetary policy, it is clear that the inward-looking Taylor rule can not be considered as a good approximation of the optimal rule since the correlation coefficients are below 0.30. When I allow for a positive degree of indexation the rule performing better is the Taylor targeting at CPI and wage inflation. In this case not only the volatility of the variables are very close to the ones obtained under the optimal rules, but also the correlations coefficients are very high (bigger than 0.95 in many cases). Therefore, going back to the question of the title, these results confirm the opportunity of targeting at CPI instead of DI.

A Derivation of $\widehat{mc}_t$

Making use of some of the equilibrium conditions defined in (2.3), the real marginal cost can be written as:

$$\begin{aligned} mc_t &= w_t - p_{H,t} - a_t \\ &= mrs_t + log(\mu_t^w) + p_t - p_{H,t} - a_t \\ &= \sigma * y_t^* + (1-\alpha)s_t + \varphi(y_t - a_t) + \alpha * s_t - a_t + log(\mu_t^w) \\ &= (\sigma - \sigma_\alpha)y_t^* + (\sigma_\alpha + \varphi)y_t - (1+\varphi)a_t + log(\mu_t^w) \end{aligned} \quad (70)$$

where μ_t^w represents the actual markup charged in each period[10]. From equation (70) we can express the level of output as:

$$y_t = \frac{mc_t}{\sigma_\alpha + \varphi} - \frac{\sigma - \sigma_\alpha}{\sigma_\alpha + \varphi} y_t^* + \frac{1+\varphi}{\sigma_\alpha + \varphi} a_t - \frac{log(\mu_t^w)}{\sigma_\alpha + \varphi} \quad (71)$$

[10]Note that with the presence of taxes that exactly offset the monopoly distortions, the wedge between the real wage and the mrs_t is do only to the presence of stickiness, whereas when $\Phi_w > 0$ then μ_t^w reflects both the presence of stickiness and the presence of monopoly power.

Let's define $\bar{y}_t$ the natural level of output, i.e. the level of output in absence of nominal rigidities:

$$\bar{y}_t = \frac{mc}{\sigma_\alpha + \varphi} - \frac{\sigma - \sigma_\alpha}{\sigma_\alpha + \varphi} y_t^* + \frac{1+\varphi}{\sigma_\alpha + \varphi} a_t + \frac{log(1 - \Phi_w)}{\sigma_\alpha + \varphi} \tag{72}$$

Then,

$$y_t - \bar{y}_t = \frac{\widehat{mc}_t}{\sigma_\alpha + \varphi} - \frac{\widehat{\mu}_t^w}{\sigma_\alpha + \varphi} \tag{73}$$

that is exactly equation (36).

B Derivation of the loss function

B.1 Derivation of $W_t - \overline{W}$

All the results in this section are derived under the assumption $\sigma = \eta = 1$. Under this assumption the relations defined in (2.3) hold exactly and I can derive a second order approximation of the utility function using first order approximation of the structural equations. I will substitute the following expression of the second order derivative: $V_{NN} = \varphi * V_N N^{-1}$. I will also use the fact that:

$$\frac{X_t - \overline{X}}{\overline{X}} = \widehat{x}_t + \frac{1}{2}\widehat{x}_t^2 + o(\|a\|^3) \tag{74}$$

Up to a second order approximation it is true that:

$$\begin{aligned} U(C_t) = \\ +U(\overline{C}) + \overline{U}_C(C_t - \overline{C}) + \frac{1}{2}\overline{U}_{CC}(C_t - \overline{C})^2 + o(\|a\|^3) \end{aligned} \tag{75}$$

Using (74) and the relations between consumption and output defined in (2.3)the previous equation becomes:

$$\begin{aligned} U(C_t) - U(\overline{C}) &= \widehat{c}_t + o(\|a\|^3) \\ &= (1-\alpha)\widehat{y}_t + o(\|a\|^3) \end{aligned} \tag{76}$$

In an analogous way it's true that:

$$\begin{aligned} E_h V(N_t(h)) = \\ V(\overline{N}) + E_h[\overline{V}_N(N_t - \overline{N})] + \frac{1}{2}E_h[\overline{V}_{NN}(N_t - \overline{N})^2] + o(\|a\|^3) \end{aligned} \tag{77}$$

that using (74) and the relation between first order and second order derivatives leads to:

$$E_h[V(N_t(h),Z_t)] = \\ V(\overline{N},0)+\overline{V}_N\overline{N}E_h\left[\widehat{n}_t(h)+\frac{1+\varphi}{2}\widehat{n}_t^2(h)\right]+o(\|a\|^3) \quad (78)$$

Combining (76) and (78) I get equation (41).

B.2 Derivation of $E_h[\widehat{n}_t(h)]$

Since in general for $A=\left[\int_0^1 A(i)^{\phi}di\right]^{\frac{1}{\phi}}$ it's true that[11] $\widehat{a}_t = E_i[\widehat{a}(i)]+\frac{1}{2}\phi * Var_i[\widehat{a}(i)]+o(\|a\|^3)$, then given the way in which aggregate labour has been defined, I can write:

$$\widehat{n}_t = E_h[\widehat{n}_t(h)]+\frac{1}{2}\frac{\theta_w-1}{\theta_w}Var_h[\widehat{n}_t(h)]+o(\|a\|^3) \quad (79)$$

Following Erceg et al. (2000), it is useful to write $\widehat{n}_t$ in function of the aggregate demand of labour by firms $N_t=\int_0^1 N_t(f)df$:

$$\widehat{n}_t = E_f[\widehat{n}_t(f)]+\frac{1}{2}Var_f[\widehat{n}_t(f)]+o(\|a\|^3) \quad (80)$$

Clearly, since $\widehat{y}_t(f)=a_t+\widehat{n}_t(f)$, then $Var_f[\widehat{n}_t(f)]=Var_f[\widehat{y}_t(f)]$ and $E_f[\widehat{n}_t(f)]=E_f[\widehat{y}_t(f)]-a_t$. Also, given the expression for aggregate output, $E_f[\widehat{y}_t(f)]=\widehat{y}_t-\frac{1}{2}\frac{\theta_p-1}{\theta_p}Var_f[\widehat{y}_t(f)]+o(\|a\|^3)$, so I can write:

$$\begin{aligned} E_h[\widehat{n}_t(h)] &= \widehat{n}_t-\frac{1}{2}\frac{\theta_w-1}{\theta_w}Var_h[\widehat{n}_t(h)]+o(\|a\|^3) \\ &= E_f[\widehat{y}_t(f)]-a_t+\frac{1}{2}Var_f[\widehat{y}_t(f)]-\frac{1}{2}\frac{\theta_w-1}{\theta_w}Var_h[\widehat{n}_t(h)]+o(\|a\|^3) \\ &= \widehat{y}_t-a_t+\frac{1}{2\theta_p}Var_f[\widehat{y}_t(f)]-\frac{1}{2}\frac{\theta_w-1}{\theta_w}Var_h[\widehat{n}_t(h)]+o(\|a\|^3) \quad (81) \end{aligned}$$

B.3 Derivation of $Var_h[w_t(h)]$

First it is useful to decompose the variance as[12]:

$$\begin{aligned} Var_h[w_t(h)] &= E_h[w_t(h)-E_hw_t(h)]^2 \\ &= \xi_w E_h[w_{t-1}(h)+\gamma_w\pi_{t-1}-E_hw_t(h)]^2 \\ &\quad +(1-\xi_w)[\widetilde{w}_t-E_hw_t(h)] \quad (82) \end{aligned}$$

[11] The reference for the results in this section is Erceg et al. (2000).

[12] In general, if X assumes value X_1 with probability α and X_2 with probability $(1-\alpha)$, then $E(X^2)=\alpha * X_1^2+(1-\alpha)X_2^2$, but the fraction of workers that can not reoptimise in t will all have a different wage, that's why, like in Erceg et al. (2000), I need to take expectations again.

Using the log-linearized expression for the aggregate wage and the result by Erceg et al. (2000) that $w_t - E_h w_t(h) = o(\|a\|^2)$ then,

$$\begin{aligned} E_h[w_{t-1}(h) + \gamma_w \pi_{t-1} - E_h w_t(h)]^2 &= E_h[w_{t-1}(h) + \gamma_w \pi_{t-1} - \xi_w E_h w_{t-1}(h) - \xi_w \gamma_w \pi_{t-1} + \\ &\quad -(1-\xi_w)\widetilde{w}_t]^2 \\ &= E_h[w_{t-1}(h) + \gamma_w \pi_{t-1} - w_t + o(\|a\|^2)]^2 \\ &= E_h[w_{t-1}(h) - E_h w_{t-1}(h) + \gamma_w \pi_{t-1} - \pi_{w,t} + o(\|a\|^2)]^2 \\ &= Var_h w_{t-1} + \pi_{w,t}^2 + \gamma_w^2 \pi_{t-1}^2 + o(\|a\|^3) \qquad (83) \end{aligned}$$

With the same arguments I have:

$$\begin{aligned} [\widetilde{w}_t - E_h w_t(h)]^2 &= [\widetilde{w}_t - w_t]^2 + o(\|a\|^3) \\ &= \left[\frac{\xi_w}{1-\xi_w}\pi_{w,t} - \frac{\xi_w}{1-\xi_w}\gamma_w \pi_{t-1}\right]^2 + o(\|a\|^3) \qquad (84) \end{aligned}$$

Substituting (83) and (84) into (82) I can write:

$$Var_h[w_t(h)] = \xi_w Var_h w_{t-1}(h) + \frac{\xi_w}{1-\xi_w}\pi_{w,t}^2 + \frac{\xi_w}{1-\xi_w}\gamma_w^2 \pi_{t-1}^2 \qquad (85)$$

Like in Woodford (2001), let's define $\triangle_t^w = Var_h[w_t(h)]$. Consequently I can rewrite (85) as:

$$\triangle_t^w = \xi_w \triangle_{t-1}^w + \frac{\xi_w}{1-\xi_w}\pi_{w,t}^2 + \frac{\xi_w}{1-\xi_w}\gamma_w^2 \pi_{t-1}^2 + o(\|a\|^3) \qquad (86)$$

Iterating backward the previous equation can be written has:

$$\triangle_t^w = \xi_w^{t+1}\triangle_{-1}^w + \sum_{s=0}^{t}\xi_w^s \frac{\xi_w}{1-\xi_w}\pi_{w,t-s}^2 + \gamma_w^2 \sum_{s=0}^{t}\xi_w^s \frac{\xi_w}{1-\xi_w}\pi_{t-1-s}^2 + o(\|a\|^3) \qquad (87)$$

Following Woodford (2001):

$$\sum_{t=0}^{\infty}\beta^t \triangle_t^w = \frac{\xi_w}{(1-\beta\xi_w)(1-\xi_w)}\sum_{t=0}^{\infty}\beta^t \pi_{w,t}^2 + \gamma_w^2 \frac{\xi_w}{(1-\beta\xi_w)(1-\xi_w)}\sum_{t=0}^{\infty}\beta^t \pi_{t-1}^2 + t.i.p. + o(\|a\|^3) \qquad (88)$$

Now it's enough to note that we can rewrite the last sum as:

$$\gamma_w^2 \frac{\xi_w}{(1-\beta\xi_w)(1-\xi_w)}\pi_{-1}^2 + \gamma_w^2 \frac{\xi_w}{(1-\beta\xi_w)(1-\xi_w)}\beta\sum_{t=0}^{\infty}\beta^t \pi_t^2 \qquad (89)$$

and π_{-1}^2 is a $t.i.p.$ like it was $\triangle_{-1}^w$. With this consideration equation (88) became equation (51) in the text.

C System of equations fully characterizing the model

With the inclusion of a monetary policy rule the following system of equations fully characterize the model:

$$\alpha s_t = \alpha s_{t-1} + \pi_t - \pi_{h,t} \tag{90}$$

$$y_t = c_t + \alpha s_t \tag{91}$$

$$y_t^n = \frac{\log(1-\alpha)}{1+\varphi} + a_t \tag{92}$$

$$y_t = a_t + n_t \tag{93}$$

$$\begin{aligned} \pi_{w,t} &= w_t + \pi_t - w_{t-1} \\ w_t &= \log(W_t/P_t) \end{aligned} \tag{94}$$

$$s_t = y_t - y_t^* \tag{95}$$

$$x_t = y_t - \overline{y}_t \tag{96}$$

$$c_t = -\left[r_t - \rho - E_t\pi_{t+1}\right] + E_t c_{t+1} \tag{97}$$

$$\pi_{w,t} = \beta E_t \pi_{w,t+1} - \lambda_w \left[w_t - c_t - \varphi n_t\right] - \xi_w \gamma_w \beta \pi_t + \gamma_w \pi_{t-1} \tag{98}$$

$$\pi_{h,t} = \beta E_t \pi_{h,t+1} + \lambda(1+\varphi)x_t + \lambda\left[w_t - c_t - \varphi n_t\right] \tag{99}$$

$$y_{t+1}^* = \rho_y y_t^* + \varepsilon_{y,t}. \tag{100}$$

$$a_{t+1} = \rho_a a_t + \varepsilon_{A,t}. \tag{101}$$

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Table 1: **Welfare losses under optimal MP and under alternative Taylor's type rules.** *Welfare losses are in percentage units of steady state consumption. For the Taylor's rules are also reported the coefficients of the policy rule minimizing the welfare losses. Moments have been computed as average over 20 simulations, each 100 periods long.*

γ_w	Optimal MP	Taylor $\phi_{p,H} - \phi_w$	Taylor $\phi_p - \phi_w$	Taylor $\phi_p - \phi_{p,H}$
0	0.0079	$\phi_{p,H} = 2.25; \phi_w = 5$ **0.0089**	$\phi_p = 1.25; \phi_w = 4.75$ 0.0095	$\phi_p = 1.25; \phi_{p,H} = 1.25$ 0.0529
0.25	0.0151	$\phi_{p,H} = 1.25; \phi_w = 5$ 0.0254	$\phi_p = 1.25; \phi_w = 5$ **0.0169**	$\phi_p = 4; \phi_{p,H} = 1.25$ 0.0594
0.45	0.0226	$\phi_{p,H} = 1.25; \phi_w = 5$ 0.0614	$\phi_p = 4.25; \phi_w = 3.5$ **0.0278**	$\phi_p = 4.5; \phi_{p,H} = 1.25$ 0.0608
0.65	0.0259	$\phi_{p,H} = 1.25; \phi_w = 3.5$ 0.1006	$\phi_p = 4.25; \phi_w = 3.75$ **0.0302**	$\phi_p = 4.5; \phi_{p,H} = 1.25$ 0.0664

Table 2: **Standard deviations of several variables under the Optimal Monetary Policy Rule and under several Taylor's type rules.** *(%) Standard deviations have been computed as average over 20 simulations, each 100 periods long.*

γ_w	Rule	$\sigma(\pi)$	$\sigma(\pi_h)$	$\sigma(\pi_w)$	$\sigma(x)$
0	Optimal	0.3289	0.2076	0.0290	0.1741
	$\pi_h - \pi_w$	0.3741	**0.1903**	0.0383	**0.3112**
	$\pi - \pi_w$	**0.2351**	0.2242	**0.0225**	0.3580
	$\pi - \pi_h$	0.3611	0.2174	0.1348	1.0501
0.25	Optimal	0.1626	0.2109	0.0292	0.5995
	$\pi_h - \pi_w$	0.2595	0.2067	0.0654	**0.5738**
	$\pi - \pi_w$	**0.1690**	**0.2061**	**0.0393**	0.6416
	$\pi - \pi_h$	0.1204	0.1383	0.1423	1.2363
0.45	Optimal	0.1151	0.2165	0.0384	0.8428
	$\pi_h - \pi_w$	0.2661	0.2002	0.1077	**0.8506**
	$\pi - \pi_w$	**0.0845**	0.2030	**0.0655**	0.9818
	$\pi - \pi_h$	0.0989	**0.1405**	0.1424	1.2092
0.65	Optimal	0.0812	0.1931	0.0470	0.9832
	$\pi_h - \pi_w$	0.2407	0.1904	0.1462	**1.0086**
	$\pi - \pi_w$	**0.0787**	0.1896	**0.0614**	1.0619
	$\pi - \pi_h$	0.0949	**0.1342**	0.1475	1.2028

Table 3: **Correlations among the simulated series obtained under the Fully Optimal Monetary Policy Rule and the ones obtained under several Taylor's type rules.**

γ_w	Rule	$\rho(\pi)$	$\rho(x)$	$\rho(\pi_w)$	$\rho(\pi_h)$
0	$\pi_h - \pi_w$	**0.2559**	**0.1671**	0.1388	**0.2829**
	$\pi - \pi_w$	0.1794	-0.4031	0.1242	0.2231
	$\pi - \pi_h$	-0.0862	-0.0887	**0.2209**	-0.0582
0.25	$\pi_h - \pi_w$	0.9268	0.8414	0.0608	0.9797
	$\pi - \pi_w$	**0.9571**	**0.9573**	-0.0550	**0.9852**
	$\pi - \pi_h$	0.0505	0.9202	**0.6105**	0.9006
0.45	$\pi_h - \pi_w$	0.8675	0.9353	0.1506	0.9699
	$\pi - \pi_w$	**0.8949**	**0.9730**	**0.8067**	**0.9892**
	$\pi - \pi_h$	0.0247	0.9620	0.6705	0.9129
0.65	$\pi_h - \pi_w$	0.7756	0.9424	0.1056	0.9295
	$\pi - \pi_w$	**0.9086**	**0.9807**	**0.8064**	**0.9876**
	$\pi - \pi_h$	0.0176	0.9721	0.6846	0.8860